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teachers, and families in the Arab Republic of Egypt



Student's Book

MATHEMATICS

Third Preparatory - First Term 2026 - 2027

Name : _____

Class : _____

School: _____

Developed and Prepared by
Educational Content Division
GPS Printing, Publishing and Distribution Co.

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2026 / 2027

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Introduction

In the framework of the continuous development of the education system, and as part of the development plan that began with the kindergarten stage and continues until the end of the secondary stage, according to Egypt's Vision 2030 and the Ministry of Education's development plan, this book has been prepared to complete and implement the development processes of educational curriculums for the preparatory stage to create a qualitative leap in the way Egypt's students are prepared to deal with future challenges.

We are pleased to present this mathematics textbook for the third preparatory grade for the first term, which includes a set of tasks and performances that help students solve mathematical problems. This aims to achieve learning outcomes that keep pace with the challenges of the 21st century, whose structure is reinforced by the rapid revolution in information and communication technology.

The curriculum aims to bring about a qualitative leap in the teaching and learning of mathematics. It also aims to provide an appropriate means to achieve the general educational goals in an integrated manner that aligns with the following:

- Emphasizing the positive role of the student in the learning process.
- Included knowledge curriculum, skills, values, positive trends needed for education, good citizenship, productive work and active participation in sustainable development programs.
- Including of modern positive trends in curriculum building, such as critical thinking skills, problem-solving skills, self-learning skills, collaborative learning, and effective communication with knowledge sources.
- Developing performance skills by focusing on self-learning and collaborative work.
- Achieving integration between mathematics and other subjects across different educational stages.
- Providing students with the opportunity to choose activities that suit their abilities, tendencies, and needs.

Asking Almighty God to make this book beneficial and hope that it will be part of a great effort to elevate Egypt to the ranks of advanced countries, ensuring a great future for all students. We wish you a very successful academic year full of achievements and success.

May God grant success,
The Authors

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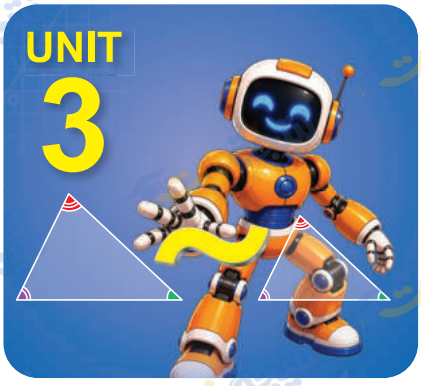
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UNIT 1

Numbers and Operations on Them

Unit Lessons

1-1 Powers of Powers

1-2 Exponential Equations



The concept of exponential equations helps speed up and develop artificial intelligence applications , which makes smart models more able to predict results based on mathematical relationships between variables.

▶ Can the concepts and rules of exponential equations be used in artificial intelligence applications?

Issues and Life Skills

- Mathematical Thinking
- Critical Thinking
- Mathematical Deduction
- Creative Thinking

Values

- Aesthetic Appreciation
- Tourism Awareness
- Teamwork
- Environmental Awareness

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Lesson

1 - 1

Powers of Powers

Learning Outcomes

- Learn the concept of a power of the power.
- Find the power of a power for a real number.
- Use the power of a power rule to simplify mathematical expressions.
- Use the power of a product and a quotient rule to solve real-life problems.

Vocabulary

- Exponent
- Base
- Power
- Power of a power

Previous Knowledge

Laws of Exponents:

For any two real numbers a and b, where $a \neq 0$ and $b \neq 0$, and two integers m and n:

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a \times b)^m = a^m \times b^m$
4. $(\frac{a}{b})^m = \frac{a^m}{b^m}$
5. $a^{-n} = \frac{1}{a^n}$
6. $a^0 = 1$

Get Ready!

The Alexandria Aquarium museum is one of the most famous educational and tourist landmarks in the city. It is located next to the Qaitbay Citadel in Anfoushi and includes a collection of glass tanks to display various fish turtles and sea creatures, in addition to taxidermied exhibits. Assuming that one of the aquarium tanks is cube-shaped with an edge length of 2^7 cm, how much water can this tank hold?



Alexandria Aquarium Museum

In this lesson, you will learn the rule of the power of a power, which will enable you to solve such real-life problems.

Think & Discuss!

The given cards have mathematical expressions. Choose all the cards that have mathematical expression equal to the one on the red card. Discuss.

$(x^4 y^3)^2$

$x^{42} y^{32}$ | $x^8 y^6$ | $x^6 y^5$

$x^3 y^4 \times x^5 y^2$ | $(x^2)^4 (y^2)^3$

Learn!

The Power of a Power Rule

$$(2^3)^4 = \underbrace{2^3 \times 2^3 \times 2^3 \times 2^3}_{4 \text{ Factors}} = 2^{3+3+3+3} = 2^{12}$$

Note that : 12 is the product of the original exponents 3 and 4 ($12 = 3 \times 4$)
That means : To find the power of a power , we multiply the exponents.

In general:

For any non-zero real number a and any two integers m and n :

$$(a^m)^n = a^{m \times n}$$

For example : • $(5^4)^2 = 5^{4 \times 2} = 5^8$ • $(7^{-2})^5 = 7^{-2 \times 5} = 7^{-10}$

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Example 1

Simplify each of the following to its simplest form :

1) $\dfrac{(2^{11})^2 \times 2^3}{(2^6)^3 \times 2^4}$ 2) $\dfrac{(x^2)^{-3} \times (x^{-1})^2}{x^{-3} \times x^{-4}}$ 3) $\dfrac{4^{x+2} \times 9^{3+x}}{6^{2x+3}}$

1) $\dfrac{(2^{11})^2 \times 2^3}{(2^6)^3 \times 2^4} = \dfrac{2^{11\times2} \times 2^3}{2^{6\times3} \times 2^4} = \dfrac{2^{22} \times 2^3}{2^{18} \times 2^4}$

$= \dfrac{2^{22+3}}{2^{18+4}} = \dfrac{2^{25}}{2^{22}} = 2^{25-22} = 2^3 = 8$

2) $\dfrac{(x^2)^{-3} \times (x^{-1})^2}{x^{-3} \times x^{-4}} = \dfrac{x^{2\times(-3)} \times x^{-1\times2}}{x^{-3} \times x^{-4}} = \dfrac{x^{-6} \times x^{-2}}{x^{-3} \times x^{-4}}$

$= \dfrac{x^{-6+(-2)}}{x^{-3+(-4)}} = \dfrac{x^{-8}}{x^{-7}} = x^{-8-(-7)} = x^{-1} = \dfrac{1}{x}$

3) $\dfrac{4^{x+2} \times 9^{3+x}}{6^{2x+3}} = \dfrac{(2^2)^{x+2} \times (3^2)^{3+x}}{(2\times3)^{2x+3}} = \dfrac{2^{2x+4} \times 3^{6+2x}}{2^{2x+3} \times 3^{2x+3}}$

$= 2^{(2x+4)-(2x+3)} \times 3^{(6+2x)-(2x+3)}$

$= 2^{2x+4-2x-3} \times 3^{6+2x-2x-3}$

$= 2^1 \times 3^3 = 2 \times 27 = 54$

The power of a product rule

For any two non-zero real numbers a and b and three integers m , n and k :

$$(a^m \times b^n)^k = a^{mk} \times b^{nk}$$

For example :

$(2^3 \times 3)^4 = 2^{3\times4} \times 3^{1\times4}$

$= 2^{12} \times 3^4$

The power of a quotient rule

For any two non-zero real numbers a and b and three integers m , n and k :

$$\left(\frac{a^m}{b^n}\right)^k = \frac{a^{mk}}{b^{nk}}$$

For example :

$\left(\dfrac{2^3}{7^2}\right)^5 = \dfrac{2^{3\times5}}{7^{2\times5}} = \dfrac{2^{15}}{7^{10}}$

Example 2

Simplify each of the following to its simplest form :

1) $(-2x^2y)^3 \times (x^{-1}y^2)^{-1}$ 2) $\left(\dfrac{2a^2b^2}{b^{-1}a^4}\right)^3$

Self-Evaluation (1)

Simplify each of the following to its simplest form :

1) $\dfrac{((\sqrt{5})^3)^2 \times \sqrt{5}}{(-\sqrt{5})^5}$

2) $\dfrac{a^4 \times (a^{-1})^4}{(a^2)^5 \times (a^5)^{-2}}$

3) $\dfrac{36^n \times 5^{2n}}{30^{2n}}$

Note that

Division by zero has no meaning, and therefore, when there are symbols in the denominator, these symbols must not be equal to zero.

Think & Discuss

1) Is $(x^3)^2 = (x^2)^3$?
2) Is $(9^2)^2 = 3^8$?
3) Is $(2^2)^3 = 2^{2^3}$?

Note that

$(a^m + b^n)^k \neq a^{mk} + b^{nk}$

$(a^m - b^n)^k \neq a^{mk} - b^{nk}$

Self-Evaluation (2)

Simplify each of the following to its simplest form :

1) $(3xy^4)^{-2} \times ((-2y)^2)^3$

2) $\left(\dfrac{4m^2n^{-3}}{n^{-4}m^3}\right)^{-2}$

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■ **Note that**

- $(-a)^n = a^n$ if n is an even integer.
- $(-a)^n = -a^n$ if n is an odd integer.

■ **Self-Evaluation (3)**

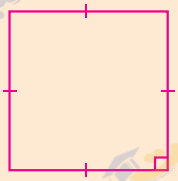
Simplify to the simplest form :

$$\frac{\left((\sqrt{5})^3\right)^{2x} \times (15)^{x-1}}{(75)^x}$$

, then find the value of the result when:

$x = 1$

■ **Self-Evaluation (4)**



$3a^2x^3\ cm$

Find the area of the square in its simplest form.

(1) $(-2x^2y)^3 \times (x^{-1}y^2)^{-1} = (-2)^3 x^{2\times3} y^3 \times x^{-1\times(-1)} y^{2\times(-1)}$

$= -8x^6y^3 \times xy^{-2} = -8x^{6+1}y^{3-2}$

$= -8x^7y$

(2) $\left(\frac{2a^2b^2}{b^{-1}a^4}\right)^3 = \frac{2^3 a^{2\times3} b^{2\times3}}{b^{-1\times3} a^{4\times3}} = \frac{8a^6b^6}{b^{-3}a^{12}}$

$= 8a^{6-12}b^{6+3} = 8a^{-6}b^9 = \frac{8b^9}{a^6}$

Example 3

Simplify to the simplest form : $\frac{9^{2n-1} \times 18^{1-4n}}{36^{2-3n}}$

, then find the numerical value of the result when $n = 2$

$\frac{9^{2n-1} \times 18^{1-4n}}{36^{2-3n}} = \frac{(3^2)^{2n-1} \times (2\times3^2)^{1-4n}}{(2^2\times3^2)^{2-3n}}$

$= \frac{3^{4n-2} \times 2^{1-4n} \times 3^{2-8n}}{2^{4-6n} \times 3^{4-6n}}$

$= 2^{(1-4n)-(4-6n)} \times 3^{(4n-2)+(2-8n)-(4-6n)}$

$= 2^{1-4n-4+6n} \times 3^{4n-2+2-8n-4+6n} = 2^{2n-3} \times 3^{2n-4}$

The value of the result when $n = 2$:

$2^{2n-3} \times 3^{2n-4} = 2^{2(2)-3} \times 3^{2(2)-4} = 2^1 \times 3^0 = 2 \times 1 = 2$

Example 4

Decoration : A room has a square floor of side length $2x^3y^2$ length units. The floor will be tiled using square tiles. The length of one side of each tile is xy^2 length units. How many tiles are needed to cover the whole floor?



∵ Area of a square = side length squared

∴ Area of the room floor $= (2x^3y^2)^2$

$= 2^2x^6y^4$

$= 4x^6y^4$ square units.

∵ Area of one tile $= (xy^2)^2$

$= x^2y^4$ square units.

∴ Number of tiles $= \frac{\text{Area of the room floor}}{\text{Area of one tile}}$

$= \frac{4x^6y^4}{x^2y^4} = 4x^{6-2} = 4x^4$ tiles

■ **Diversified Strategies**

Number of tiles $= \frac{\text{Area of the room floor}}{\text{Area of one tile}}$

$= \frac{(2x^3y^2)^2}{(xy^2)^2}$

$= \left(\frac{2x^3y^2}{xy^2}\right)^2$

$= (2x^2)^2$

$= 4x^4$

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Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

1. Which of the following is equal to $(a^4)^7$?
   (a) a^3 (b) a^{11} (c) a^{28} (d) a^{47}

2. What is the value of $(3^8)^0$?
   (a) 3^8 (b) 3^{80} (c) 1 (d) 24

3. Which of the following is equal to $(3^4)^2 \times 3^5$?
   (a) 3^{11} (b) 3^{13} (c) 3^{40} (d) 9^{11}

4. What is the result of $\frac{(x^3)^5}{(x^8)^2}$ in its simplest form?
   (a) $\frac{1}{x}$ (b) x (c) x^{-2} (d) x^{31}

5. What is the result of $x^2 \times (x^5)^2$ in its simplest form?
   (a) x^9 (b) x^{12} (c) x^{14} (d) x^{20}

6. What is the result of $(-a^3 b)^2$ in its simplest form?
   (a) $a^3 b^3$ (b) $-a^5 b^3$ (c) $a^6 b^2$ (d) $-a^6 b^2$

7. If $2^x = 5$, what is the value of 8^x ?
   (a) 125 (b) 32 (c) 27 (d) 15

8. What is the result of $\left(\frac{3a^2}{2b}\right)^2$ in its simplest form?
   (a) $\frac{9a^4}{4b^3}$ (b) $\frac{9a^4}{4b^2}$ (c) $\frac{3a^4}{2b^2}$ (d) $\frac{3a^4}{2b^3}$

▸ **Spot the mistake:**

9. Khaled and Asmaa both simplified $(a^2)^3 \times a^4$ to its simplest form.
   Their solutions are shown.
   Which one is correct ? Discuss.

Khaled's solution :
$(a^2)^3 \times a^4 = a^5 \times a^4 = a^{20}$

Asmaa's solution :
$(a^2)^3 \times a^4 = a^6 \times a^4 = a^{10}$

Second Applying Concepts

10. **Find each of the following in its simplest form :**

1. $(a^4)^3 \times a^7$
2. $(a^5)^2 \times (a^3)^3$
3. $(\sqrt{5})^3 \times ((-\sqrt{5})^2)^3$
4. $((-2)^3)^2 \times (-\sqrt{2})^{-4}$
5. $\frac{(a^2)^3}{a^4}$
6. $\frac{(a^4)^3}{(a^2)^5}$
7. $(((\sqrt{3})^3)^{-2})^{-1}$
8. $\frac{((-\sqrt{7})^2)^3 \times ((\sqrt{7})^3)^{-1}}{((\sqrt{7})^2)^{-2}}$

11. **Simplify each of the following to its simplest form :**

1. $(4a^4 b^3 c)^2$
2. $(-2m^5 n^4)^3$
3. $\left(\frac{2r^2 s^{-1}}{r^3 s}\right)^{-2}$
4. $(c^3)^2 \times (-3c^7)^2$
5. $(-3x^2 y^3 z)^2 \times ((-x^2 y)^3)^2$
6. $\frac{5qp^{-2}}{(q^{-1}p)^3}$
7. $\frac{(2a^5)^3 \times (3a^3)^2}{(6a^7)^2 \times (a^{-1})^3}$
8. $\frac{(16x^2 y^{-2})^0}{(4x^0 y^{-6})^{-2}}$
9. $\left(\frac{5^0 c^2 d^3 f}{2c^4 d^{-4}}\right)^{-3}$

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(12) Simplify each of the following to its simplest form , then find the value of the result when $x = 1$:

① $\frac{4^x \times 2^{x-4}}{16^{x-2}}$

② $\frac{(4)^x \times (49)^{x-1}}{(98)^x}$

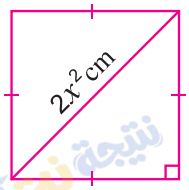
(13) Prove that : $\frac{27^{x-1} \times 8^x}{(2\sqrt{2})^{2x} \times (3\sqrt{3})^{2x}} = \frac{1}{27}$

(14) If $3^a = 2$, find the value of : $9^a + (27)^a$

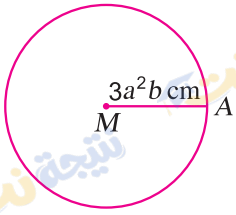
Third Analysis and Subjects Integration

(15) **Geometry:** Find the area of each figure in its simplest form :

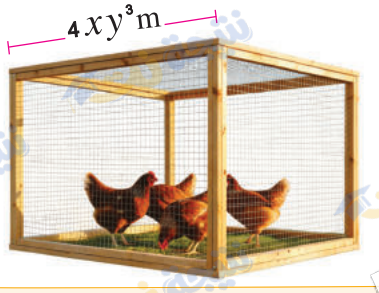
①



②



(16) **Animal Wealth:** A chicken coop is in the shape of a cube. The length of its edge is $4xy^3$ meters , find the volume of the coop in its simplest form.



Poultry is one of the most important sectors of animal wealth in Egypt. It is an important source of animal protein.



Reducing fuel consumption helps reduce air pollution and protect the environment.

(17) **Energy Sources :** A cylindrical barrel is used to store fuel. The radius of its base is $2kw^2$ meters and its height is $4k$ meters. Find the volume of the barrel in its simplest form.

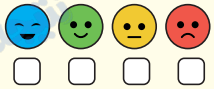
Creative Thinking

(18) If $10^x = 5^{x+1}$, find the value of 4^x

(19) If $a = 5^{30}$, $b = 3^{45}$, $c = 2^{60}$, arrange a , b and c in an ascending order.

***Evaluate** your understanding!*

How well do you understand powers of powers ?
Tick the right box



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Lesson

1 - 2

Exponential Equations

Learning Outcomes

- Solve exponential equations using the laws of exponents.
- Use exponential equations to solve real-life problems.

Vocabulary

- Exponential equation

Note that

You can use factorization to write numbers in exponential form:

125	5
25	5
5	5
1	

$125 = 5^3$

Get Ready!

Bacteria are single - celled living organisms. They cannot be seen with the naked eye, but they can be seen only under a microscope. Bacteria reproduce in a way called binary fission. In this process, one cell divides into two cells every one hour.
If we start with one bacterial cell, after how many hours will the number of cells become 1024 cells?

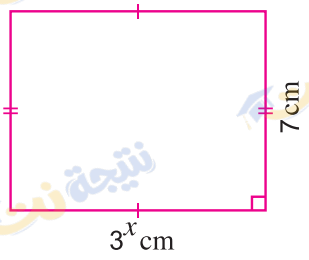


Bacteria under a microscope

In this lesson, you will learn the concept of an exponential equation and how to solve it using the laws of exponents, which will enable you to solve such real-life problems.

Think & Discuss!

If the area of the given rectangle is $63\ cm^2$ what is the value of x ? Discuss.



Learn!

Exponential Equation

An exponential equation is an equation that contains a variable in the exponent.

Examples of exponential equations:

- $2^x = 8$
- $3^{x-4} = \frac{1}{9}$

Examples of equations that are not exponential:

- $x^2 = 9$
- $x - 7 = 3^2$

This year, we will study exponential equations in the form $a^x = b$ where $a > 0 , a \neq 1$

Solving Exponential Equations

To solve exponential equations, we use the property of equality of powers.

If $a^x = a^y$ where $a > 0 , a \neq 1$, then $x = y$

Examples : • If $3^x = 3^4$, then $x = 4$

• If $5^x = 125$, then $5^x = 5^3$ therefore $x = 3$

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Example 1

Solve each of the following equations in R :

1) $3^{x+1} = 81$ 2) $2^{x-7} = \frac{1}{32}$ 3) $(\frac{2}{3})^{x+2} = \frac{27}{8}$

1) $\because 3^{x+1} = 81$

Rewrite 81 in exponential form so that the base is 3

$\therefore 3^{x+1} = 3^4$ $(81 = 3^4)$

When the bases are equal, the exponents are equal

$\therefore x + 1 = 4$ $\therefore x = 4 - 1 = 3$

2) $\because 2^{x-7} = \frac{1}{32}$

Rewrite 32 in exponential form so that the base is 2

$\therefore 2^{x-7} = \frac{1}{2^5}$ $(32 = 2^5)$

$\therefore 2^{x-7} = 2^{-5}$

When the bases are equal, the exponents are equal

$\therefore x - 7 = -5$ $\therefore x = 7 - 5 = 2$

3) $\because (\frac{2}{3})^{x+2} = \frac{27}{8}$ $\therefore (\frac{2}{3})^{x+2} = \frac{3^3}{2^3}$

$\therefore (\frac{2}{3})^{x+2} = (\frac{3}{2})^3$

$\therefore (\frac{2}{3})^{x+2} = (\frac{2}{3})^{-3}$

$\therefore x + 2 = -3$ $\therefore x = -2 - 3 = -5$

Note that

If a and b are two non-zero real numbers and n is an integer,

then : $(\frac{a}{b})^n = (\frac{b}{a})^{-n}$

For example : $(\frac{3}{2})^3 = (\frac{2}{3})^{-3}$

Example 2

Solve each of the following equations in R :

1) $9^{x+2} = 3^x$ 2) $(125)^{x+1} = (625)^{x-1}$

3) $(\frac{3}{2})^{5-3x} = (\frac{4}{9})^{2x-7}$ 4) $2^{x^2} = 16^{3-x}$

1) $\because 9^{x+2} = 3^x$

Rewrite 9 in exponential form so that the base is 3 $(9 = 3^2)$

$\therefore (3^2)^{x+2} = 3^x$

Use the power of a power rule

$\therefore 3^{2x+4} = 3^x$

When the bases are equal, the exponents are equal

$\therefore 2x + 4 = x$

$\therefore 2x - x = -4$

$\therefore$ $x = -4$

2) $\because (125)^{x+1} = (625)^{x-1}$

Rewrite 125 and 625 in exponential form so that they have the same base, which is 5

$\therefore (5^3)^{x+1} = (5^4)^{x-1}$

Use the power of a power rule

$\therefore 5^{3x+3} = 5^{4x-4}$

When the bases are equal, the exponents are equal

$\therefore 3x + 3 = 4x - 4$

$\therefore 4x - 3x = 3 + 4$

$\therefore$ $x = 7$

Self-Evaluation (1)

Solve each of the following equations in R :

1) $2^{x-2} = 8$

2) $(\frac{1}{2})^{2x-1} = 32$

3) $\frac{125}{27} = (\frac{3}{5})^{x+2}$

Checking the validity of the solution

You can check the validity of the solution for equation 1 as follows :

Substitute the value of x with 3 in the original equation.

$3^{x+1} = 81$

$3^{3+1} \stackrel{?}{=} 81$

$3^4 \stackrel{?}{=} 81$

$81 = 81$ ✓

The statement is correct

So, the solution is 3

▸ Try to verify the validity of the solution for equations 2 , 3 yourself.

Self-Evaluation (2)

Solve each of the following equations in R :

1) $27^{x-1} = 3^{2x}$

2) $64^{x+1} = 4^{x+7}$

3) $\frac{2^{x^2}}{16^x} = 32$

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③ $\because (\frac{3}{2})^{5-3x} = (\frac{4}{9})^{2x-7}$

$\therefore (\frac{3}{2})^{5-3x} = (\frac{2^2}{3^2})^{2x-7}$

$\therefore (\frac{3}{2})^{5-3x} = ((\frac{2}{3})^2)^{2x-7}$

$\therefore (\frac{3}{2})^{5-3x} = ((\frac{3}{2})^{-2})^{2x-7}$

$\therefore (\frac{3}{2})^{5-3x} = (\frac{3}{2})^{-4x+14}$

$\therefore 5-3x = -4x+14$

$\therefore 4x-3x = 14-5$

$\therefore \boxed{x=9}$

④ $\because 2^{x^2} = 16^{3-x}$

$\therefore 2^{x^2} = (2^4)^{3-x}$

$\therefore 2^{x^2} = 2^{12-4x}$

$\therefore x^2 = 12-4x$

$\therefore x^2+4x-12=0$

$\therefore (x+6)(x-2)=0$

$\therefore x+6=0$ *or* $x-2=0$

$\boxed{x=-6}$ | $\boxed{x=2}$

Note:

- If $\boxed{a^x=1}$ where $a>0$, $a\neq 1$, then $\boxed{x=0}$
- If $\boxed{a^x=b^x}$ where $a>0$, $b>0$, $a\neq b$, then $\boxed{x=0}$

Example 3

Solve each of the following equations in R :

1 $4^{x^2-9}=1$

2 $2^{x+1}=3^{x+1}$

① $\because 4^{x^2-9}=1$

$\therefore x^2-9=0$ $\therefore x^2=9$

$\therefore x=\pm\sqrt{9}$ $\therefore \boxed{x=\pm 3}$

② $\because 2^{x+1}=3^{x+1}$

$\therefore x+1=0$

$\therefore \boxed{x=-1}$

Example 4

Find the value of n in each of the following :

1 $\frac{2^n\times 9^{n+1}}{18^n}=3^n$

2 $\frac{12^{n-1}}{2^{n-1}\times 3^{n-1}}=1$

① $\because \frac{2^n\times 9^{n+1}}{18^n}=3^n$

$\therefore \frac{2^n\times (3^2)^{n+1}}{(2\times 3^2)^n}=3^n$

$\therefore \frac{2^n\times 3^{2n+2}}{2^n\times 3^{2n}}=3^n$

$\therefore 3^{2n+2-2n}=3^n$

$\therefore 3^2=3^n$ $\therefore \boxed{n=2}$

② $\because \frac{12^{n-1}}{2^{n-1}\times 3^{n-1}}=1$

$\therefore \frac{(3\times 2^2)^{n-1}}{2^{n-1}\times 3^{n-1}}=1$

$\therefore \frac{3^{n-1}\times 2^{2n-2}}{2^{n-1}\times 3^{n-1}}=1$

$\therefore 2^{2n-2-n+1}=1$ $\therefore 2^{n-1}=1$

$\therefore n-1=0$ $\therefore \boxed{n=1}$

■ Self-Evaluation ③

Solve each of the following equations in R :

1 $2^{x^2}\times 2^x=1$

2 $(\frac{1}{2})^{2a-1}=5^{1-2a}$

■ Self-Evaluation ④

Find the value of x in each of the following :

1 $\frac{4^x\times 9^{2-x}}{6^{2x}}=1$

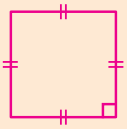
2 $\frac{2^x\times 3^x}{12^x}=\frac{1}{2}$

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■ Self-Evaluation (5)



8^{x+1} cm



$6\times2^{x+2}$ cm

If the perimeter of the equilateral triangle is equal to the perimeter of the square, what is the side length of each shape?

Example 5

In the given figure :
A cardboard box is in the shape of a cuboid. Its volume is $1m^3$.
Find the dimensions of the box.



$\because$ The volume of the box $= \ell \times w \times h$

where : $\ell = 2^{x+2}$, $w = 2^{x+1}$, $h = 2^x$

$\therefore 2^{x+2} \times 2^{x+1} \times 2^x = 1$ $\quad \therefore 2^{x+2+x+1+x} = 1$ $\quad \therefore 2^{3x+3} = 1$

$\therefore 3x + 3 = 0$ $\quad \therefore 3x = -3$ $\quad \therefore x = \frac{-3}{3} = -1$

$\therefore$ $\boxed{\ell = 2^{-1+2} = 2}$, $\boxed{w = 2^{-1+1} = 2^0 = 1}$, $\boxed{h = 2^{-1} = \frac{1}{2}}$

$\therefore$ The dimensions of the box are: $2\ m$, $1\ m$, $\frac{1}{2}\ m$.

Lesson Assessment



First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

(1) If $2^{x-3} = 1$, what is the value of x?
(a) 0 (b) 1 (c) 2 (d) 3

(2) If $2^x = 8$, what is the value of x?
(a) 1 (b) 2 (c) 3 (d) 4

(3) If $3^{-x} = 3$, what is the value of x?
(a) -3 (b) -1 (c) 1 (d) 3

(4) If $3^{x+2} = 3^{2x-1}$, what is the value of x?
(a) 2 (b) 3 (c) 4 (d) 5

(5) If $5^{x-1} = 3^{x-1}$, what is the value of x?
(a) 0 (b) 1 (c) 3 (d) 5

(6) If $9^x = 27$, what is the value of x?
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{1}{3}$ (d) 3

(7) If $5^{2x-1} = \left(\frac{1}{5}\right)^{-7}$, what is the value of x?
(a) 2 (b) 3 (c) 4 (d) 5

(8) If $3^x + 3^x = 6$, what is the value of x?
(a) 1 (b) 2 (c) 3 (d) 9

Second Applying Concepts

(9) **Solve each of the following equations in R :**

① $5^{2x-1} = 125$ ② $3^{2x-3} = \frac{1}{27}$ ③ $4^{1-2x} = 2$

④ $\left(\frac{1}{8}\right)^{x-2} = 128$ ⑤ $\left(\frac{2}{5}\right)^{2x+1} = \frac{8}{125}$ ⑥ $\left(\frac{3}{5}\right)^{2x+1} = \frac{125}{27}$

⑦ $7^{2x+3} = 7^{x-4}$ ⑧ $7^{x+2} = 2^x \times 2^{-x}$ ⑨ $9\times5^{x-1} = 25\times3^{x-1}$

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10 **Find the solution set in R for each of the following equations :**

① $(216)^{x} = \frac{1}{6^{3}}$ ② $(\frac{1}{4})^{2x-1} = 8^{x+3}$ ③ $32^{x-3} = 8^{2x+1}$

④ $\frac{36^{x+1}}{6^{x-1}} = \frac{1}{216}$ ⑤ $8^{x-1} \times 4^{2x+1} = 64$ ⑥ $\frac{8^{x} \times 9^{x}}{18^{x}} = 64$

11 If $\frac{49^{n} \times 25^{2n} \times 3^{4n}}{7^{-n} \times 15^{4n}} = 343$, find the value of 6^{2n}

▸ **Discuss**

12 Marwan and Mariam both solved the equation :

$16^{x+1} = 64^{x-1}$

Which one followed the correct method of solution?

Marwan's solution :

$16^{x+1} = 64^{x-1}$

$(2^{4})^{x+1} = (2^{6})^{x-1}$

$2^{4x+4} = 2^{6x-6}$

$4x + 4 = 6x - 6$

$4 + 6 = 6x - 4x$

$10 = 2x \quad x = 5$

Mariam's solution :

$16^{x+1} = 64^{x-1}$

$(4^{2})^{x+1} = (4^{3})^{x-1}$

$4^{2x+2} = 4^{3x-3}$

$2x + 2 = 3x - 3$

$3x - 2x = 2 + 3$

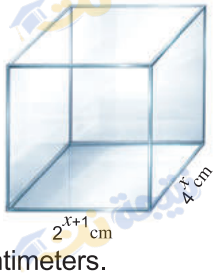
$x = 5$

Third Analysis and Subjects Integration

13 **Volumes :**

A glass box is in the shape of a cube. The lengths of the sides of its base are 2^{x+1} cm , 4^{x} cm as shown in the figure

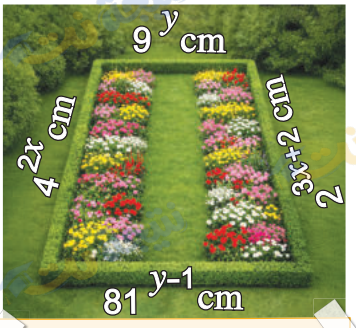
Find its volume in cubic centimeters.



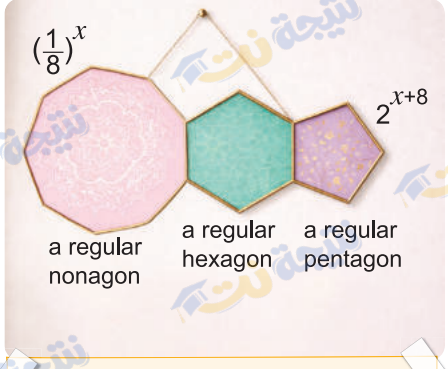
14 **Gardens :**

A flower bed is in the shape of a rectangle. Its dimensions are shown in the figure.

Calculate the perimeter of the flower bed.



Flowers give gardens and homes an elegant appearance that reflects taste and beauty



Taking care of decoration makes a place more comfortable, organized, and elegant

15 **Decoration :**

In the given figure :

A decorative shape is hanged on a wall. It consists of three regular polygons, a pentagon, a hexagon and a nonagon.

What is the value of x ?

Creative Thinking

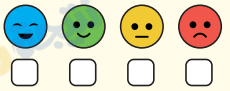
16 Find the value of x in each of the following :

① $2^{x} + 2^{x+1} + 2^{x+2} = 112$

② $2 \times 3^{x+1} - 3^{x} = 5^{x+1} - 2 \times 5^{x}$

***Evaluate** your understanding!*

How well do you understand exponential equations? Tick the right box



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Unit One Assessment

Evaluate your understanding!
How well do you understand Unit One? Tick the right box

☐ ☐ ☐ ☐

▸ Choose the correct answer from the given ones :

(1) Which of the following is equal to $(a^2)^4 \times a^5$?

(a) a^{11} (b) a^{13} (c) a^{40} (d) a^{30}

(2) If $\frac{3^x \times 27^x}{81^x} = 2^{-2x}$, what is the value of x ?

(a) 0 (b) 1 (c) 2 (d) -1

(3) If $3^x = b$, which of the following is equal to 9^{x+1} ?

(a) $27\,b^2$ (b) $3\,b^2$ (c) $9\,b^2$ (d) $3\,b$

(4) If $4^x = 8$, what is the value of x ?

(a) 1 (b) 2 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

▸ Complete each of the following :

(5) If $7^{x+1} = 5^{x+1}$, then $x =$ .....

(6) The result of $\frac{(-2a^2)^3}{2a}$ in its simplest form is .....

(7) If $5^{x-3} = 1$, then $2^x =$ .....

(8) If $2^a + 2^a + 2^a = 24$, then $a =$ .....

▸ Answer each of the following questions :

(9) If $2^x = 5$, find the value of $8^x - 3 \times 2^{x+2}$

(10) Simplify : $\frac{4^x \times 9^{2-x}}{6^{2x}}$ to its simplest form, then find the value of the result when $x = 1$

(11) Find in R the solution set of the equation : $5^{x^2-4} = 125^x$

(12) Find the value of m if : $3^{-2m+1} \times 9^{2m} = \frac{1}{27}$



Unit One Activity — Exponential Equations

▸ Activity Aim :

Understand how to solve exponential equation and the methods of solution.

▸ Implementation Steps :

Work with one of your classmates and try to do the following :

1. Use any source, such as (book , internet , artificial intelligence application,...) to write a real-life problem that depends on exponential growth. (Such as : virus spread , doubling social media followers , bacteria reproduction .....)
2. Rewrite the problem in your own words, then write the equation that represents it in the form $a^x = b$
3. Use what you have learned to solve the equation you wrote, then verify your solution.
4. Explain the steps to solve the problem, highlighting how exponents laws were used.

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UNIT 2

Algebra

Unit Lessons

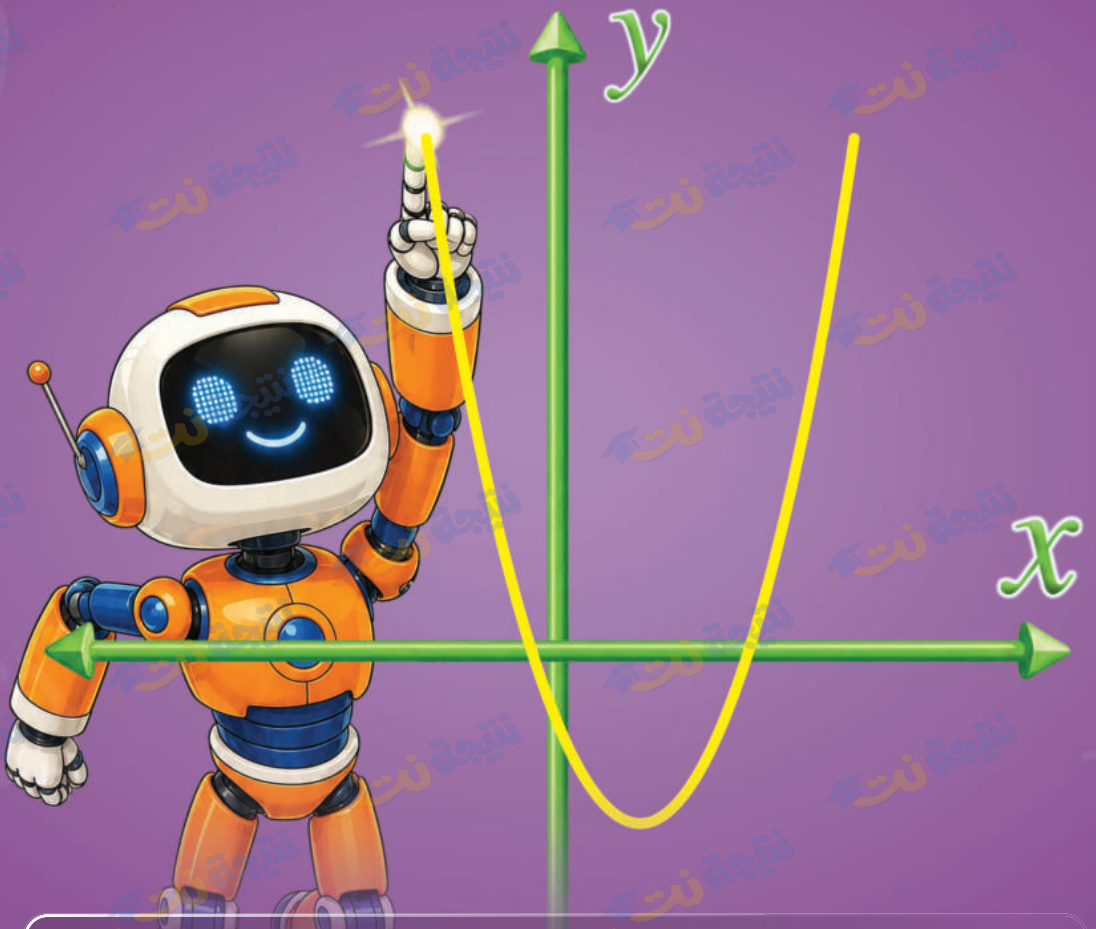
2-1 Quadratic Function

2-2 Set of Real Zeros of Polynomial Functions

2-3 Rational Algebraic Functions

2-4 Common Domain and Equivalent Rational Algebraic Functions

2-5 Operations on Rational Algebraic Fractions



The concept of rational algebraic functions contributes to artificial intelligence by representing nonlinear relationships between data. This helps machine learning algorithms improve the accuracy of decision-making and analyze mathematical patterns more efficiently.

▶ Can the concepts and laws of rational algebraic functions be used in artificial intelligence applications?

Issues and Life Skills

- Mathematical Thinking
- Critical Thinking
- Mathematical Deduction
- Creative Thinking

Values

- Health Awareness
- Teamwork
- Tourism Awareness

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Lesson

2 - 1 Quadratic Function

Learning Outcomes

- Learn the standard form of the quadratic function.
- Learn the vertex form of the quadratic function.
- Determine the vertex of the curve and the axis of symmetry of a quadratic function.
- Represent a quadratic function graphically.
- Find the maximum or minimum value of a quadratic function.
- Use the graphical representation of a quadratic function to solve some real-life problems.

Vocabulary

- Quadratic function
- Parabola
- Maximum value
- Minimum value
- y-intercept
- Vertex form

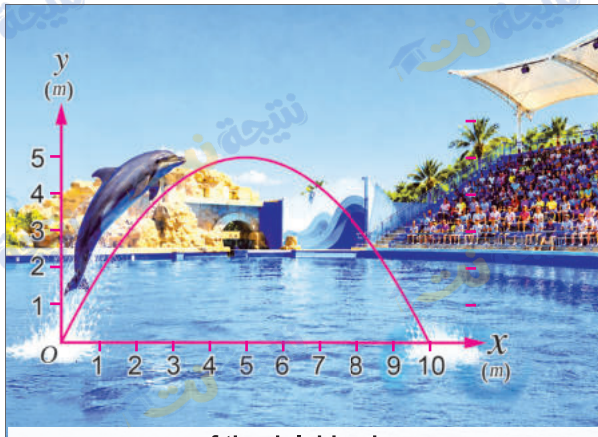
Note that

The domain of the quadratic function is the set of real numbers R.

Note that

The y-intercept is the y-coordinate of the intersection point of the curve with the y-axis.
And to find the y-intercept of the function f, where $f(x) = a x^2 + b x + c$ we substitute $x = 0$
Therefore $f(0) = c$

Get Ready!



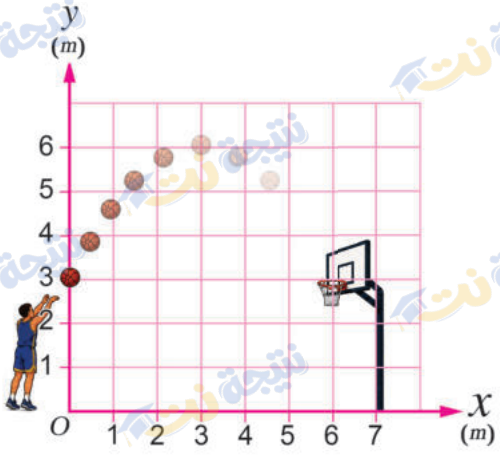
one of the dolphin shows

Dolphins are among the smartest marine creatures. They are distinguished by their ability to swim quickly and jump high above the surface of the water. Dolphin shows are considered among the most attractive marine shows for visitors, as dolphins display amazing skills in jumping and acrobatic movements. During one of the shows, a dolphin jumped out of the water, and its path was represented by the function f where $f(x) = -0.2 x^2 + 2 x$
What is the maximum height reached by the dolphin?

In this lesson, you will learn the concept of the quadratic function, its properties, and how to represent it graphically, which will enable you to solve such real-life problems.

Think & Discuss!

In the given figure :
If the ball moves along the curved path of the function f where :
$f(x) = -\frac{1}{3}(x-3)^2 + 6$,
will the ball go in the basket ?
Why ? Discuss.



Learn!

Quadratic Function

The quadratic function is a second-degree function in one variable. It is represented by a curve called a "parabola". This curve is symmetric about a vertical straight line called the axis of symmetry, which passes through the vertex of the curve.

The standard form of the quadratic function

The standard form of the quadratic function f is $f(x) = a x^2 + b x + c$ where $a \neq 0$

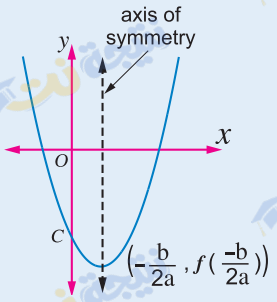
- The x-coordinate of the vertex of the curve : $x = \frac{-b}{2a}$
- The y-coordinate of the vertex of the curve : $y = f\left(\frac{-b}{2a}\right)$
- The vertex of the curve : $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
- The y-intercept equals c

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Graphical Representation of the Quadratic Function in Standard Form

If $a > 0$

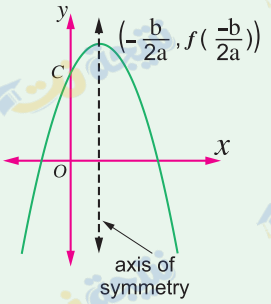
The curve opens upward.
The function has a minimum value at the vertex of the curve.



The minimum value of the function is $f\left(\frac{-b}{2a}\right)$

If $a < 0$

The curve opens downward.
The function has a maximum value at the vertex of the curve.



The maximum value of the function is $f\left(\frac{-b}{2a}\right)$

Example 1

Represent graphically the function f in each of the following, showing the axis of symmetry on the graph, then find the minimum or maximum value of the function :

1) $f(x) = x^2 - 2x - 3$ 2) $f(x) = -x^2 + 6x - 5$

1) To draw the quadratic function f, follow these steps :

- Find the x-coordinate of the vertex of the curve

$\because a = 1 \ , \ b = -2 \ , \ c = -3$

$\therefore x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = 1$

- Choose some real values that are symmetric around the x-coordinate of the vertex of the curve :

$-2 \ , -1 \ , 0 \ , 1 \ , 2 \ , 3 \ , 4$

The x-coordinate of the vertex of the curve

- Find the y-coordinate of the vertex of the curve and the selected values as shown in the table :

x	$f(x) = x^2 - 2x - 3$	$(x , f(x))$
-2	$f(-2) = (-2)^2 - 2(-2) - 3 = 5$	$(-2 , 5)$
-1	$f(-1) = (-1)^2 - 2(-1) - 3 = 0$	$(-1 , 0)$
0	$f(0) = (0)^2 - 2(0) - 3 = -3$	$(0 , -3)$
1	$f(1) = (1)^2 - 2(1) - 3 = -4$	$(1 , -4)$ ← Vertex of the curve
2	$f(2) = (2)^2 - 2(2) - 3 = -3$	$(2 , -3)$
3	$f(3) = (3)^2 - 2(3) - 3 = 0$	$(3 , 0)$
4	$f(4) = (4)^2 - 2(4) - 3 = 5$	$(4 , 5)$

Note that

The curve of the quadratic function is symmetric about the axis of symmetry, which is the vertical straight line passing through the vertex of the curve. The points lying on the two opposite sides of the curve with respect to the axis of symmetry are equidistant from the axis on the right and on the left.

Self-Evaluation 1

Represent graphically the function f where $f(x) = 3x - x^2$, showing the axis of symmetry on the graph, then find the minimum or maximum value of the function.

Think & Discuss

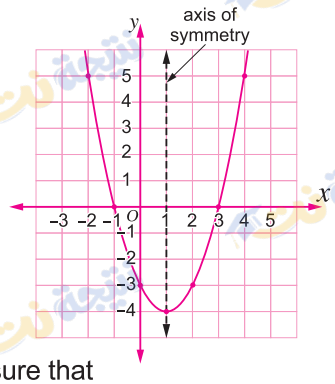
1) Does the point $(6 , 21)$ lie on the curve of the function f, where $f(x) = x^2 - 2x - 3$?

2) Does the point $(-1 , -10)$ lie on the curve of the function f, where $f(x) = -x^2 + 6x - 5$?

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- Plot the points in the coordinate plane, then draw a curve passing through the points.

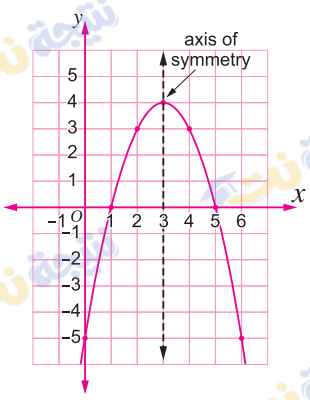
The curve opens upward, so the function has a minimum value, which is -4 (the y-coordinate of the vertex of the curve).



(2) Follow the same previous steps, making sure that the graphical representation includes the vertex of the curve.

$$x = \frac{-b}{2a} = \frac{-6}{2(-1)} = 3$$

x	$f(x)$	$(x, f(x))$	
0	-5	$(0, -5)$	
1	0	$(1, 0)$	
2	3	$(2, 3)$	
3	4	$(3, 4)$	← Vertex of the curve
4	3	$(4, 3)$	
5	0	$(5, 0)$	
6	-5	$(6, -5)$	



The curve opens downward, so the function has a maximum value, which is 4

Example 2

In each of the following, find the vertex of the curve of the quadratic function, the point of intersection of the curve with the y-axis, and the minimum or maximum value of the function :

(1) $f(x) = x^2 - 4x + 3$ (2) $g(x) = -x^2 + 9$

(1) $\because a = 1 \ , \ b = -4 \ , \ c = 3$

$\therefore \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$

$\therefore f(2) = (2)^2 - 4(2) + 3 = -1$

$\therefore$ The vertex of the curve is $(2, -1)$

$\because c = 3$

$\therefore$ The point of intersection of the curve with the y-axis is $(0, 3)$

$\because a > 0$

$\therefore$ The curve opens upward

$\therefore$ The minimum value of the function $= -1$

(2) $\because a = -1 \ , \ b = 0 \ , \ c = 9$

$\therefore \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$

$\therefore f(0) = -(0)^2 + 9 = 9$

$\therefore$ The vertex of the curve is $(0, 9)$

$\because c = 9$

$\therefore$ The point of intersection of the curve with the y-axis is $(0, 9)$

$\because a < 0$

$\therefore$ The curve opens downward

$\therefore$ The maximum value of the function $= 9$

Using Technology

- Different programs can be used to represent and study the quadratic function, such as GeoGebra.
- Try it yourself use GeoGebra to represent the two functions in example (1)

Self-Evaluation (2)

In each of the following, find the vertex of the curve of the quadratic function, the point of intersection of the curve with the y-axis, and the minimum or maximum value of the function :

(1) $f(x) = x^2 - 2x$

(2) $g(x) = 1 - x^2$

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The vertex form of the quadratic function

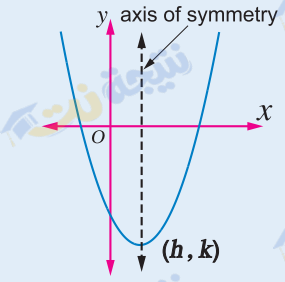
The vertex form of the quadratic function f is :

$f(x) = a(x-h)^2 + k$ where a, h, k are real numbers, $a \neq 0$ and the vertex of the curve is (h, k)

Graphical Representation of the Quadratic Function in Vertex Form

If $a > 0$

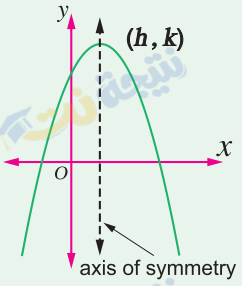
The curve opens upward.
The function has a minimum value at the vertex of the curve.



The minimum value of the function is $f(h) = k$

If $a < 0$

The curve opens downward.
The function has a maximum value at the vertex of the curve.



The maximum value of the function is $f(h) = k$

Think & Discuss

Hagar wrote that the vertex of the curve of the function f, where $f(x) = (x+2)^2 + 4$ is the point $(2, 4)$, is what Hagar wrote correct ? Discuss.

Self-Evaluation (3)

Represent graphically the function f, where $f(x) = (x-2)^2 - 4$, showing the axis of symmetry of the function on the graph, then find the maximum or minimum value of the function.

Example 3

Represent graphically the function f where $f(x) = 5 - 2(x+1)^2$, showing the axis of symmetry of the function on the graph, then find the maximum or minimum value of the function.

$\because f(x) = 5 - 2(x+1)^2$ $\therefore f(x) = -2(x-(-1))^2 + 5$

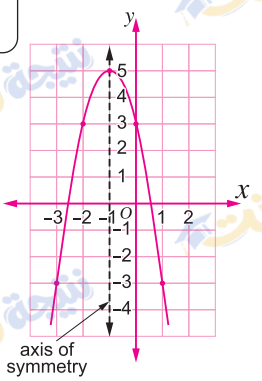
$\therefore a = -2$, $h = -1$, $k = 5$

$\therefore$ The vertex of the curve is: $(-1, 5)$

x	$f(x) = 5 - 2(x+1)^2$	$(x, f(x))$
-3	$f(-3) = 5 - 2(-3+1)^2 = -3$	$(-3, -3)$
-2	$f(-2) = 5 - 2(-2+1)^2 = 3$	$(-2, 3)$
-1	$f(-1) = 5 - 2(-1+1)^2 = 5$	$(-1, 5)$ ← Vertex of the curve
0	$f(0) = 5 - 2(0+1)^2 = 3$	$(0, 3)$
1	$f(1) = 5 - 2(1+1)^2 = -3$	$(1, -3)$

$\because a = -2$ $\therefore a < 0$

$\therefore$ The curve opens downward, and the function has a maximum value, which is 5



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Example 4

Determine the vertex of the curve and the maximum or minimum value of the function f where $f(x) = 2(x-3)^2 + 1$, then find the value of b if the point $(b, 9)$ lies on the curve.

$\because f(x) = 2(x-3)^2 + 1$ $\quad \therefore h = 3 \ , \ k = 1$

$\therefore$ The vertex of the curve is $(3, 1)$

$\because a = 2$ $\quad \therefore a > 0$

$\therefore$ The curve opens upward.

$\therefore$ The minimum value of the function $= 1$

$\because$ The point $(b, 9)$ lies on the curve.

$\therefore f(b) = 9$ $\quad \therefore 2(b-3)^2 + 1 = 9$

$\therefore 2(b-3)^2 = 9 - 1 = 8$ $\quad \therefore (b-3)^2 = \frac{8}{2} = 4$

$\therefore b - 3 = \pm\sqrt{4} = \pm 2$

$\therefore b - 3 = 2$ or $b - 3 = -2$

$\therefore b = 2 + 3 = 5$ $\quad b = -2 + 3 = 1$

Self-Evaluation (4)

Determine the vertex of the curve and the maximum or minimum value of the function f where:

$f(x) = -(x+1)^2 + 3$

If the point $(m, -1)$ lies on the curve, find the value of m

Example 5

A football player kicks a ball upward from the ground. Its height above the ground, $h(t)$, in meters, after (t) seconds is determined by the relation: $h(t) = -4.9\,t^2 + 19.6\,t$. Represent the relation graphically, and find the maximum height and the time at which the ball reaches this maximum height.



$\because a = -4.9 \ , \ b = 19.6 \ , \ c = 0$

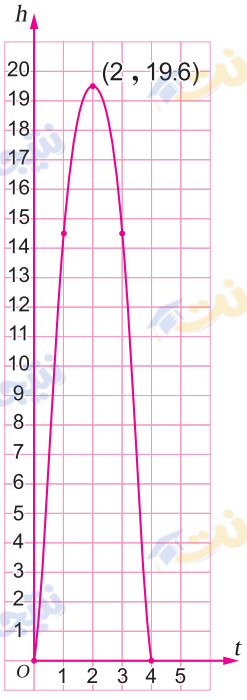
$\therefore \frac{-b}{2a} = \frac{-19.6}{2(-4.9)} = 2$

Note that:
Do not choose negative values for the variable t

t	$h(t)$	$(t, h(t))$
0	0	(0 , 0)
1	14.7	(1 , 14.7)
2	19.6	(2 , 19.6)
3	14.7	(3 , 14.7)
4	0	(4 , 0)

$\because$ The vertex of the curve is: $(2, 19.6)$

$\therefore$ The maximum height the ball reaches $= 19.6$ meters, and the time needed to reach the maximum height $= 2$ seconds.



Self-Evaluation (5)

If the function h, where $h(x) = -0.5\,x^2 + 2x + 6$ represents the relation between the height of a car $h(x)$, in meters, as it moves on a mountain slope and the horizontal distance x, in meters, draw the curve of the function and state the maximum height at which the car begins to descend.



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Technology Activity — Using the Calculator to Draw a Quadratic Function

A scientific calculator can be used to create the table used to draw a quadratic function.

The following steps show how to create the table for drawing the function f,

where $f(x) = x^2 - 2x - 3$ using the calculator.

1. First, the calculator must be set to (*Table*) mode by pressing the key 

, then choosing (Table) mode.

2. Enter the data: Write the rule of the given function by pressing the following keys:



3. Press the key $=$, then at the beginning of the interval Start, write $(-)$ 2, then press the key $=$

4. Write the number 4 at the end of the interval, END ?, then press the key $=$

5. Determine the step length of the interval, STEP ? and choose the number 1, then press the key $=$.

Thus, the table is created as shown on the given screen.

	x	$F(x)$
1	-2	5
2	-1	0
3	0	-3

You can move using the key  upward and downward.

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Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

1. What is the vertex of the curve of the function f, where $f(x) = 2(x-3)^2 + 1$?

   (a) $(2, 3)$ (b) $(2, 1)$ (c) $(-3, 1)$ (d) $(3, 1)$

2. If (h, k) is the vertex of the curve of the function f, where $f(x) = 2x^2 - 8x + 7$, what is the value of h ?

   (a) 2 (b) 4 (c) 6 (d) 8

3. What is the point of intersection of the curve of the function f, where $f(x) = x^2 - 3x - 5$ with the y-axis?

   (a) $(0, -5)$ (b) $(0, -3)$ (c) $(0, 3)$ (d) $(0, 5)$

4. What is the minimum value of the function f, where $f(x) = x^2 - 6x + 14$?

   (a) -6 (b) 5 (c) 9 (d) 14

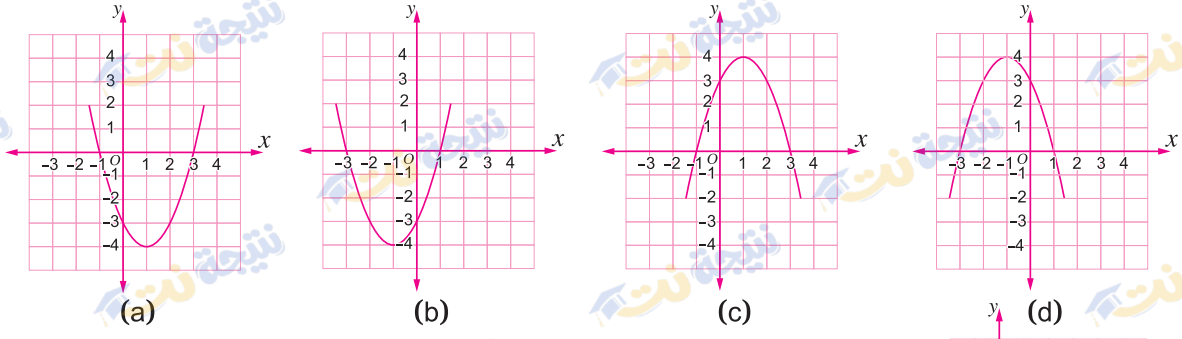
5. If the function f, where $f(x) = (a+4)x^3 + x^2 - 5x + 6$ is a quadratic function, what is the value of a ?

   (a) -4 (b) -5 (c) 5 (d) 6

6. If -2 is the x-coordinate of the vertex of the curve of the function f, where $f(x) = ax^2 + 12x + 7$, what is the value of a ?

   (a) 3 (b) 2 (c) -2 (d) -3

7. Which curve represents the function f, where $f(x) = x^2 - 2x - 3$?



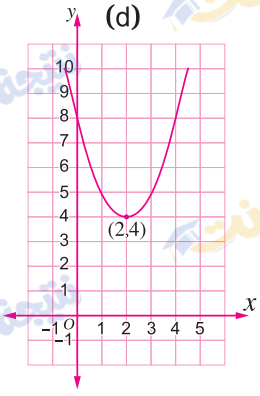
8. If the given curve represents the quadratic function f, what is the rule of the function f ?

   (a) $f(x) = (x+2)^2 + 4$

   (b) $f(x) = (x+4)^2 + 2$

   (c) $f(x) = (x-2)^2 + 4$

   (d) $f(x) = (x-4)^2 + 2$



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Second Applying Concepts

9 Represent graphically each of the following, showing on the graph the axis of symmetry of the function f, and find the maximum or minimum value of the function :

① $f(x) = x^2 - 4x + 7$

② $f(x) = x^2 - 3$

③ $f(x) = 2x - x^2$

④ $f(x) = 6 + 2x - x^2$

⑤ $f(x) = (x-1)^2 + 2$

⑥ $f(x) = (x+3)^2 - 1$

Third Analysis and Subjects Integration

10 **Sports :** A football player kicked a ball into the air. The height of the ball above the ground, h in meters, at any time t : in seconds, can be found using the function h , where : $h(t) = -(t-4)^2 + 16$

① Draw the graph of the function for $0 \leq t \leq 8$

② Use the graphical representation to find the maximum height reached by the ball and the time needed for it to return to the ground.





11 **Archery :** A person wants to shoot an arrow over a horizontal tree branch that is 15 meters above the ground. The height of the arrow above the ground, h in meters, after t seconds from being shot is given by the function h , where: $h(t) = -2t(t-6)$

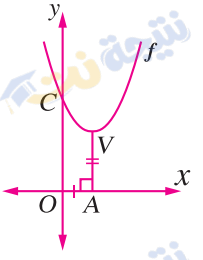
① Draw the graph of the height of the arrow for $0 \leq t \leq 6$

② Use the graphical representation to find the maximum height reached by the arrow. Did the arrow pass over the branch?

Creative Thinking

12 In the given figure :

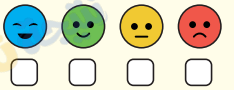
The function f, where $f(x) = x^2 - 2x + k + 1$ is represented graphically by a curve whose vertex is V. If $OA = \frac{1}{2} VA$, find the value of k and the coordinates of the point C.



13 If f is a function, where $f(x) = x^2 - 10x + k$ and the vertex of the curve V is 13 length units away from the origin, what are the possible values of k ?

***Evaluate** your understanding!*

How well do you understand quadratic function?
Tick the right box.



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Lesson

2 - 2

Set of Real Zeros of Polynomial Functions

Learning Outcomes

- Find the set of real zeros of polynomial functions algebraically.
- Find the set of real zeros of polynomial functions represented graphically.
- Use the real zeros of polynomial functions to solve some real-life problems.

Vocabulary

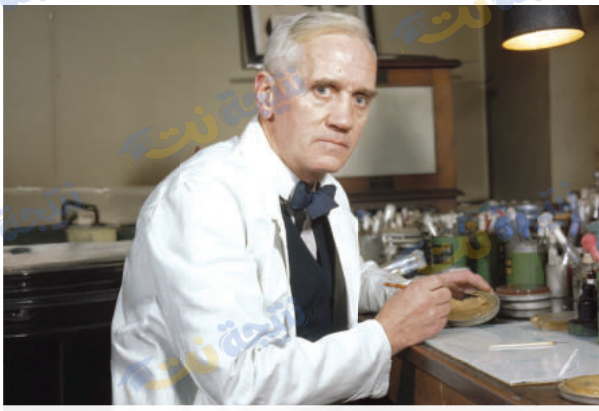
- Function
- Polynomial function
- Zeros of function
- Solution of equation

Think with your classmates

Is each of the following functions a polynomial function or not?

1. $f(x) = x^4$
2. $f(x) = 4^x$
3. $f(x) = \frac{1}{x}$
4. $f(x) = -2x^{-3}$

Get Ready!



The British scientist Alexander Fleming

The British scientist Alexander Fleming is considered one of the most prominent scientists in the history of medicine, as he discovered the first antibiotic , penicillin, in 1928.
The effectiveness of antibiotics depends on time , as their concentration in the human body decreases over time after they are taken until it completely vanishes. A patient takes a dose of an antibiotic, and the amount of medicine remaining in his body , in milligrams , after t hours is expressed by the function c , where :

$$c(t) = 9 - \frac{1}{4}t^2$$

After how many hours does the effect of this antibiotic completely vanishe from the patient's body ?

In this lesson, you will learn the concept of real zeros of polynomial functions , which will enable you to solve such real-life problems.

Think & Discuss!

If $f(x) = 5x^3 - 20x$, find $f(0)$ and $f(2)$ What do you notice ?
Can you find another value at which $f(x) = 0$? Discuss.

Learn!

Polynomial Function

The function $f: R \longrightarrow R$ which is in the form :

$$f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$$

where $a_0, a_1, a_2, \cdots, a_n$ are real numbers and all exponents of the variable x are positive integers , and $a_n \neq 0$ is called a polynomial function in one variable of degree n.

Examples of polynomial functions :

- $f(x) = x^3 - \sqrt{2}x + 1$: a polynomial of the third degree (cubic function).
- $f(x) = 2x - x^2$: a polynomial of the second degree (quadratic function).
- $f(x) = 9 - 4x$: a polynomial of the first degree (linear function).
- $f(x) = 7$: a polynomial of degree zero (constant function).
- $f(x) = 0$: the zero polynomial, and it has no degree.

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Set of Real Zeros of a Polynomial Function

The real zeros of a function are the real values of x that make $f(x)$ equal to zero.

So that : The set of real zeros of the function f is $\{x : f(x) = 0 , x \in R\}$.

For example : If $f(x) = (x-5)^2$, then 5 is a zero of the function f because : $f(5) = (5-5)^2 = 0$

To find the real zeros of the function f, we solve the equation : $f(x) = 0$

Example 1

Find the set of real zeros of each of the following:

1. $f(x) = 3x - 6$
2. $p(x) = x^2 - 25$
3. $g(x) = x^2 - x - 12$
4. $f(x) = x^3 + 2x^2 - 15x$
5. $p(x) = 3x^2 + 12$
6. $g(x) = 27x^3 + 125$

1) By setting : $3x - 6 = 0$

$\therefore x = \frac{6}{3} = 2$

$\therefore$ The set of zeros of the function $f = \{2\}$

2) By setting : $x^2 - 25 = 0$

$\therefore x^2 = 25 \quad \therefore x = \pm 5$

$\therefore$ The set of zeros of the function $p = \{-5 , 5\}$

3) By setting : $x^2 - x - 12 = 0$

$\therefore (x+3)(x-4) = 0$

$\therefore x = -3$ or $x = 4$

$\therefore$ The set of zeros of the function $g = \{-3 , 4\}$

4) By setting :

$x^3 + 2x^2 - 15x = 0$

$\therefore x(x^2 + 2x - 15) = 0$

$\therefore x(x+5)(x-3) = 0$

$\therefore x = 0$ or $x = -5$ or $x = 3$

$\therefore$ The set of zeros of the function $f = \{0 , -5 , 3\}$

5) By setting : $3x^2 + 12 = 0$

$\therefore 3x^2 = -12$

$\therefore x^2 = -4$

$\therefore x = \pm\sqrt{-4}$

there are no real roots

$\therefore$ The set of zeros of the function p is $\varnothing$

6) By setting : $27x^3 + 125 = 0$

$\therefore x^3 = \frac{-125}{27}$

$\therefore x = \sqrt[3]{\frac{-125}{27}} = \frac{-5}{3}$

$\therefore$ The set of zeros of the function $g = \left\{\frac{-5}{3}\right\}$

Note that

If f is a polynomial function and a is a real number such that $f(a) = 0$, then:

- a is one of the zeros of the polynomial function f.
- $x = a$ is one of the solutions of the equation $f(x) = 0$
- $(x - a)$ is one of the factors of $f(x)$

Self-Evaluation (1)

Find the set of real zeros of each of the following:

1. $p(x) = 5x - 10$
2. $g(x) = 2x^2 - 18$
3. $f(x) = x^3 - 8$
4. $g(x) = x^3 + 2x^2 - 8x$

Note that

- If f where $f(x) = a$, where a is a real number not equal to zero (a constant polynomial function) , then the set of zeros of the function f is $\varnothing$.
- If f where $f(x) = 0$ a zero polynomial function, then the set of zeros of the function f is R

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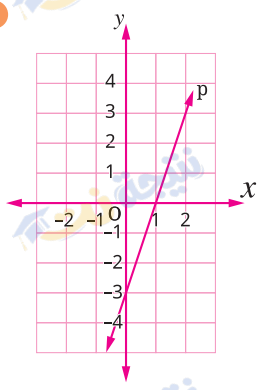
Finding the Real Zeros of a Polynomial Function Represented Graphically

If the curve of a polynomial function f intersects the X-axis at the point $(a, 0)$, then a is a zero of the function f.

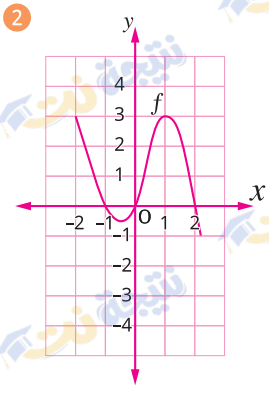
Example 2

Find the set of real zeros of each of the functions represented by the following curves :

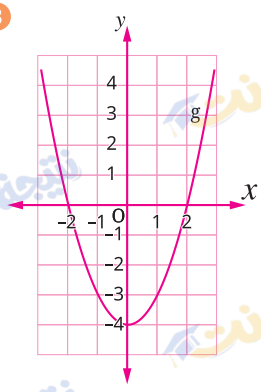
1



2



3



(1) ∵ The curve of the function p intersects the X-axis at $X = 1$

∴ The set of zeros of the function p is $\{1\}$.

(2) ∵ The curve of the function f intersects the X-axis at :

$X = -1 , X = 0 , X = 2$

∴ The set of zeros of the function f is $\{-1 , 0 , 2\}$.

(3) ∵ The curve of the function g intersects the X-axis at :

$X = -2 , X = 2$

∴ The set of zeros of the function g is $\{-2 , 2\}$.

Example 3

If -3 is one of the zeros of the polynomial function f, where : $f(X) = 2X^2 + bX + 3$, find the value of b , then find $f(4)$.

∵ -3 is one of the zeros of the function f, where: $f(X) = 2X^2 + bX + 3$

∴ $f(-3) = 0$

∴ $2(-3)^2 + b(-3) + 3 = 0$

∴ $-3b = -21$ ∴ $b = 7$

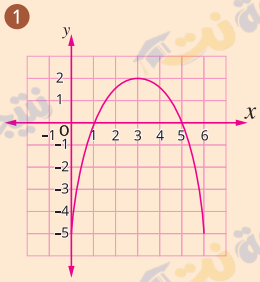
∴ $f(X) = 2X^2 + 7X + 3$

∴ $f(4) = 2(4)^2 + 7(4) + 3 = 63$

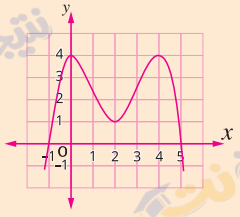
Self-Evaluation (2)

Find the set of real zeros of each of the functions represented by the following curves :

1



2



Note that

The number of different real zeros of a polynomial function is the number of points of intersection of the curve representing the function with the X-axis.

Self-Evaluation (3)

If **0** and **4** are the zeros of the polynomial function f, where : $f(X) = X^2 + aX + b$, find the value of each of a and b, then find : $f(-2)$

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Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

(1) What is the set of zeros of the function f, where $f(x) = -3x$?

(a) $\{3\}$ (b) $\{-3\}$

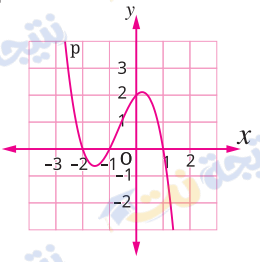
(c) $\{0\}$ (d) $R - \{0\}$

(2) What is the set of real zeros of the function p, where $p(x) = x^2 + 25$?

(a) $\{5\}$ (b) $\{5 , -5\}$

(c) $\{0\}$ (d) $\varnothing$

(3) What are the real zeros of the function p shown in the graph ?



(a) $2 , 0$

(b) $-1 , 0 , 1$

(c) $-2 , 2 , 1$

(d) $-2 , -1 , 1$

(4) Which of the following has the set of zeros $\{-2 , 0\}$?

(a) $f(x) = x^2 - 2x$

(b) $f(x) = x^3 + 8$

(c) $f(x) = x^2 + 2x$

(d) $f(x) = x^2 - 4$

(5) What is the set of zeros of the function g, where $g(x) = 3$?

(a) $\{3\}$ (b) $\{0\}$

(c) $\varnothing$ (d) R

(6) If $(x + 3)$ is one of the factors of $p(x)$, which of the following is correct ?

(a) $p(0) = 3$ (b) $p(0) = -3$

(c) $p(3) = 0$ (d) $p(-3) = 0$

(7) If 1 is one of the zeros of the function p , which of the following is correct ?

(a) $p(0) = 1$ (b) $p(0) = -1$

(c) $p(1) = 0$ (d) $p(-1) = 0$

(8) If $\{-3 , 3\}$ is the set of zeros of the function f , where $f(x) = x^2 + k$, what is the value of k ?

(a) ± 3 (b) 9

(c) -9 (d) ± 9

Second Applying Concepts

(9) **Find the set of real zeros of each of the following :**

① $f(x) = 2x - 16$

② $f(x) = x^2 - 36$

③ $g(x) = 4x^2 + 9$

④ $p(x) = x$

⑤ $p(x) = x^3 - x$

⑥ $p(x) = x^2 - 3x + 2$

⑦ $p(x) = 3x^2 - 12x$

⑧ $p(x) = x^3 + 64$

⑨ $f(x) = 2x^2 - 7x + 6$

(10) If -2 is one of the zeros of the function f, where $f(x) = x^2 + bx - 14$, find the value of b.

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(11) If -7 is one of the zeros of the function p, where $p(x) = 2x^2 + 6x + c$, find the value of c and the set of zeros of p.

(12) If 3 is one of the zeros of the function g, where $g(x) = x^2 - (k-5)x - 5k$, find the value of k, then find the set of zeros of g.

Third Analysis and Subjects Integration



Tourism is considered one of the most important pillars of the economy in Egypt, as it is an important source of foreign currency.

(13) **Tourism :** If the number of daily views of a short film promoting tourism in Egypt follows the function :

$p(t) = -3t^2 + 18t + 165$

where t is the number of days since the film was posted, on which day will the number of views reach zero?

Creative Thinking

(14) **Find the set of real zeros of each of the following :**

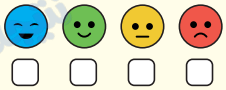
① $f(x) = x^3 - 5x^2 - 4x + 20$

② $p(x) = 2x^3 - x^2 - 8x + 4$

③ $g(x) = x^2 - 2x - 6$

***Evaluate** your understanding!*

How well do you understand the set of real zeros of polynomial functions? Tick the right box



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Lesson

2 - 3

Rational Algebraic Functions

Learning Outcomes

- Learn the concept of the rational algebraic function.
- Identify the domain of a rational algebraic function.
- Simplify a rational algebraic fraction to its simplest form.
- Identify the set of zeros of a rational algebraic function.
- Use rational algebraic functions to solve some real-life problems.

Vocabulary

- Rational algebraic fraction
- Rational algebraic function
- Domain
- Zeros of function

Not that

The algebraic fraction, in general, may contain a radical or fractional exponent or polynomials in numerator and denominator, but in the rational algebraic function , both numerator and denominator must be polynomials.

Get Ready!

Al-Galala Road in Egypt is considered one of the engineering achievements that amazed the world. It was carved through mountains and rocks over a length of 82 kilometers. The road starts after the end of the Kattameya Road and continues to Zaafarana. A car travels $(x^2 + 8x + 12)$ kilometers in a time of $(x^2 - x - 6)$ hours. Find the speed of the car in its simplest form, knowing that :
speed = distance ÷ time



Al - Galala Road

In this lesson, you will learn rational algebraic functions, which will enable you to solve such real-life problems.

Think & Discuss!

What are the values of x that make the function f undefined, where :
$f(x) = \frac{x^2}{x^2 - 9}$?

Hassan, Hanan, and Amgad answered the previous question, and their answers were as follows :

Hassan	Hanan	Amgad
f is undefined at $x = 9$	f is undefined at $x = \pm 3$	f is defined for all real values of x

Which of them has the correct answer ? Discuss.

Learn!

Rational Algebraic Fractions

A rational algebraic fraction is a ratio between two polynomials, and it is defined at all real numbers except the set of zeros of the denominator.
That is, if $p(x)$ and $q(x)$ are polynomials, then :
$\frac{p(x)}{q(x)}$ is called a rational algebraic fraction, where $q(x) \neq 0$

For example :

$\frac{x^2 - 1}{x + 2}$ is a rational algebraic fraction defined at all real numbers except at $x = -2$

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Example 1

Find the set of values of x that make the rational algebraic fraction undefined in each of the following :

1) $\frac{x-3}{x^2-3x+2}$ 2) $\frac{x+4}{x^2+16}$

1) By setting : $x^2-3x+2=0$

$\therefore (x-2)(x-1)=0$

$\therefore x-2=0$ or $x-1=0$

$\therefore x=2$ | $x=1$

$\therefore$ The set of zeros of the denominator $=\{1, 2\}$

$\therefore$ The fraction is undefined when $x \in \{1, 2\}$

2) By setting : $x^2+16=0$

$\therefore x^2=-16$

$\therefore x=\pm\sqrt{-16} \notin R$

$\therefore$ The set of zeros of the denominator $=\varnothing$

$\therefore$ The set of values of x that make the fraction undefined $=\varnothing$

Rational Algebraic Functions

If $\frac{p(x)}{q(x)}$ is a rational algebraic fraction, then:

The function f, where : $f(x)=\frac{p(x)}{q(x)}$ is a rational algebraic function whose domain is the set of real numbers R except the set of zeros of the denominator. The domain of the function f is denoted by the symbol D_f

So that : $D_f = R - \{x : q(x)=0\}$

Example 2

Determine the domain of each of the following rational algebraic functions :

1) The function f, where : $f(x)=\frac{x^2-25}{x-5}$, then find, if possible : $f(5)$, $f(-5)$

2) The function g, where : $g(x)=\frac{x^2-2x}{x^2+x-6}$, then find, if possible : $g(-3)$, $g(0)$

1) By setting : $x-5=0$

$\therefore x=5$

$\therefore$ The set of zeros of the denominator $=\{5\}$

$\therefore D_f = R-\{5\}$

$\therefore f(5)$ is undefined because $5 \notin D_f$

$\because -5 \in D_f$ $\quad \therefore f(-5)=\frac{(-5)^2-25}{(-5)-5}=\frac{0}{-10}=0$

Self-Evaluation 1

Find the set of values of x that make the rational algebraic fraction undefined in each of the following :

1) $\frac{2x-6}{x-3}$ 2) $\frac{x^2+x}{x^2-x-2}$

Checking the validity of the solution

You can check the validity of the solution in part 1 as follows :

By substituting the value $x=1$:

$\therefore \frac{x-3}{x^2-3x+2}$

$=\frac{1-3}{1^2-3(1)+2}$

$=\frac{-2}{0}$ (undefined)

Check yourself by substituting the value $x=2$

Discuss

Are polynomial functions considered rational algebraic functions or not ?

Self-Evaluation 2

Determine the domain of the function r, where :

$r(x)=\frac{2x+1}{x^2-4x-5}$

, then find, if possible :

$r(-1)$, $r(0)$, $r(5)$

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■ **Discuss**

What is the domain of the rational algebraic function if the set of zeros of the denominator is equal to $\varnothing$?

② By setting : $x^2 + x - 6 = 0$

$\therefore (x-2)(x+3) = 0$

$\therefore x - 2 = 0$ or $x + 3 = 0$

$\therefore x = 2$ | $x = -3$

$\therefore$ The set of zeros of the denominator $= \{-3, 2\}$

$\therefore D_g = R - \{-3, 2\}$

$\therefore g(-3)$ is undefined because $-3 \notin D_g$

$\because 0 \in D_g$ $\quad \therefore g(0) = \dfrac{(0)^2 - 2(0)}{(0)^2 + (0) - 6} = 0$

Simplifying Rational Algebraic Functions

A rational algebraic function can be simplified by canceling the common factors between the numerator and the denominator, while the function keeps its domain before cancellation.

■ **Self-Evaluation** ③

Put the function f in its simplest form, where :

$f(x) = \dfrac{x^3 - 64}{x^2 - 3x - 4}$,

showing its domain, then find, if possible :

$f(4)$, $f(-1)$, $f(0)$

Example 3

Put the function f in its simplest form, where : $f(x) = \dfrac{x^2 + 2x - 15}{x^2 - 5x + 6}$, showing its domain, then find, if possible : $f(1)$, $f(2)$, $f(3)$

– Factorize each of the numerator and the denominator :

$f(x) = \dfrac{(x-3)(x+5)}{(x-2)(x-3)}$

– Find the set of zeros of the denominator by setting the denominator equal to zero.

$\because (x-2)(x-3) = 0$

$\therefore x - 2 = 0$ or $x - 3 = 0$

$\therefore x = 2$ | $x = 3$

$\therefore$ The set of zeros of the denominator $= \{2, 3\}$

– Find the domain of the function f

$D_f = R - \{2, 3\}$

– Cancel the common factors between the numerator and the denominator to simplify the function f

$\because f(x) = \dfrac{(x-3)(x+5)}{(x-2)(x-3)}$ $\quad \therefore \boxed{f(x) = \dfrac{x+5}{x-2}}$

$\therefore f(1) = \dfrac{1+5}{1-2} = -6$

$\because 2 \notin D_f , 3 \notin D_f$

$\therefore$ It is not possible to determine the values of $f(2)$ and $f(3)$ using the function f

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Zeros of the Rational Algebraic Function

If f is a rational algebraic function, where : $f(x)=\frac{p(x)}{q(x)}$,
then the set of zeros of the function f is the set of zeros of the numerator $p(x)$ except the set of zeros of the denominator $q(x)$
So that : The set of zeros of the function $f=\{x : p(x)=0, q(x)\neq 0\}$

Example 4

Find the set of zeros of the function f, where : $f(x)=\frac{x^2-7x+12}{x^2-x-6}$

$\because f(x)=\frac{x^2-7x+12}{x^2-x-6}$ $\quad\therefore f(x)=\frac{(x-3)(x-4)}{(x-3)(x+2)}$

$\therefore$ The set of zeros of the numerator $=\{3, 4\}$

The set of zeros of the denominator $=\{3, -2\}$

$\therefore$ The set of zeros of the function f is :

$\{3, 4\}-\{3, -2\}=\{4\}$

Check your answer

Substitute the value $x=4$ in : $f(x)=\frac{x^2-7x+12}{x^2-x-6}$
You will find that : $f(4)=0$

Example 5

1. If $f(x)=\frac{x^2-4}{x-2}$, find the solution set of the equation : $f(x)=5$ in R
2. If $g(x)=\frac{x^2+2x-3}{x-1}$, find the solution set of the equation : $g(x)=4$ in R

1) $\because f(x)=\frac{x^2-4}{x-2}$

$\therefore f(x)=\frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$

$\therefore D_f=R-\{2\}$

$\therefore f(x)=x+2$

$\because f(x)=5$

$\therefore x+2=5$

$\therefore x=5-2=3\in D_f$

$\therefore$ The solution set $=\{3\}$

2) $\because g(x)=\frac{x^2+2x-3}{x-1}$

$\therefore g(x)=\frac{(x+3)\cancel{(x-1)}}{\cancel{(x-1)}}$

$\therefore D_g=R-\{1\}$

$\therefore g(x)=x+3$

$\because g(x)=4$

$\therefore x+3=4$

$\therefore x=4-3=1\notin D_g$

$\therefore$ The solution set $=\varnothing$

Remember that

The set of zeros of the function f is the values of x that make $f(x)=0$

Self-Evaluation 4

Find the set of zeros of the function f, where :

$f(x)=\frac{x^3+x^2-2x}{x^3+2x^2}$

Self-Evaluation 5

1. If $f(x)=\frac{x^2-3x}{x-3}$, find the solution set of the equation : $f(x)=3$ in R
2. If $g(x)=\frac{x^2+2x-8}{x^2+4x}$, find the solution set of the equation : $g(x)=-1$ in R

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Lesson Assessment

First — Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones :**

(1) Which of the following values makes the algebraic fraction $\frac{x-1}{x+2}$ undefined ?

(a) 1 (b) 2 (c) -1 (d) -2

(2) Which of the following values makes the function f, where $f(x)=\frac{x+3}{x-2}$, equal to zero?

(a) -3 (b) 3 (c) -2 (d) 2

(3) What is the domain of the function g, where $g(x)=\frac{x^2-x-2}{x^2+4}$?

(a) $R-\{2\}$ (b) $R-\{-2\}$

(c) $R-\{-4\}$ (d) R

(4) What is the domain of the function f, where $f(x)=\frac{x^2-25}{7}$?

(a) R (b) $\{7\}$

(c) $R-\{7\}$ (d) $\{-5, 5\}$

(5) What is the set of zeros of the function f, where $f(x)=\frac{x^2-2x}{x^2+2x}$?

(a) $\{0\}$ (b) $\{2\}$ (c) $\{-2\}$ (d) $\{2, 0\}$

(6) If the domain of the function f, where $f(x)=\frac{2x}{x-a}$, is equal to $R-\{-2\}$, what is the value of a ?

(a) 1 (b) -1 (c) 2 (d) -2

(7) If the set of zeros of the function f, where $f(x)=\frac{x^2-x+a}{x+2}$ is $\{3\}$, what is the value of a ?

(a) -6 (b) -2 (c) 3 (d) 6

▸ **Spot the mistake :**

(8) Amal and Magdy simplified the function f to the simplest form, where : $f(x)=\frac{4x+12}{x^2-x-12}$ and determined its domain.

Which of the two solutions is correct ? Why?

Amal's solution :

$\because f(x)=\frac{4x+12}{x^2-x-12}$

$\therefore f(x)=\frac{4(x+3)}{(x+3)(x-4)}$

$\therefore D_f=R-\{-3, 4\}$

$\therefore f(x)=\frac{4}{x-4}$

Magdy's solution :

$\because f(x)=\frac{4x+12}{x^2-x-12}$

$\therefore f(x)=\frac{4(x+3)}{(x+3)(x-4)}$

$\therefore f(x)=\frac{4}{x-4}$

$\therefore D_f=R-\{4\}$

Second — Applying Concepts

(9) **Put the function f in its simplest form, showing its domain, in each of the following :**

① $f(x)=\frac{2x-5}{5-2x}$

② $f(x)=\frac{x-4}{x^2-16}$

③ $f(x)=\frac{x^2-3x+2}{3x-3}$

④ $f(x)=\frac{6x+12}{2x^2+7x+6}$

⑤ $f(x)=\frac{16+16x+3x^2}{x^2-3x-28}$

⑥ $f(x)=\frac{4x^2-8x-5}{15-x-2x^2}$

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⑩ **Find the set of zeros of the function f in each of the following :**

① $f(x) = \dfrac{x+2}{x-2}$

② $f(x) = \dfrac{x^2-9}{x+3}$

③ $f(x) = \dfrac{2x-6}{x^2-x-6}$

④ $f(x) = \dfrac{2x^2+5x-3}{x^2+x-6}$

⑤ $f(x) = \dfrac{x^2-3x+2}{x^2-7x+12}$

⑥ $f(x) = \dfrac{4x^2-4x-3}{9-3x-2x^2}$

⑪ If the domain of the function f, where : $f(x) = \dfrac{x+6}{x^2-ax+36}$ is $R - \{6\}$, find the value of a

⑫ If $f(x) = \dfrac{x+a}{x-b}$ and $f(2)$ is undefined, and $f(5) = 2$, find the value of each of a and b

⑬ If $f(x) = \dfrac{2x^2+7x-15}{2x-3}$, find the solution set of the equation : $f(x) = 10$ in R

⑭ If $g(x) = \dfrac{x^2-5x+6}{x-2}$, find the solution set of the equation : $g(x) = -1$, where $x \in R$

Third Analysis and Subjects Integration

⑮ **Agriculture :** In the given figure, two beds of flower in the shape of a rectangle with the same area.

Express k as a function of x and put it in the simplest form, then find :

① the value of k when $x = 2$

② the value of x when $k = 4$



Creative Thinking

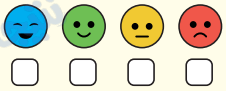
⑯ **Open Question :** Write a rational algebraic function whose domain is $R - \{4, -2\}$

⑰ If the volume of box A is $(x^3 - 2x^2 - 3x)$ cubic units, and the volume of box B is $(x^3 + 2x^2 - 15x)$ cubic units, what is the ratio of the volume of box A to the volume of box B in simplest form ?

Can the ratio between their volumes be $\dfrac{1}{2}$?

***Evaluate** your understanding!*

How well do you understand rational algebraic functions? Tick the right box.



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Lesson

2 - 4

Common Domain and Equivalent Rational Algebraic Functions

Learning Outcomes

- Find the common domain of two or more rational algebraic functions.
- Learn the conditions for the equality of rational algebraic functions.

Vocabulary

- Common domain
- Equivalent functions

Discuss

If f and g are two rational algebraic functions, and Domain of $f = R - \{1, 2\}$, Domain of $g = R - \{2, 3\}$ which of the following statements is correct :

1. The common domain $= R - \{2\}$
2. The common domain $= R - \{1, 2, 3\}$

Get Ready!

If the average age of the students in a girls' school is equal to $\left(\frac{x^2 - 5x}{x^2 - 8x + 15}\right)$ years and the average age of the students in a boys' school is equal to $\left(\frac{x^2 - x}{x^2 - 4x + 3}\right)$ years, do the two schools have the same average ages for all values of x?



In this lesson , you will learn the concept of the common domain of rational algebraic functions and the conditions for the equality of rational algebraic functions, which will enable you to solve such real-life problems.

Think & Discuss!

If $f(x) = \frac{x^2 - 4}{x - 2}$, $g(x) = x + 2$, is $f(2) = g(2)$? Why ?

Learn!

Common Domain of Rational Algebraic Functions

The common domain of two functions is the intersection of their domains, that is, the set of values for which the two functions are defined together.

For any two rational algebraic functions f and g , if :

The domain of f is $R - Z_1$, where Z_1 is the set of zeros of the denominator of $f(x)$, and the domain of g is $R - Z_2$, where Z_2 is the set of zeros of the denominator of $g(x)$, then the common domain of the two functions f and $g = D_f \cap D_g = R - (Z_1 \cup Z_2)$

So that

The common domain of two rational algebraic functions is equal to all real numbers except the set of zeros of the two denominators.

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■ Self-Evaluation (1)

Find the common domain of the two functions f and g, where :

$f(x)=\frac{x^2+7x}{x^2-49}$,

$g(x)=\frac{x-7}{x^2-10x+21}$

Example 1

Find the common domain of the two functions f and g, where :

$f(x)=\frac{x^2-4x+3}{x^2-x-6}$, $g(x)=\frac{x^2-9}{x^2-5x+6}$

By setting $x^2-x-6=0$

$\therefore (x-3)(x+2)=0$

$\therefore x-3=0$ or $x+2=0$

$\therefore x=3$ | $x=-2$

$\therefore D_f=R-\{-2,3\}$

By setting $x^2-5x+6=0$

$\therefore (x-2)(x-3)=0$

$\therefore x-2=0$ or $x-3=0$

$\therefore x=2$ | $x=3$

$\therefore D_g=R-\{2,3\}$

$\therefore$ The common domain $=R-\{-2,2,3\}$

■ Self-Evaluation (2)

Find the common domain of the functions f, g and p, where :

$f(x)=\frac{x+4}{2}$,

$g(x)=\frac{2}{x^2-4}$,

$p(x)=\frac{7x}{x^2-2x}$

Example 2

Find the common domain of the functions f, g and p, where :

$f(x)=\frac{x^3+1}{x^2-1}$, $g(x)=\frac{5}{x^2+4}$, $p(x)=\frac{2x^2-7x+6}{2x+3}$

By setting $x^2-1=0$

$\therefore (x-1)(x+1)=0$

$\therefore x-1=0$ or $x+1=0$

$\therefore x=1$ | $x=-1$

$\therefore D_f=R-\{-1,1\}$

By setting $x^2+4=0$

$\therefore x^2=-4$ $\quad\therefore x=\pm\sqrt{-4}\notin R$

$\therefore D_g=R$

By setting $2x+3=0$

$\therefore 2x=-3$ $\quad\therefore x=\frac{-3}{2}$

$\therefore D_p=R-\{\frac{-3}{2}\}$

$\therefore$ The common domain of the functions f, g and $p=R-\{\frac{-3}{2},-1,1\}$

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Equality of Rational Algebraic Functions

Two rational algebraic functions are equivalent if the following two conditions are satisfied together :

1. The domains are identical, that is, the two functions have exactly the same domain.
2. The rules are equivalent, that is, when the two rules are simplified to the simplest form, they become identical.

So that

For any two rational algebraic functions f and g, $\boxed{f = g}$, if the two conditions are satisfied:

1. $D_f = D_g$
2. $f(x) = g(x)$ for every $x \in (D_f \cap D_g)$

Example 3

If $f(x) = \frac{x^2 - 3x}{x^2 - 6x + 9}$, $g(x) = \frac{4x}{4x - 12}$, then prove that : $f = g$

$\because f(x) = \frac{x^2 - 3x}{x^2 - 6x + 9} = \frac{x(x-3)}{(x-3)(x-3)}$

$\therefore D_f = R - \{3\}$, $f(x) = \frac{x}{x-3}$

$\because g(x) = \frac{4x}{4x - 12} = \frac{4x}{4(x-3)}$ $\therefore D_g = R - \{3\}$, $g(x) = \frac{x}{x-3}$

$\therefore$ $\boxed{\text{Domain of } f = \text{Domain of } g}$, $\boxed{f(x) = g(x)}$ for every $x \in R - \{3\}$

$\therefore$ $\boxed{f = g}$

Example 4

If $f(x) = \frac{x}{x^2 - 2x}$, $g(x) = \frac{x^2 + 2x + 4}{x^3 - 8}$,

then prove that : $f(x) = g(x)$ for every x belonging to the common domain of the two functions f and g , and find this domain.

$\because f(x) = \frac{x}{x^2 - 2x} = \frac{x}{x(x-2)}$

$\therefore D_f = R - \{0, 2\}$, $f(x) = \frac{1}{x-2}$,

, $\because g(x) = \frac{x^2 + 2x + 4}{x^3 - 8} = \frac{x^2 + 2x + 4}{(x-2)(x^2 + 2x + 4)}$

$\therefore D_g = R - \{2\}$, $g(x) = \frac{1}{x-2}$

Note that

Domain of $f \neq$ Domain of g

$\therefore$ The common domain of the two functions f , $g = R - \{0, 2\}$

$\therefore f(x) = g(x)$ for every x belonging to the common domain $R - \{0, 2\}$

Self-Evaluation (3)

If $f(x) = \frac{x+2}{x^2 + 2x}$,

$g(x) = \frac{x^2 + 2x}{x^3 + 2x^2}$,

then prove that : $f = g$

Self-Evaluation (4)

If $f(x) = \frac{4}{4x - 20}$,

$g(x) = \frac{x+5}{x^2 - 25}$,

then prove that :

$f(x) = g(x)$

for every x belonging to the common domain, and find this domain.

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Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones :**

(1) What is the common domain of the two functions f and g, if $f(x)=\frac{3}{x+2}$ and $g(x)=\frac{-1}{x-2}$?

(a) $R-\{2\}$ (b) $R-\{-2\}$ (c) $R-\{-2,2\}$ (d) R

(2) If $f(x)=\frac{x}{x(x-3)}$ and $g(x)=\frac{1}{x-3}$,
which of the following sets expresses the set of values of x at which : $f(x)=g(x)$?

(a) $R-\{3\}$ (b) $R-\{0\}$ (c) $R-\{0,3\}$ (d) R

(3) If $f(x)=\frac{x}{x-k}$ and $g(x)=\frac{-2}{x+3}$ and the common domain of the two functions f and g is $R-\{-3\}$, what is the value of k ?

(a) 3 (b) -3 (c) 0 (d) -2

▸ **Spot the mistake :**

(4) Doaa and Iman answered the following question :

If $f(x)=\frac{x^2-1}{x^2-4x-5}$ and $g(x)=\frac{x^2-x}{x^2-5x}$, is $f=g$?

Doaa's answer :

$\because f(x)=\frac{(x-1)(x+1)}{(x+1)(x-5)}$

$\therefore D_f=R-\{-1,5\}$, $f(x)=\frac{x-1}{x-5}$

$\because g(x)=\frac{x(x-1)}{x(x-5)}$

$\therefore D_g=R-\{0,5\}$, $g(x)=\frac{x-1}{x-5}$

$\because D_f\neq D_g$, $f(x)=g(x)$

$\therefore f\neq g$

Iman's answer :

$\because f(x)=\frac{(x-1)(x+1)}{(x+1)(x-5)}$

$\therefore f(x)=\frac{x-1}{x-5}$

$\because g(x)=\frac{x(x-1)}{x(x-5)}$

$\therefore g(x)=\frac{x-1}{x-5}$

$\because f(x)=g(x)$

$\therefore f=g$

Which of them has the correct answer ? Discuss.

Second Applying Concepts

(5) **Find the common domain of the two functions f and g in each of the following :**

① $f(x)=\frac{x+1}{x-3}$, $g(x)=\frac{x-1}{x-2}$

② $f(x)=\frac{x^2-x-20}{x^2-3x-10}$, $g(x)=\frac{x^2-3x-10}{x^2-x-6}$

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6 Find the common domain of the functions f , g and k where :

$$f(x)=\frac{x+2}{3}\ ,\ g(x)=\frac{3}{x^2-9}\ ,\ k(x)=\frac{5x}{x^2-3x}$$

7 **Prove that $f=g$ in each of the following :**

① $f(x)=\frac{x}{x^2-4}$, $g(x)=\frac{3x}{3x^2-12}$

② $f(x)=\frac{x^2-2x}{x^2-4x+4}$, $g(x)=\frac{x}{x-2}$

③ $f(x)=\frac{x^2+2x}{x^2+4x+4}$, $g(x)=\frac{6x}{6x+12}$

8 **In each of the following, is $f=g$? Why ?**

① $f(x)=\frac{x^2-5x+4}{x^2-16}$, $g(x)=\frac{x^2+2x-3}{x^2+7x+12}$

② $f(x)=\frac{x^2-3x}{x^2-5x+6}$, $g(x)=\frac{2x-6}{2x^2-10x+12}$

9 **Determine the values of x that make $f(x)=g(x)$ for the two functions f , g in each of the following :**

① $f(x)=\frac{x-1}{x^2-1}$, $g(x)=\frac{1}{x+1}$

② $f(x)=\frac{x+1}{x-3}$, $g(x)=\frac{x^2+x}{x^2-3x}$

③ $f(x)=\frac{x+4}{x^2-16}$, $g(x)=\frac{x+1}{x^2-3x-4}$

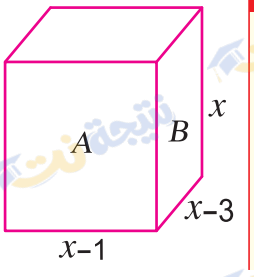
Third Analysis and Subjects Integration

10 **Proportion :**

If x , x^2-3x , $x+3$ and x^2-9 are four proportional quantities, what is the set of possible values of x ?

Creative Thinking

11 **In the given figure :** A box in the shape of a cuboid. If the function f is defined as the ratio of the area of face A to the lateral area, and the function g is defined as the ratio of the area of face B to the lateral area, then write each of $f(x)$ and $g(x)$, showing the domain of each of the two functions f and g and find the common domain of the two functions.



***Evaluate** your understanding!*

How well do you understand the common domain and equivalent rational algebraic functions?
Tick the right box.



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Lesson

2 - 5

Operations on Rational Algebraic Fractions

Learning Outcomes

- Perform operations on rational algebraic fractions.
- Learn the concept of the additive inverse and the multiplicative inverse of a rational algebraic fraction.
- Learn the properties of operations on rational algebraic fractions.

Vocabulary

- Rational algebraic fractions
- Reciprocal of the function

Previous Knowledge

Adding and subtracting rational numbers :

If $\frac{a}{b}$, $\frac{c}{b}$, $\frac{c}{d}$ are rational numbers, then :

1. $\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$

For example :

$\frac{1}{7} + \frac{4}{7} = \frac{5}{7}$

2. $\frac{a}{b} \pm \frac{c}{d} = \frac{a\,d}{b\,d} \pm \frac{c\,b}{b\,d} = \frac{a\,d \pm c\,b}{b\,d}$

For example :

$\frac{3}{7} - \frac{1}{3} = \frac{3 \times 3}{7 \times 3} - \frac{1 \times 7}{7 \times 3} = \frac{9}{21} - \frac{7}{21} = \frac{2}{21}$

Get Ready!



In order for Khaled to get an excellent grade in the mathematics exam in the final year of his university studies, the sum of his scores must be at least 8.
If his score in the first-term exam is $\frac{3x^2+16x+21}{x^2+5x+6}$ and his score in the second-term exam is $\frac{5x^2+34x+45}{x^2+7x+10}$, will Khaled get an excellent grade ?

In this lesson, you will learn the operations on rational algebraic fractions, which will enable you to solve such real-life problems.

Think & Discuss!

Sameh wrote : If $f(x) = \frac{x}{x+3} + \frac{3}{x+3}$, then $f(x) = 1$ for all real values of x. Is what Sameh wrote correct? Why? Discuss.

Learn!

First Adding and Subtracting Rational Algebraic Fractions

Adding and subtracting rational algebraic fractions are similar to adding and subtracting rational numbers, by following these steps :

1. Factorize the numerator and denominator of each fraction, then simplify each fraction, if possible.
2. Find a common denominator, then perform the addition or subtraction operation.
3. Put the final result in its simplest form.

For example : Assuming that the following rational algebraic fractions are defined, then :

1. $\frac{2x-1}{x+1} + \frac{x+3}{x+1} = \frac{2x-1+x+3}{x+1} = \frac{3x+2}{x+1}$

2. $\frac{x^2-x}{x^2+2x-3} + \frac{3}{x+3} = \frac{x\,(x-1)}{(x+3)\,(x-1)} + \frac{3}{x+3} = \frac{x}{x+3} + \frac{3}{x+3}$
$= \frac{x+3}{x+3} = 1$

3. $\frac{2}{x} - \frac{2}{1+x} = \frac{2\,(1+x)}{x\,(1+x)} - \frac{2\,(x)}{x\,(1+x)} = \frac{2+2x-2x}{x\,(1+x)} = \frac{2}{x\,(1+x)}$

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Example 1

Find $f(x)$ in its simplest form, stating the domain of f, in each of the following :

1 $f(x)=\dfrac{x^2-4}{x^2-x-6}+\dfrac{x^2-5x+4}{x^2-4x+3}$

2 $f(x)=\dfrac{2x-2}{x^2-3x+2}-\dfrac{3x+3}{x^2-x-2}$

Note that

The domain of the function of adding or subtracting two algebraic fractions is all real numbers except the values of x that make the denominators of the two fractions equal to zero.

1 Factorize each algebraic fraction :

$$f(x)=\frac{(x-2)(x+2)}{(x-3)(x+2)}+\frac{(x-1)(x-4)}{(x-1)(x-3)}$$

Find the domain of f : $D_f=R-\{3,-2,1\}$

Simplify each algebraic fraction and add, then simplify the result :

$$f(x)=\frac{x-2}{x-3}+\frac{x-4}{x-3}=\frac{2x-6}{x-3}=\frac{2(x-3)}{x-3}=2$$

2 Factorize each algebraic fraction :

$$f(x)=\frac{2(x-1)}{(x-1)(x-2)}-\frac{3(x+1)}{(x-2)(x+1)}$$

Find the domain of f : $D_f=R-\{1,2,-1\}$

Simplify each algebraic fraction and subtract :

$$f(x)=\frac{2}{x-2}-\frac{3}{x-2}=\frac{-1}{x-2}$$

Example 2

Find $f(x)$ in its simplest form, stating the domain of f, where :

$$f(x)=\frac{2x+4}{x^2-4}+\frac{3x+3}{x^2-1},$$

then find $f(3)$, $f(-1)$ if possible.

$$f(x)=\frac{2(x+2)}{(x+2)(x-2)}+\frac{3(x+1)}{(x+1)(x-1)}$$

$D_f=R-\{-2,2,-1,1\}$

$$f(x)=\frac{2}{x-2}+\frac{3}{x-1}=\frac{2(x-1)+3(x-2)}{(x-2)(x-1)}$$

$$=\frac{2x-2+3x-6}{(x-2)(x-1)}=\frac{5x-8}{(x-2)(x-1)}$$

$$f(3)=\frac{5\times3-8}{(3-2)(3-1)}=3.5$$

$f(-1)$ is undefined because $-1\notin D_f$

Self-Evaluation (1)

Find $f(x)$ in its simplest form, stating the domain of f in each of the following :

1 $f(x)=\dfrac{x^2+2x}{x^2+5x}+\dfrac{x^2-5x+6}{x^2+2x-15}$

2 $f(x)=\dfrac{x^2+5x+4}{x^2-4x-5}-\dfrac{x^2+3x}{x^2-5x}$

Note that

The domain of the function must be found before simplification.

Self-Evaluation (2)

Find $f(x)$ in its simplest form, stating the domain of f where :

$f(x)=\dfrac{x+2}{x^2+2x}-\dfrac{x+2}{x^2-4}$

Then find : $f(4)$, $f(-2)$

Properties of Addition of Rational Algebraic Fractions

1 Commutativity.
2 Associativity.
3 Zero is the additive identity.
4 The additive inverse exists.

Note that

The rational algebraic fraction $\dfrac{p(x)}{q(x)}$ has an additive inverse in the form :

$-\dfrac{p(x)}{q(x)}$ or $\dfrac{-p(x)}{q(x)}$ or $\dfrac{p(x)}{-q(x)}$

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Second Multiplying Rational Algebraic Fractions

Multiplying rational algebraic fractions is similar to multiplying rational numbers, by following these steps :

1. Factorize the numerator and denominator of each fraction completely, if possible.
2. Cancel the common factors between the numerator and denominator of each fraction, and between the numerator of one fraction and the denominator of the other fraction.
3. Perform the multiplication operation and put the final result in its simplest form.

For example : Assuming that the following rational algebraic fractions are defined, then :

1. $\frac{2x}{x-1} \times \frac{2x-2}{x^2+3x} = \frac{2x}{x-1} \times \frac{2(x-1)}{x(x+3)} = \frac{4}{x+3}$
2. $\frac{x^2-25}{x^2+5x} \times \frac{3x^2+18x}{x^2+x-30} = \frac{(x-5)(x+5)}{x(x+5)} \times \frac{3x(x+6)}{(x+6)(x-5)} = 3$

Example 3

Find $f(x)$ in its simplest form, stating the domain of f, in each of the following :

1. $f(x) = \frac{x^2+x-20}{x-3} \times \frac{2x^2-6x}{x^2+3x-10}$
2. $f(x) = \frac{x^2+x-6}{x^2+7x+12} \times \frac{x^2+6x+8}{4-x^2}$

Note that

The domain of the function of the product of two algebraic fractions is all real numbers except the values of x that make the denominators of the two fractions equal to zero.

(1) Factorize each algebraic fraction :

$$f(x) = \frac{(x+5)(x-4)}{x-3} \times \frac{2x(x-3)}{(x+5)(x-2)}$$

Find the domain of f : $D_f = R - \{3, -5, 2\}$

Simplify and perform the multiplication operation :

$$f(x) = \frac{2x(x-4)}{x-2}$$

(2) Factorize each algebraic fraction :

$$f(x) = \frac{(x+3)(x-2)}{(x+3)(x+4)} \times \frac{(x+4)(x+2)}{(2-x)(2+x)}$$

Find the domain of f : $D_f = R - \{-3, -4, 2, -2\}$

Simplify and perform the multiplication operation, then simplify the result :

$$f(x) = \frac{x-2}{2-x} = \frac{x-2}{-(x-2)} = -1$$

Note that

$x+2 = 2+x$

$2-x = -(x-2)$

Previous Knowledge

Multiplying rational numbers :

If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers,

then $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

For example :

$\frac{1}{2} \times \frac{3}{5} = \frac{1 \times 3}{2 \times 5} = \frac{3}{10}$

Self-Evaluation (3)

Find $f(x)$ in its simplest form, stating the domain of f, in each of the following :

1. $f(x) = \frac{x^2+3x}{x^2-3x-4} \times \frac{x^2-5x+4}{x^2+2x-3}$
2. $f(x) = \frac{x^2+2x-15}{9-x^2} \times \frac{x^2-3x-18}{x^2-7x+6}$

Properties of Multiplication of Rational Algebraic Fractions

1. Commutativity.
2. Associativity.
3. One is the multiplicative identity.
4. The multiplicative inverse exists for every rational algebraic fraction that is not equal to zero.

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Reciprocal of a Rational Algebraic Function

If $f(x)$ is a rational algebraic function, then the reciprocal of the rational algebraic function $g(x)$ is $g(x) = [f(x)]^{-1} = \frac{1}{f(x)}$

The domain of the reciprocal function is all real numbers except the values of x that make the numerator and denominator of the function $f(x)$ equal to zero.

For example :

If $f(x) = \frac{x-2}{x+3}$ is a function whose domain is $R - \{-3\}$, then the reciprocal of the function is $g(x)$ where

$$g(x) = [f(x)]^{-1} = \frac{1}{f(x)} = \frac{x+3}{x-2}$$

and the domain of the function g is $R - \{-3, 2\}$

Example 4

Find, in its simplest form, $g(x)$ the reciprocal of the function $f(x)$ where $f(x) = \frac{x^2+x-2}{x^2-3x-10}$, stating the domain of g, then find $g(-2)$

If $g(x) = \frac{1}{2}$, find the value of x

$\because f(x) = \frac{x^2+x-2}{x^2-3x-10} = \frac{(x-1)(x+2)}{(x+2)(x-5)}$

$\therefore g(x) = [f(x)]^{-1} = \frac{1}{f(x)} = \frac{(x+2)(x-5)}{(x-1)(x+2)}$

$\therefore D_g = R - \{1, -2, 5\}$, $g(x) = \frac{x-5}{x-1}$

$g(-2)$ is undefined because $-2 \notin D_g$

Put $g(x) = \frac{1}{2}$

$\therefore \frac{x-5}{x-1} = \frac{1}{2}$ $\quad \therefore 2x - 10 = x - 1$ $\quad \therefore x = 9 \in D_g$

Third Dividing Rational Algebraic Fractions

Dividing rational algebraic fractions is similar to dividing rational numbers. That is, dividing one fraction by another is changed into multiplying the first fraction by the reciprocal of the second fraction

For example : Assuming that the following rational algebraic fractions are defined and the second fraction is not equal to zero, then :

1. $\frac{3}{x} \div \frac{x-2}{x} = \frac{3}{x} \times \frac{x}{x-2} = \frac{3}{x-2}$

2. $\frac{x^2+12x+20}{x^2-3x-10} \div \frac{x^2+10x}{x-5} = \frac{(x+2)(x+10)}{(x+2)(x-5)} \div \frac{x(x+10)}{x-5}$

$= \frac{(x+2)(x+10)}{(x+2)(x-5)} \times \frac{x-5}{x(x+10)} = \frac{1}{x}$

Note that

The rational algebraic fraction $\frac{p(x)}{q(x)}$ has a multiplicative inverse in the form : $\frac{q(x)}{p(x)}$ and this inverse is defined for every $p(x) \neq 0$, $q(x) \neq 0$

Self-Evaluation (4)

Find, in its simplest form, $g(x)$ the reciprocal of the function $f(x)$ where : $f(x) = \frac{x^2-x-2}{x^2-5x+6}$, stating the domain of g, then find $g(2)$

If $g(x) = \frac{1}{3}$, find the value of x

Previous Knowledge

Dividing rational numbers :

If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then :

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

(where $\frac{c}{d} \neq 0$)

For example :

$$\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$$

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■ Self-Evaluation (5)

Find $f(x)$ in its simplest form, stating the domain of f in each of the following :

1) $f(x)=\dfrac{x^2-x-2}{x^2-4x-5}\div\dfrac{4-x^2}{x^2-3x-10}$

2) $f(x)=\dfrac{x^3-27}{x^2+x-12}\div\dfrac{x^2+3x+9}{2x+8}$

Example 5

Find $f(x)$ in its simplest form, stating the domain of f in each of the following :

1) $f(x)=\dfrac{x^2+2x+1}{1-x^2}\div\dfrac{x^2+5x+4}{x^2+3x-4}$

2) $f(x)=\dfrac{2x^2+5x+2}{2x^2+3x-2}\div\dfrac{3x^2+13x+4}{2x^2+7x-4}$

1) Factorize each algebraic fraction :

$$f(x)=\frac{(x+1)(x+1)}{(1-x)(1+x)}\div\frac{(x+1)(x+4)}{(x-1)(x+4)}$$

Find the domain of f : $D_f=R-\{1,-1,-4\}$

Change the division into multiplication, by multiplying by the multiplicative inverse of the divisor :

$$f(x)=\frac{(x+1)(x+1)}{(1-x)(1+x)}\times\frac{(x-1)(x+4)}{(x+1)(x+4)}$$

Perform the multiplication operation, then simplify the result :

$$=\frac{x-1}{1-x}=\frac{x-1}{-(x-1)}=-1$$

2) $f(x)=\dfrac{(2x+1)(x+2)}{(2x-1)(x+2)}\div\dfrac{(3x+1)(x+4)}{(2x-1)(x+4)}$

$D_f=R-\{\frac{1}{2},-2,-4,\frac{-1}{3}\}$

$$f(x)=\frac{(2x+1)(x+2)}{(2x-1)(x+2)}\times\frac{(2x-1)(x+4)}{(3x+1)(x+4)}=\frac{2x+1}{3x+1}$$

■ Note that

The domain of the quotient of two algebraic fractions is all real numbers except the values of x that make the denominators of the two fractions equal to zero and the values of x that make the numerator of the divisor fraction equal to zero.

Lesson Assessment



First Measuring Conceptual Understanding



▸ **Choose the correct answer from the given ones :**

1) $\dfrac{2}{x-2}-\dfrac{x}{x-2}=$ .....

(a) -1 (b) $\dfrac{2-x}{2(x-2)}$

(c) $\dfrac{x}{x-2}$ (d) $\dfrac{x+2}{x-2}$

2) $\dfrac{1}{x-2}+\dfrac{1}{x+2}=$ .....

(a) $\dfrac{1}{(x-2)(x+2)}$ (b) $\dfrac{2}{(x-2)(x+2)}$

(c) $\dfrac{4}{(x-2)(x+2)}$ (d) $\dfrac{2x}{(x-2)(x+2)}$

3) Which of the following does not represent the additive inverse of the algebraic fraction $\dfrac{x+2}{x-3}$?

(a) $-\dfrac{x+2}{x-3}$ (b) $\dfrac{x+2}{3-x}$

(c) $\dfrac{x-3}{x+2}$ (d) $\dfrac{-x-2}{x-3}$

4) What is the domain of the reciprocal of the function f where $f(x)=\dfrac{x-2}{x+3}$?

(a) $R-\{2\}$ (b) $R-\{-2\}$

(c) $R-\{2,-3\}$ (d) $R-\{-2,3\}$

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(5) $\frac{x}{x-1} \times \frac{x^2-1}{x^2} =$ .....

(a) $\frac{x}{x-1}$ (b) $\frac{x-1}{x}$

(c) $\frac{x}{x+1}$ (d) $\frac{x+1}{x}$

(6) What is the domain of the function f where $f(x) = \frac{x-2}{x+3} \div \frac{x}{x-5}$?

(a) $R - \{-3\}$ (b) $R - \{2, -3, 0, 5\}$

(c) $R - \{2, -3, 5\}$ (d) $R - \{-3, 5, 0\}$

(7) If $g(x)$ is the reciprocal of the function f where $f(x) = \frac{x-5}{x+7}$ and $g(k) = \frac{3}{2}$, what is the value of k ?

(a) 2 (b) 3

(c) 29 (d) -29

(8) If $f(x) = \frac{4}{3x} + \frac{5}{3x}$, then $[f(x)]^{-1} =$ .....

(a) $\frac{3}{x}$ (b) $\frac{x}{3}$

(c) $\frac{6}{x}$ (d) $\frac{x}{6}$

(9) **Spot the mistake in each of the following :**

$$\frac{x}{x+1} - \frac{5}{x+3} = \frac{x-5}{(x+1)(x+3)}$$

$$\frac{\cancel{2}}{x-\cancel{2}} \times \frac{5x}{x^2-\cancel{5x}} = \frac{1}{x} \times \frac{1}{x^2} = \frac{1}{x^3}$$

$$\frac{-4}{2-x} + \frac{2x}{x-2} = \frac{-4}{2-x} - \frac{2x}{2-x} = \frac{-4-2x}{2-x} = \frac{-2(2-x)}{2-x} = -2$$

Second Applying Concepts

(10) **Find each of the following in its simplest form (assuming that all the following rational algebraic fractions are defined) :**

① $\frac{5x}{x-2} - \frac{10}{x-2}$ ② $\frac{2}{x-5} + \frac{x^2-27}{x-5}$ ③ $\frac{x}{3} - \frac{4}{x}$

④ $\frac{1}{x-3} + \frac{1}{x-4}$ ⑤ $\frac{x}{x+3} \div \frac{x^2}{x^2+3x}$ ⑥ $\frac{3x-6}{5x-20} \times \frac{10x-40}{27x-54}$

(11) **Find $f(x)$ in its simplest form, stating the domain of f in each of the following :**

① $f(x) = \frac{x^2+x}{x^2+3x+2} + \frac{2x}{x^2+2x}$ ② $f(x) = \frac{x^2+6x}{x^2+5x} + \frac{x^2+2x-8}{x^2+3x-10}$

③ $f(x) = \frac{1}{x-4} - \frac{8}{x^2-16}$ ④ $f(x) = \frac{x^2-3x+2}{x^2-4} - \frac{x^2-3x}{x^2-x-6}$

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⑤ $f(x)=\frac{3x+15}{x^2+7x+10}-\frac{x+5}{x^2+6x+5}$

⑥ $f(x)=\frac{x^2+2x+4}{x^3-8}+\frac{x^2-x-12}{x^2+x-6}$

⑦ $f(x)=\frac{x^2+x-20}{x^2+2x-15}\times\frac{x^2+4x-21}{x^2+3x-28}$

⑧ $f(x)=\frac{x^2-11x+28}{x^2-13x+42}\times\frac{x^2+7x+10}{20-x-x^2}$

⑨ $f(x)=\frac{2x^2+5x+2}{2x^2+7x+3}\times\frac{x^2-7x-30}{x^2-6x-40}$

⑩ $f(x)=\frac{3x-15}{x+4}\div\frac{5-x}{6x+24}$

⑪ $f(x)=\frac{x^2-2x}{2x^2-x-6}\div\frac{2x^2-3x}{4x^2-9}$

⑫ $f(x)=\frac{6x^2+13x+6}{4x^2-9}\div\frac{6x^2+x-2}{4x^2-1}$

12 Find $f(x)$ in its simplest form, stating the domain of f where $f(x)=\frac{x^2-x-20}{x^2+2x-8}+\frac{3x+6}{x^2-4}$ then find, if possible $f(-2)$, $f(5)$

13 Find $f(x)$ in its simplest form, stating the domain of f where $f(x)=\frac{x^2-4}{x^2-x-2}\div\frac{x^2+5x+6}{x^2+2x-3}$ then find, if possible $f(3)$, $f(1)$, $f(-4)$

If $f(k)=\frac{3}{4}$, find the value of k

Third Analysis and Subjects Integration

14 A rhombus has an area of $\frac{x^2-3x-4}{x^2-x-2}$ square units and a height of $\frac{x^2-7x+12}{x^2-2x}$ length units. Find the length of its base in terms of x in its simplest form.

Creative Thinking

15 If $\frac{x^2+ax-b}{x^2-2x-15}\times\frac{x^2-9x+20}{x^2-16}=\frac{x-7}{x+3}$, find the value of : $a+b$

16 If the domain of the function f, where $f(x)=\frac{a}{x}+\frac{1}{b-x}$ is $R-\{0,4\}$, and $f(2)=3$, find the value of each of a , b

***Evaluate** your understanding!*

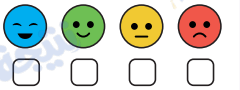
How well do you understand operations on rational algebraic fractions? Tick the right box



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Unit Two Assessment

Evaluate your understanding!
How well do you understand Unit Two? Tick the right box

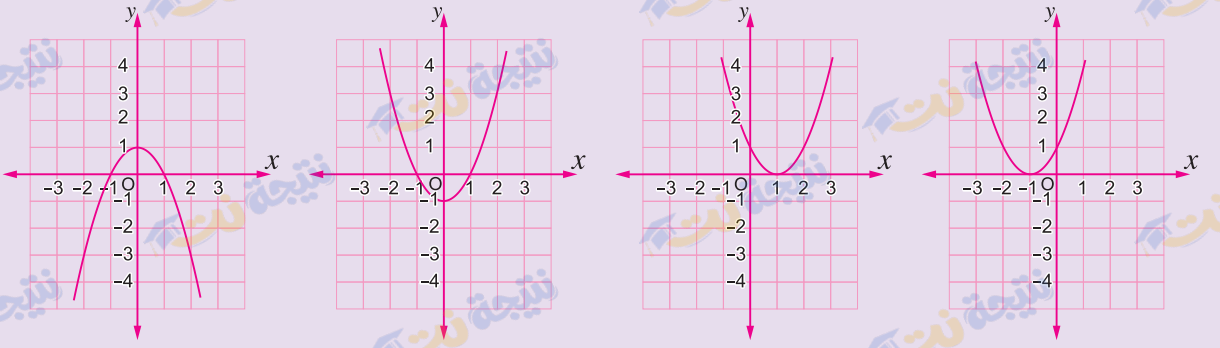


▸ Choose the correct answer from the given ones :

(1) What is the set of real zeros of the function f, where $f(x) = x^2 + 4$?

(a) $\{-2\}$ (b) $\{2\}$ (c) $\{-2, 2\}$ (d) $\varnothing$

(2) Which graph represents the function f, where $f(x) = x^2 - 1$?



(a) (b) (c) (d)

(3) What is the domain of the function f, where $f(x) = \dfrac{x^2 - 3x}{x^2 - 4x + 3}$?

(a) R (b) $R - \{0, 1, 3\}$ (c) $R - \{1, 3\}$ (d) $R - \{0, 3\}$

(4) What is the set of real zeros of the function f, where $f(x) = \dfrac{x^2 - x}{x^2 - 1} \times \dfrac{x^2 + x}{x^2}$?

(a) $\varnothing$ (b) R (c) $\{0, 1, -3\}$ (d) $\{0, 1\}$

▸ Complete each of the following :

(5) If $f(x) = \dfrac{x^2 - x - 2}{x^2 + x - 6}$, then the set of values of x that make $f(x) = \dfrac{3}{5}$ is .....

(6) The vertex of the curve of the function f, where $f(x) = (x - 3)^2 + 1$, is .....

(7) The common domain of the two functions f_1, f_2, where $f_1(x) = \dfrac{x - 7}{x + 1}$, $f_2(x) = \dfrac{x + 3}{x(x - 7)}$ is .....

(8) The function f, where $f(x) = x^2 + 3x - 4$ intersects the y-axis at the point .....

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▸ **Answer the following questions :**

(9) If $f_1(x) = \frac{x^2}{x^3 - x^2}$, $f_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$, prove that : $f_1 = f_2$

(10) Find $f(x)$ in its simplest form, stating the domain of f , if $f(x) = \frac{x}{x^2 + 2x} + \frac{x - 2}{x^2 - 4}$

(11) If $p(x) = \frac{x^2 - 25}{5x - x^2}$, find the domain of $[p(x)]^{-1}$

(12) Represent graphically the function f , where $f(x) = (x - 1)^2 + 3$, showing the axis of symmetry on the graph, and find the maximum or minimum value of the function.



Unit Two Activity — The Quadratic Function

▸ **Activity Aim :**

Understanding the quadratic function, and using technology to graph it and study its curve.

▸ **Implementation Steps :**

In cooperation with one of your classmates, try to do the following:

1. Search using different sources, such as (your school library books, the Internet, or one of the artificial intelligence applications ...), for situations in different fields that can be represented by a quadratic function, such as : projectile motion, bridge and arch design, or designing water sprinkler systems in agricultural lands.
2. Choose one of them, then use an interactive program such as GeoGebra to draw this function.
3. Use what you have learned about the quadratic function and graph the curve of the function, then identify the vertex of the curve and explain its meaning in the context of the problem.
4. Explain the solution steps, emphasizing how to draw the curve of the quadratic function when interactive programs are not available.

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UNIT 3

Geometry

Unit Lessons

3-1 Similarity of Polygons
3-2 Similarity of Triangles
3-3 Dilation
3-4 Trigonometric Ratios of an Acute Angle
3-5 Trigonometric Ratios of Some Special Angles
3-6 The Relation between the Slopes of Two Parallel and Perpendicular Straight Lines
3-7 Equation of the Straight Line



The equation of the straight line is one of the mathematical foundations that help artificial intelligence systems understand patterns, make accurate decisions, and reach solutions to some geometric problems.

▶ Can geometric laws be used to reach more complex solutions in artificial intelligence applications?

Issues and Life Skills
- Mathematical Communication
- Critical Thinking
- Creative Thinking
- Sustainable Development

Values
- Environmental Awareness
- Civilizational Awareness
- Aesthetic Appreciation
- Teamwork

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Lesson

3 - 1

Similarity of Polygons

Learning Outcomes

- Learn the conditions for the similarity of two polygons.
- Recognize the similarity ratio.
- Use the properties of similarity to solve some real-life problems.

Vocabulary

- Similarity
- Similarity ratio

Note that

When writing two similar polygons, they must be written in the same order of the corresponding vertices.

Polygon $A\ B\ C\ D$
$\sim$
Polygon $E\ F\ G\ H$

Notation in Mathematics

The symbol $\sim$ is used to express the similarity of polygons.

Get Ready!

The Ancient Egyptians excelled in sculpting huge statues of their kings to decorate temples. To promote tourism, the Egyptians today make small souvenir models that accurately imitate those original statues.
In front of you is the statue of King Thutmose III, one of the greatest rulers of the New Kingdom in ancient Egypt. What is the height of a small model of it that has a width of **5** cm?

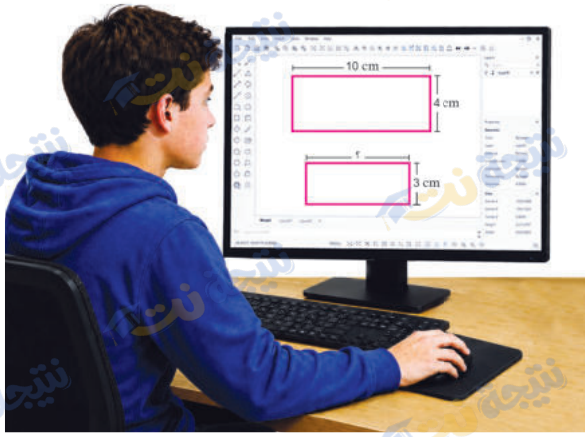


Statue of King Thutmose III

In this lesson, you will learn the conditions for the similarity of polygons, which will enable you to solve such real-life problems.

Think & Discuss!

Marwan used a geometric drawing program to obtain a smaller copy of the rectangle. What is the length of the rectangle in the smaller copy?



Learn!

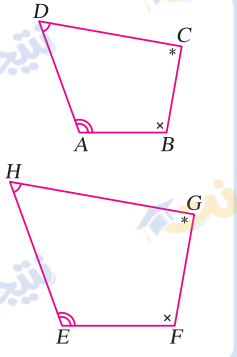
Similarity of Two Polygons

Two polygons are similar if the following two conditions are satisfied together :

1. Their corresponding angles are equal in measure.
2. The lengths of their corresponding sides are proportional.

In the two polygons $ABCD$ and $EFGH$ if :

1. $m(\angle A) = m(\angle E)$, $m(\angle B) = m(\angle F)$
   $m(\angle C) = m(\angle G)$, $m(\angle D) = m(\angle H)$
2. $\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE} = k$



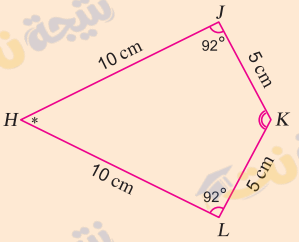
Then polygon $ABCD \sim$ polygon $EFGH$
where k is the similarity ratio of polygon $ABCD$ to polygon $EFGH$

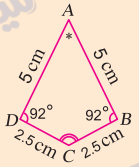
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Self-Evaluation (1)

In the following figure :

Prove that the two polygons are similar, and write the similarity ratio of polygon $ABCD$ to polygon $HLKJ$

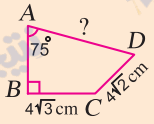


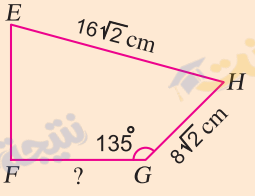


Self-Evaluation (2)

In the following figure :

If $ABCD \sim EFGH$



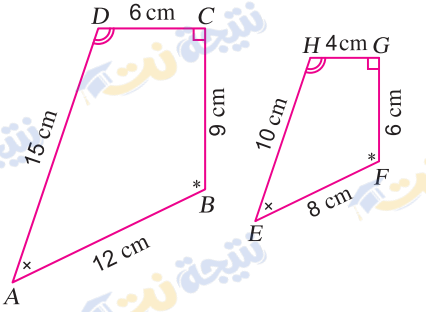


1. Find : $m(\angle C)$, $m(\angle D)$
2. Find the length of each of $\overline{FG}$, $\overline{AD}$

Example (1)

In the given figure :

Prove that the two polygons $ABCD$ and $EFGH$ are similar and write the similarity ratio of polygon $ABCD$ to polygon $EFGH$



$\because$ In the two polygons $ABCD$ and $EFGH$:

(1) $m(\angle A) = m(\angle E)$, $m(\angle B) = m(\angle F)$, $m(\angle C) = m(\angle G)$,
$m(\angle D) = m(\angle H)$

(2) $\frac{AB}{EF} = \frac{12}{8} = \frac{3}{2}$, $\frac{BC}{FG} = \frac{9}{6} = \frac{3}{2}$, $\frac{CD}{GH} = \frac{6}{4} = \frac{3}{2}$, $\frac{DA}{HE} = \frac{15}{10} = \frac{3}{2}$

$\therefore \frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE} = \frac{3}{2}$

$\therefore$ Polygon $ABCD \sim$ polygon $EFGH$

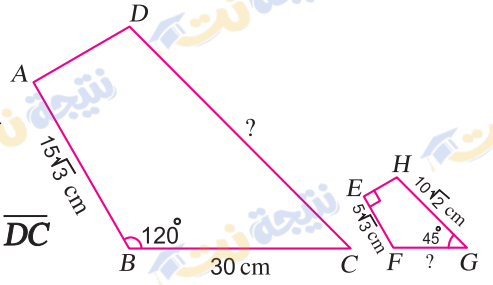
and the similarity ratio of polygon $ABCD$ to polygon $EFGH$ is $\frac{3}{2}$

Example (2)

In the given figure :

If polygon $ABCD \sim$ polygon $EFGH$

1. Find : $m(\angle C)$, $m(\angle H)$
2. Find the length of each of $\overline{FG}$ and $\overline{DC}$



(1) $\because ABCD \sim EFGH$

$\therefore m(\angle A) = m(\angle E) = 90°$, $m(\angle B) = m(\angle F) = 120°$,

$m(\angle C) = m(\angle G) = 45°$

$\therefore m(\angle D) = m(\angle H) = 360° - (90° + 120° + 45°) = 105°$

(2) $\because ABCD \sim EFGH$

$\therefore \frac{AB}{EF} = \frac{BC}{FG}$ $\quad \therefore \frac{15\sqrt{3}}{5\sqrt{3}} = \frac{30}{FG}$

$\therefore FG = \frac{5\sqrt{3} \times 30}{15\sqrt{3}} = 10$ cm.

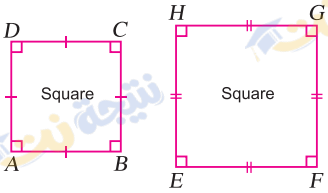
$\frac{AB}{EF} = \frac{DC}{HG}$ $\quad \therefore \frac{15\sqrt{3}}{5\sqrt{3}} = \frac{DC}{10\sqrt{2}}$

$\therefore DC = \frac{15\sqrt{3} \times 10\sqrt{2}}{5\sqrt{3}} = 30\sqrt{2}$ cm.

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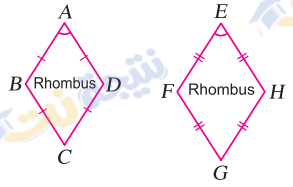
Conditions for the Similarity of Special Quadrilaterals

All squares are similar.



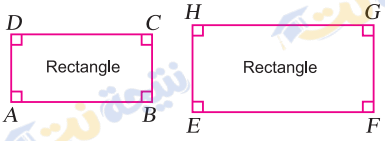
$ABCD \sim EFGH$

Two rhombuses are similar if the measures of two corresponding angles are equal.



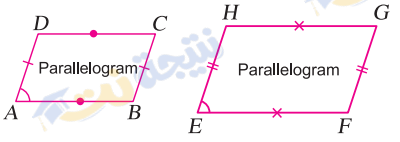
If $m(\angle A) = m(\angle E)$, then $ABCD \sim EFGH$

Two rectangles are similar if their corresponding dimensions are proportional.



If $\frac{AB}{EF} = \frac{BC}{FG}$, then $ABCD \sim EFGH$

Two parallelograms are similar if the measure of an angle in one of them is equal to the measure of an angle in the other, and the lengths of the sides containing these two angles are proportional.

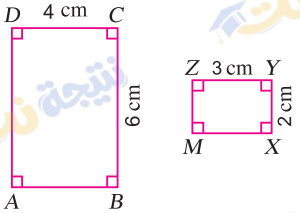


If $m(\angle A) = m(\angle E)$, $\frac{AB}{EF} = \frac{AD}{EH}$, then $ABCD \sim EFGH$

Example 3

In each of the following, prove that the two polygons are similar, and write the similarity ratio :

1 Rectangle $ABCD$ to rectangle $XYZM$



1 ∵ In the two rectangles $ABCD$ and $XYZM$:

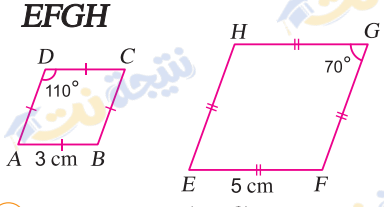
$\frac{BC}{YZ} = \frac{6}{3} = 2$, $\frac{CD}{ZM} = \frac{4}{2} = 2$

∴ $\frac{BC}{YZ} = \frac{CD}{ZM} = 2$

∴ $ABCD \sim XYZM$

The similarity ratio of rectangle $ABCD$ to rectangle $XYZM$ is 2

2 Rhombus $ABCD$ to rhombus $EFGH$



2 In rhombus $ABCD$:

$m(\angle C) = 180° - 110° = 70°$

∴ In the two rhombuses $ABCD$ and $EFGH$:

$m(\angle C) = m(\angle G) = 70°$

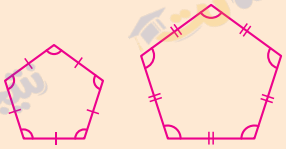
∴ $ABCD \sim EFGH$

∵ $\frac{AB}{EF} = \frac{3}{5}$

∴ The similarity ratio of rhombus $ABCD$ to rhombus $EFGH$ is $\frac{3}{5}$

Note that

Regular polygons that have the same number of sides are always similar.



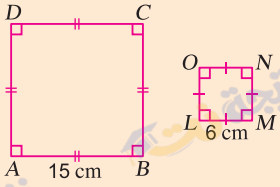
Discuss with your classmates

Discuss with your classmates the conditions for the similarity of special quadrilaterals, and how you can conclude them from the two previously mentioned conditions for the similarity of two polygons.

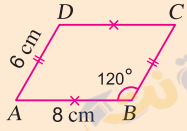
Self-Evaluation (3)

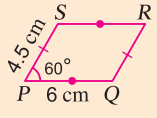
In each of the following, prove that the two polygons are similar, and write the similarity ratio :

1 Square $ABCD$ to square $LMNO$



2 Parallelogram $ABCD$ to parallelogram $PQRS$





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Note :

If polygon $ABCD \sim$ polygon $EFGH$,

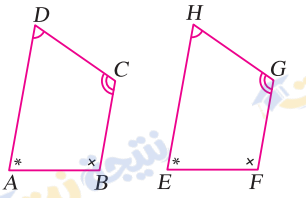
then $\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE} = k$

and from the properties of proportion,

you conclude that :

$$\frac{AB + BC + CD + DA}{EF + FG + GH + HE} = k$$

$$\therefore \frac{\text{Perimeter of polygon } ABCD}{\text{Perimeter of polygon } EFGH} = k \text{ (similarity ratio)}$$



That is :

The ratio between the perimeters of two similar polygons is equal to the ratio between the lengths of any two corresponding sides.

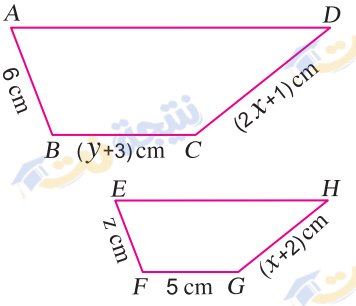
Example 4

In the given figure :

If polygon $ABCD \sim$ polygon $EFGH$,

$\frac{\text{Perimeter of polygon } ABCD}{\text{Perimeter of polygon } EFGH} = \frac{3}{2}$,

find the value of each of : x , y , z



$\because$ Polygon $ABCD \sim$ polygon $EFGH$,

$$\frac{\text{Perimeter of polygon } ABCD}{\text{Perimeter of polygon } EFGH} = \frac{3}{2}$$

$\therefore \frac{AB}{EF} = \frac{3}{2}$ $\quad \therefore \frac{6}{z} = \frac{3}{2}$ $\quad \therefore z = \frac{2 \times 6}{3} = 4$

$\frac{BC}{FG} = \frac{3}{2}$ $\quad \therefore \frac{y+3}{5} = \frac{3}{2}$ $\quad \therefore y + 3 = \frac{5 \times 3}{2}$

$\therefore y + 3 = 7.5$ $\quad \therefore y = 7.5 - 3 = 4.5$

$\frac{CD}{GH} = \frac{3}{2}$ $\quad \therefore \frac{2x+1}{x+2} = \frac{3}{2}$

$\therefore 4x + 2 = 3x + 6$ $\quad \therefore 4x - 3x = 6 - 2$ $\quad \therefore x = 4$

Collaborative Activity

Cooperate with one of your classmates in measuring the dimensions of your school playground. Then, draw in your notebook a rectangle similar to this playground, and write the similarity ratio between this rectangle and your school playground.

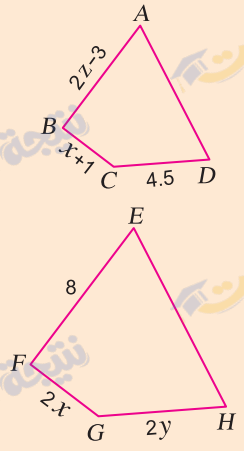


Self-Evaluation (4)

In the following figure :

If $ABCD \sim EFGH$,

$\frac{\text{Perimeter of } ABCD}{\text{Perimeter of } EFGH} = \frac{3}{4}$



find the value of each of x , y , z

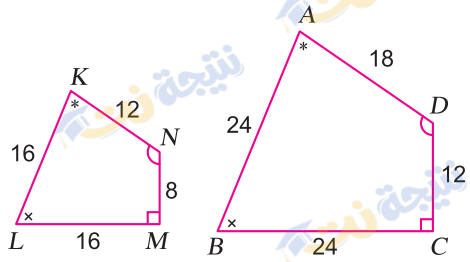
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Lesson Assessment

First **Measuring Conceptual Understanding**

▸ **Choose the correct answer from the given ones:**

1 In the following figure :

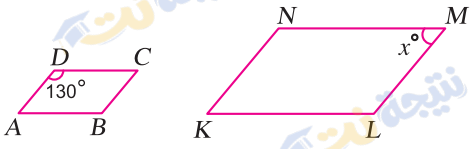


What is the similarity ratio of polygon $KLMN$ to polygon $ABCD$?

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) $\frac{1}{2}$ (d) $\frac{3}{1}$

3 In the following figure :

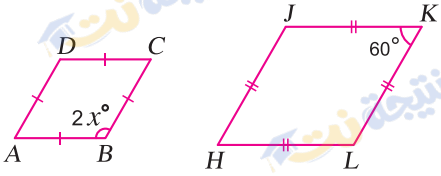


Parallelogram $ABCD \sim$ parallelogram $KLMN$, what is the value of $\mathcal{X}$?

(a) 50 (b) 130

(c) 60 (d) 70

5 In the following figure :

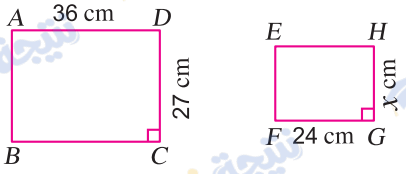


If rhombus $ABCD \sim$ rhombus $HLKJ$, what is the value of $\mathcal{X}$?

(a) 60 (b) 120

(c) 100 (d) 80

2 In the following figure :

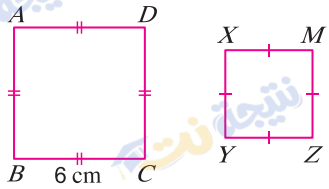


If rectangle $ABCD \sim$ rectangle $EFGH$, what is the value of $\mathcal{X}$?

(a) 27 (b) 9

(c) 18 (d) 24

4 In the following figure :

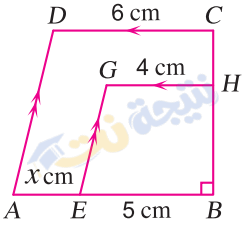


If $\frac{\text{Perimeter of square } ABCD}{\text{Perimeter of square } XYZM} = \frac{3}{2}$, what is the area of square $XYZM$?

(a) 36 cm^2 (b) 24 cm^2

(c) 16 cm^2 (d) 8 cm^2

6 In the following figure :



If polygon $ABCD \sim$ polygon $EBHG$, what is the value of $\mathcal{X}$?

(a) 5 (b) 2.5

(c) 7.5 (d) 4

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7 Two similar polygons , the ratio between the lengths of two corresponding sides is **2 : 3** , and the perimeter of the smaller polygon is **34** cm. What is the perimeter of the larger polygon ?

(a) **17** cm (b) **34** cm (c) **51** cm (d) **68** cm

▸ **Spot the mistake :**

8 The teacher asked his students the following question :

Are all regular polygons similar ?

Ahmed's and Nourhan's answers were as shown in the given figure. Which of the two answers is correct ? Why ?

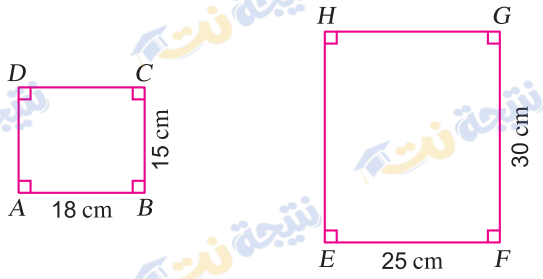
Ahmed's answer :
Yes, all regular polygons are similar.

Nourhan's answer :
Only regular polygons that have the same number of sides are similar.

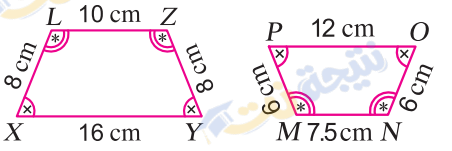
Second Applying Concepts

9 Determine which of the following pairs of polygons are similar and which are not :

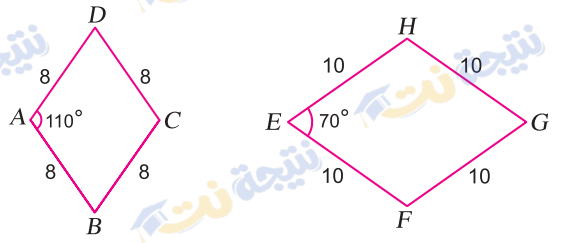
①



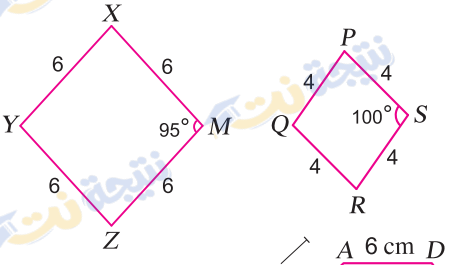
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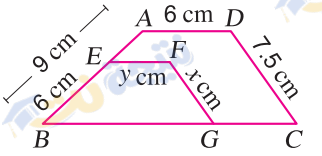
③



④

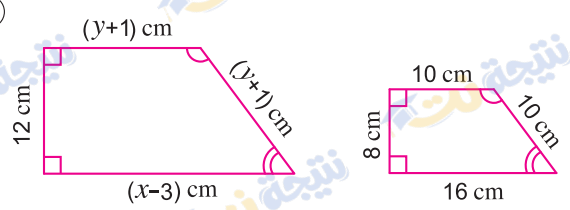


10 In the given figure : If polygon ABCD ~ polygon EBGF, find the value of each of x , y

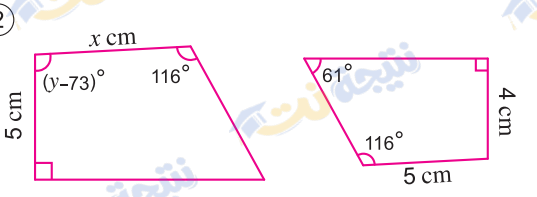


11 If each pair of the following pairs of polygons is similar, find the value of each of x , y

①



②



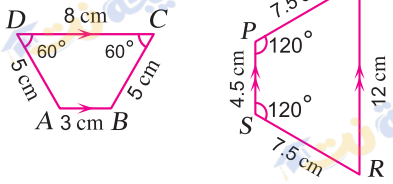
12 Two similar polygons , the lengths of the sides of one of them are **3** cm, **5** cm, **6** cm, **8** cm, **10** cm, and the perimeter of the other is **80** cm. Find the lengths of the sides of the second polygon.

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13 In the given figure :

Prove that :

polygon $ABCD \sim$ polygon $SPQR$



Third Analysis and Subjects Integration

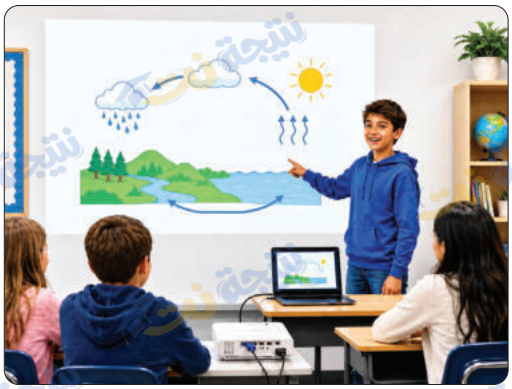
14 **Entertainment :** A rectangular cinema screen has a width of **45** feet and a height of **25** feet.

Explain which of the television screen formats shown in the picture you think will display the most complete scene from a movie shown on the cinema screen.



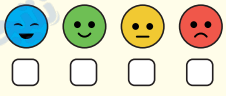
15 **Technology :** Ahmed displayed a presentation on the wall of his classroom. The width of his computer screen is **13.25** inches and its height is **10.6** inches, while the image displayed on the wall has a width of **53** inches and a height of **42.4** inches. Are the two figures similar?

If they are similar, what is the similarity ratio of the computer screen to the displayed image?



***Evaluate** your understanding!*

How well do you understand similarity of polygons?
Tick the right box



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Lesson

3 - 2

Similarity of Triangles

Learning Outcomes

- learn the conditions for the similarity of two triangles.
- Use the similarity of triangles to find angles measures and sides lengths.
- Prove that two triangles are similar.
- Use the similarity of two triangles to solve some real-life problems.

Vocabulary

- Similarity of triangles

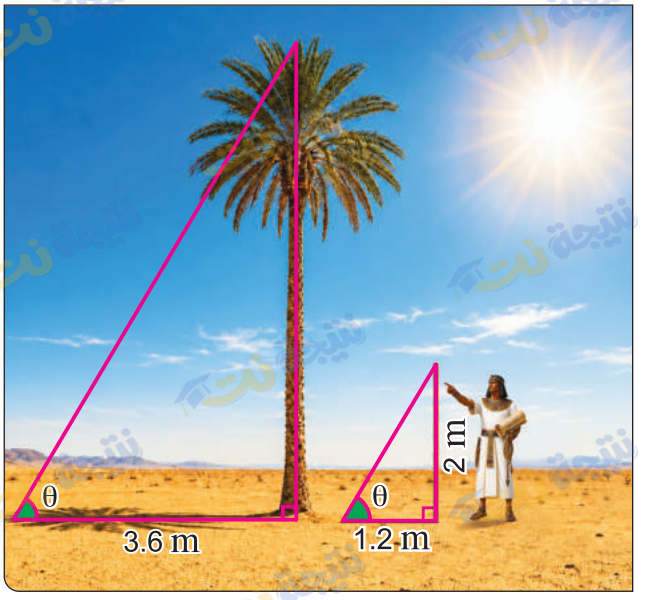
Note that

In this case, from similarity you can conclude that :

- $m(\angle C) = m(\angle F)$
- $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Get Ready!

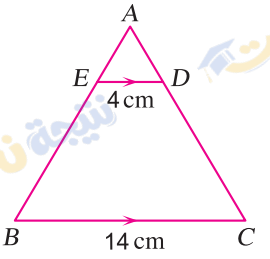
The ancient Egyptians were among the first civilizations to use shadow to calculate the heights of tall objects. This idea based on knowing the length of the shadow of a known-height object, such as a stick, and the shadow of an object with unknown height to be found. In the picture, a person uses a vertical stick on the ground to find the height of a palm tree. Can you find the height of the palm tree?



In this lesson, you will learn the concept of triangle similarity and the cases of similarity of two triangles, which will enable you to solve such real-life problems.

Think & Discuss!

In the given figure : If you know that the perimeter of ΔABC is 49 cm, what is the perimeter of ΔAED? Which of the following answers is correct?



Ali's answer:	Maryam's answer:
Perimeter of $\Delta AED = 35$ cm	Perimeter of $\Delta AED = 14$ cm

Learn!

Cases of Similarity of Two Triangles

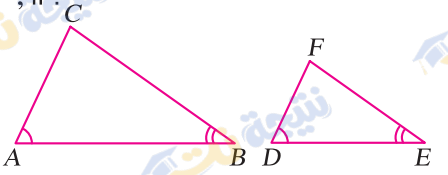
Case 1: Two Angles (AA)

If two angles in one triangle are congruent with two angles in another triangle, then the two triangles are similar.

In the two triangles ABC , DEF , if :

- $m(\angle A) = m(\angle D)$
- $m(\angle B) = m(\angle E)$

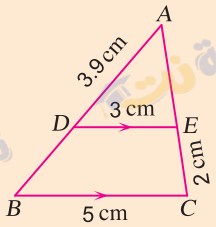
, then $\Delta ABC \sim \Delta DEF$



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Self-Evaluation (1)

In the following figure :



$D \in \overline{AB}$, $\overline{DE} // \overline{BC}$,

$E \in \overline{AC}$

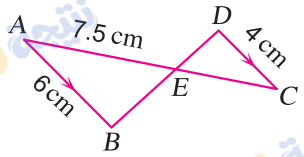
Prove that :

$\Delta ABC \sim \Delta ADE$,

then find the lengths of :

$\overline{BD}$, $\overline{AE}$

Example (1)



In the given figure :

$\overline{AB} // \overline{DC}$, $\overline{AC} \cap \overline{BD} = \{E\}$

$AB = 6$ cm, $AE = 7.5$ cm, $DC = 4$ cm, $BD = 7$ cm

Prove that : $\Delta ABE \sim \Delta CDE$ and find the lengths of : $\overline{EC}$, $\overline{DE}$, $\overline{BE}$

$\because \overline{AB} // \overline{CD}$

$\therefore m(\angle A) = m(\angle C)$ (Alternating Angles) ,

$m(\angle B) = m(\angle D)$ (Alternating Angles)

$\therefore \Delta ABE \sim \Delta CDE$

From the similarity :

$\frac{AB}{CD} = \frac{BE}{DE} = \frac{AE}{CE}$

$\because BE + DE = 7$ $\quad \therefore BE = 7 - DE$

$\therefore \frac{6}{4} = \frac{7 - DE}{DE} = \frac{7.5}{CE}$ $\quad \therefore \frac{6}{4} = \frac{7.5}{CE}$ $\quad \therefore CE = \frac{4 \times 7.5}{6} = 5$ cm

$\frac{6}{4} = \frac{7 - DE}{DE}$ $\quad \therefore 6\,DE = 28 - 4\,DE$ $\quad \therefore 10\,DE = 28$

$\therefore DE = 2.8$ cm $\quad \therefore BE = 7 - 2.8 = 4.2$ cm

Note that

In this case, from similarity you can conclude that :

- $m(\angle A) = m(\angle D)$
- $m(\angle B) = m(\angle E)$
- $m(\angle C) = m(\angle F)$

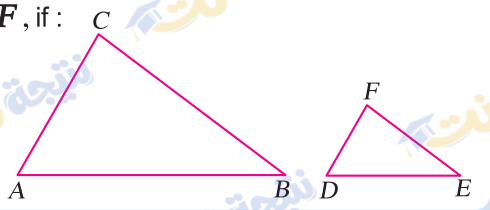
Case 2: Three Sides (SSS)

If the lengths of the corresponding sides of two triangles are proportional, then the two triangles are similar.

In the two triangles ABC , DEF , if :

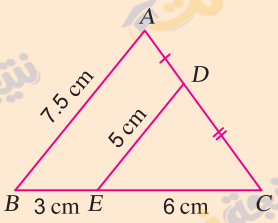
$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

, then $\Delta ABC \sim \Delta DEF$



Self-Evaluation (2)

In the following figure :



If $DC = 2\,AD$, prove that :

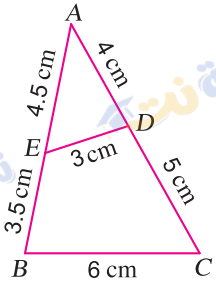
1. $\Delta ABC \sim \Delta DEC$
2. $\overline{DE} // \overline{AB}$

Example (2)

In the given figure:

Prove that : $\Delta ABC \sim \Delta ADE$

showing the similarity ratio of ΔABC to ΔADE



$\because \frac{AB}{AD} = \frac{4.5 + 3.5}{4} = 2$, $\frac{BC}{DE} = \frac{6}{3} = 2$, $\frac{AC}{AE} = \frac{4 + 5}{4.5} = 2$

$\therefore \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE} = 2$ $\quad \therefore \Delta ABC \sim \Delta ADE$

$\therefore$ The similarity ratio of ΔABC to $\Delta ADE = \frac{2}{1}$

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Case 3: Two Sides and the Included Angle (SAS)

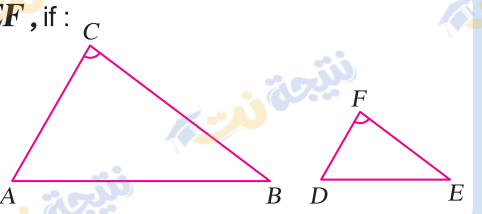
If an angle of one triangle is congruent with an angle of another triangle and the lengths of the sides containing these angles are proportional, then the two triangles are similar.

In the two triangles ABC , DEF , if :

$m(\angle C) = m(\angle F)$

, $\frac{AC}{DF} = \frac{BC}{EF}$

, then $\Delta ABC \sim \Delta DEF$



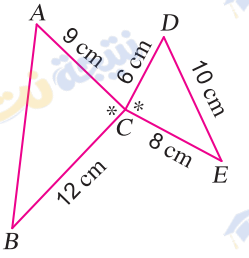
Example 3

In the given figure :

$m(\angle ACB) = m(\angle DCE)$

Use the lengths shown to prove that :

$\Delta ABC \sim \Delta DEC$, then find the length of $\overline{AB}$



$\because \frac{AC}{DC} = \frac{9}{6} = \frac{3}{2}$, $\frac{BC}{EC} = \frac{12}{8} = \frac{3}{2}$ $\therefore \frac{AC}{DC} = \frac{BC}{EC}$

$\because m(\angle ACB) = m(\angle DCE)$ $\therefore \Delta ABC \sim \Delta DEC$

From similarity :

$\frac{AB}{DE} = \frac{AC}{DC} = \frac{BC}{EC} = \frac{3}{2}$

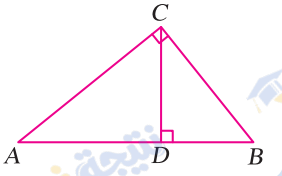
$\therefore \frac{AB}{DE} = \frac{3}{2}$ $\therefore \frac{AB}{10} = \frac{3}{2}$ $\therefore AB = \frac{3 \times 10}{2} = 15$ cm

Similarity in right-angled triangles

In a right-angled triangle, the perpendicular from the right angle to the hypotenuse divides the triangle into two similar triangles, each of which is similar to the original triangle.

If ΔABC is right-angled at C , $\overline{CD} \perp \overline{AB}$

, then $\Delta ACB \sim \Delta ADC \sim \Delta CDB$



By using Case 1 of triangle similarity :

In the two triangles ADC , ACB :

$m(\angle ADC) = m(\angle ACB)$ (each of which is a right angle)

$m(\angle A) = m(\angle A)$ (common angle)

$\therefore$ $\Delta ADC \sim \Delta ACB$ .... (1)

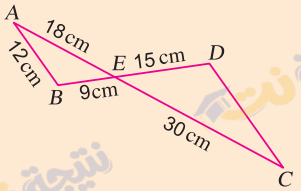
Note that

In this case, from similarity you can conclude that :

- $m(\angle A) = m(\angle D)$
- $m(\angle B) = m(\angle E)$
- $\frac{AC}{DF} = \frac{AB}{DE} = \frac{CB}{FE}$

Self-Evaluation (3)

In the following figure :



$\overline{AC} \cap \overline{DB} = \{E\}$

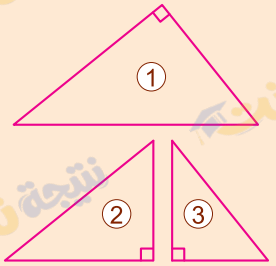
Use the lengths shown in the figure to prove that :

$\Delta ABE \sim \Delta CDE$

, then find the length of $\overline{DC}$

Note that

If two triangles are similar to a third one , then they are similar.



If $\Delta 1 \sim \Delta 2$, $\Delta 1 \sim \Delta 3$, then $\Delta 2 \sim \Delta 3$

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In the two triangles CDB , ACB

$m(\angle CDB) = m(\angle ACB)$ (each of which is a right angle)

$m(\angle B) = m(\angle B)$ (common angle)

$\therefore \Delta CDB \sim \Delta ACB$ .... ②

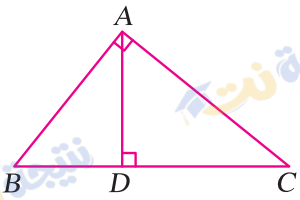
From ① , ② we conclude: $\Delta ADC \sim \Delta ACB \sim \Delta CDB$

Example 4

In the given figure :

ΔABC is right-angled at A , $\overline{AD} \perp \overline{BC}$

Prove that : $(AD)^2 = BD \times DC$



$\because m(\angle BAC) = 90°$, $\overline{AD} \perp \overline{BC}$

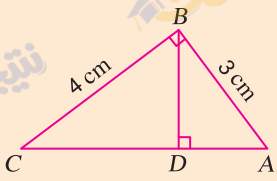
$\therefore \Delta ADB \sim \Delta CDA$

From similarity :

$\frac{AD}{CD} = \frac{DB}{DA}$ $\therefore (AD)^2 = DB \times CD$

Self-Evaluation 4

In the following figure :



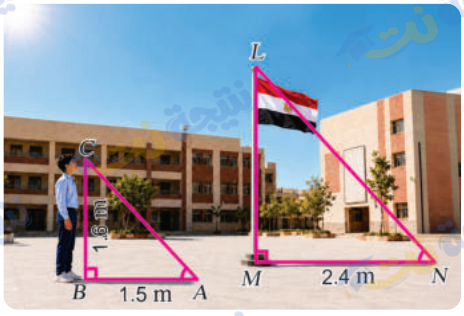
$D \in \overline{AC}$, $\overline{BD} \perp \overline{AC}$

Find the lengths of :

$\overline{AD}, \overline{DC}$

Example 5

A student, 1.6 m tall, used the equality of the sun-ray angles on the ground to find the height of the school flagpole.
If the student's shadow is 1.5 m and the flagpole's shadow is 2.4 m, what is the height of the flagpole?



$\because$ In the two triangles ABC and NML :

$m(\angle A) = m(\angle N)$, $m(\angle B) = m(\angle M) = 90°$

$\therefore \Delta ABC \sim \Delta NML$

From similarity :

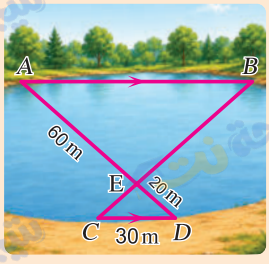
$\frac{AB}{NM} = \frac{BC}{ML}$ $\therefore \frac{1.5}{2.4} = \frac{1.6}{ML}$

$\therefore ML = \frac{2.4 \times 1.6}{1.5} = 2.56$

So the height of the flagpole = 2.56 m

Self-Evaluation 5

In the following figure :



What is the distance between the two trees A and B on opposite sides of the lake?

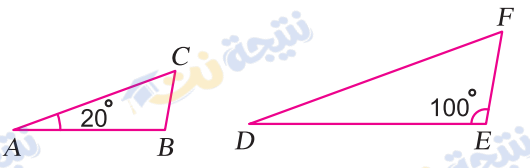
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Lesson Assessment

First **Measuring Conceptual Understanding**

▸ **Choose the correct answer from the given ones:**

(1) In the following figure :

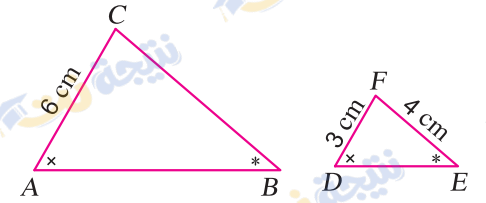


If $\Delta ABC \sim \Delta DEF$, what is the measure of $\angle F$?

(a) 20° (b) 60°

(c) 80° (d) 100°

(2) In the following figure :

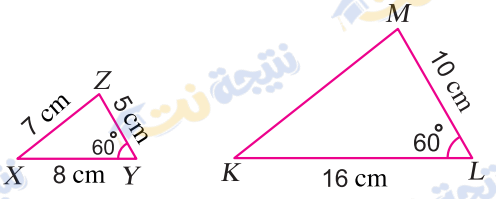


What is the length of $\overline{BC}$?

(a) 4 cm (b) 6 cm

(c) 8 cm (d) 10 cm

(3) In the following figure :

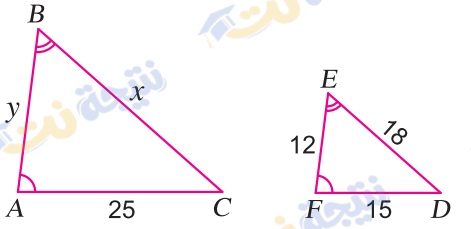


What is the length of $\overline{KM}$?

(a) 7 cm (b) 8 cm

(c) 14 cm (d) 16 cm

(4) In the following figure :

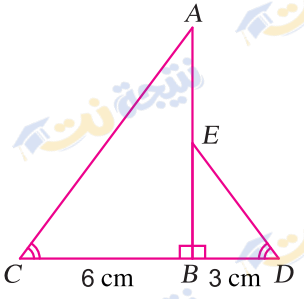


What is the value of $x + y$?

(a) 18 (b) 30

(c) 45 (d) 50

(5) In the following figure :

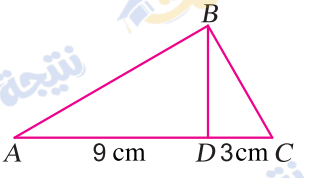


If the perimeter of $\Delta ABC = 24$ cm, what is the perimeter of ΔEBD?

(a) 8 cm (b) 12 cm

(c) 24 cm (d) 48 cm

(6) In the following figure :



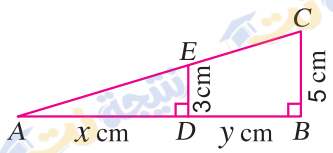
If $\Delta ABC \sim \Delta BDC$, what is the length of $\overline{BC}$?

(a) 6 cm (b) $3\sqrt{3}$ cm

(c) 9 cm (d) $2\sqrt{3}$ cm

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7 In the following figure :

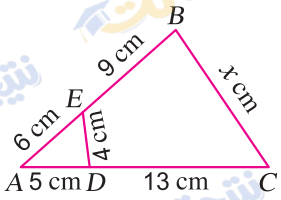


What is the ratio equal to $\frac{x}{y}$?

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$

(c) $\frac{3}{8}$ (d) $\frac{3}{2}$

8 In the following figure :



What is the value of X ?

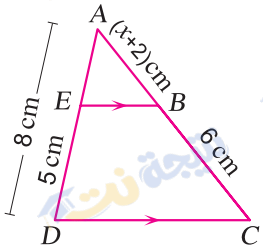
(a) 12 (b) 8

(c) 16 (d) 24

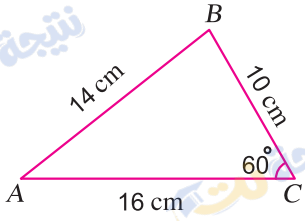
Second Applying Concepts

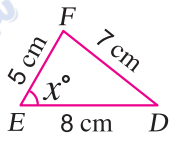
9 In each of the following, prove that the two triangles are similar, then find the value of X :

① ΔADC , ΔAEB

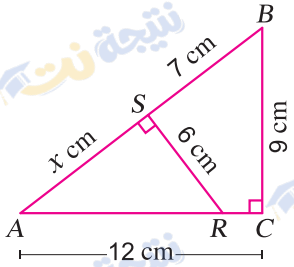


② ΔABC , ΔDFE

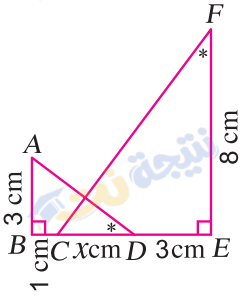




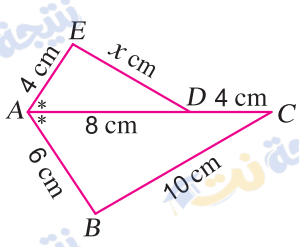
③ ΔABC , ΔARS



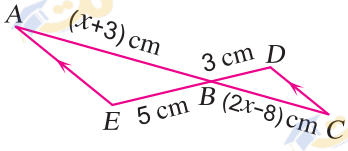
④ ΔABD , ΔCEF



⑤ ΔABC , ΔAED



⑥ ΔAEB , ΔCDB

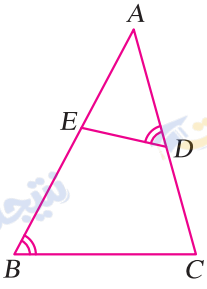


10 In the given figure :

$D \in \overline{AC}$, $E \in \overline{AB}$, $m(\angle ADE) = m(\angle B)$

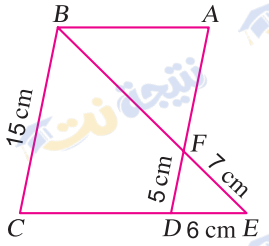
Prove that : $\Delta ABC \sim \Delta ADE$

, then prove that : $AB \times AE = AC \times AD$



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(11) In the following figure :

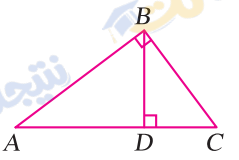


$ABCD$ is a parallelogram, $E \in \overrightarrow{CD}$

Prove that : $\Delta\, ABF \sim \Delta\, DEF$

and find the perimeter of $\Delta\, ABF$

(12) In the following figure :



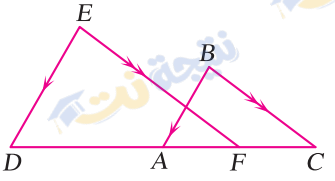
ABC is a triangle , $m(\angle ABC) = 90^\circ$,

$\overline{BD} \perp \overline{AC}$

Prove that : $\Delta\, ADB \sim \Delta\, BDC$

If $BD = 4.8$ cm , $AD = 6.4$ cm , then find the length of $\overline{DC}$

(13) In the following figure :



$\overline{AB} \,//\, \overline{DE}$, $\overline{BC} \,//\, \overline{EF}$,

The points D , A , F , C are collinear

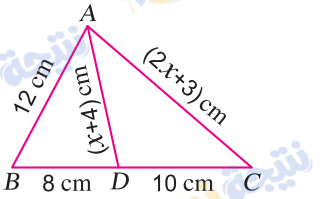
Prove that : $\Delta\, ABC \sim \Delta\, DEF$

If $EF : BC = 3 : 2$

and the perimeter of $\Delta\, ABC = 24$ cm,

then find the perimeter of $\Delta\, DEF$

(14) In the following figure :



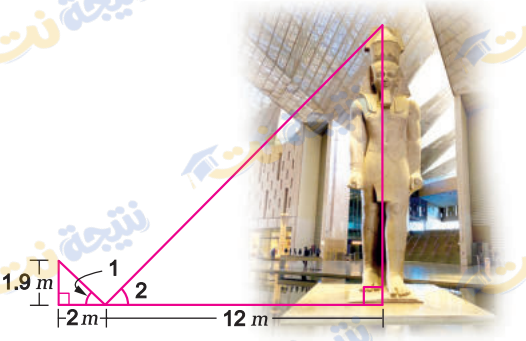
$D \in \overline{BC}$

If $AB = 12$ cm, $BD = 8$ cm, $DC = 10$ cm,

prove that : $\Delta\, ABD \sim \Delta\, CBA$, then find the value of x

Third Analysis and Subjects Integration

(15) **Tourism :** A man wanted to know the height of the statue of King Ramses II in the Grand Egyption Museum. He placed a mirror horizontally on the ground 12 m from the base of the statue and stepped back 2 m until he could see the top of the statue in the mirror. If the man's height is 1.9 m and $m(\angle 1) = m(\angle 2)$, what is the height of the statue?



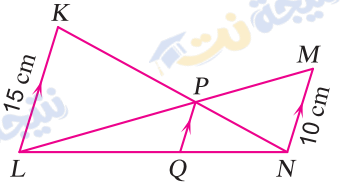
Creative Thinking

(16) In the given figure :

$\overline{KL} \,//\, \overline{PQ} \,//\, \overline{MN}$

If $KL = 15$ cm, $MN = 10$ cm,

find the length of $\overline{PQ}$



***Evaluate** your understanding!*

How well do you understand similarity of triangles?
Tick the right box

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Lesson

3 - 3

Dilation

Learning Outcomes

- Learn the concept of dilation.
- Learn the positive dilation.
- Use coordinates to enlarge or reduce a given figure.

Vocabulary

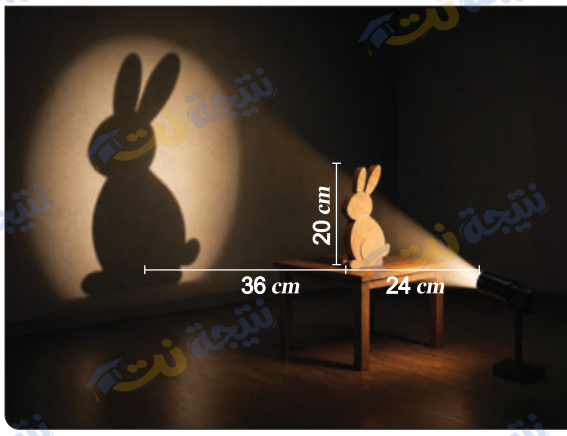
- Dilation
- Center of dilation
- Dilation factor
- Enlargement
- Reduction

Note that

We will study dilation with positive dilation factors only.

Get Ready!

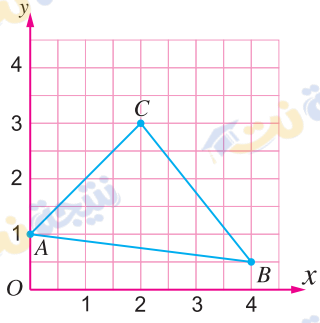
When an opaque object is placed between a light source and a wall, an image of the object is formed on the wall, and its dimensions change as the object moves closer to or farther from the light source.
In the picture, a rabbit-shaped model **20** cm tall is **24** cm from the light source and **36** cm from the wall. What is the height of the rabbit in the image?



In this lesson, you will learn the concept of dilation, which will enable you to solve such real-life problems.

Think & Discuss!

Hani used a geometric drawing program to draw the triangle ABC. If he wants to enlarge it with a ratio of **2 : 1**, which of the following are the coordinates of the vertices of the triangle $A`B`C`$ the image of the triangle ABC?



$A`(0,2)$	$A`(2,3)$
$B`(8,1)$	$B`(6,2.5)$
$C`(4,6)$	$C`(4,5)$

Learn!

Concept of Dilation

- A dilation is a geometric transformation that enlarges or reduces a figure by a specific ratio, which is the ratio of the length of a side in the image to the corresponding side length of the original figure, this ratio is called the dilation factor.
- The figure and its image under dilation are similar, meaning; the measures of corresponding angles remain unchanged, while the lengths of corresponding sides change by a constant ratio.

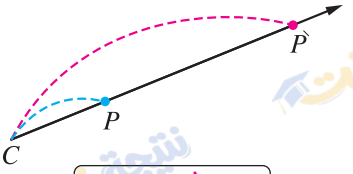
Definition of Dilation

A dilation with center C and dilation factor k is a geometric transformation that maps each point P in the plane to a point $P`$, such that :

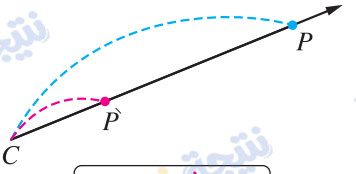
1. The point C maps to itself.
2. If P does not coincide with C, and $\overline{CP}$ is not congruent to $\overline{CP`}$, then $CP` = k\,(CP)$
   The dilation is an enlargement if $k > 1$, and a reduction if $0 < k < 1$

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The point $P`$ is the image of the point P under a dilation with center C and dilation factor k



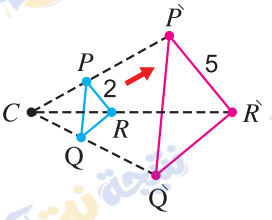
$k = \frac{CP`}{CP} > 1$
Enlargement



$k = \frac{CP`}{CP} < 1$
Reduction

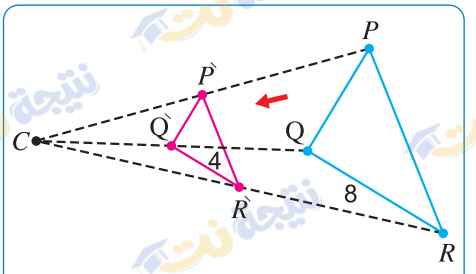
Note that for an enlargement, $P \in \overline{CP`}$, while for a reduction, $P` \in \overline{CP}$

Examples :



$\Delta\ P`R`Q`$ is the image of $\Delta\ PRQ$ under a dilation with center C, dilation factor $k = \frac{5}{2}$

Note : $\Delta\ P`R`Q`$ is an enlargement of $\Delta\ PRQ$



$\Delta\ P`R`Q`$ is the image of $\Delta\ PRQ$ under a dilation with center C, dilation factor $k = \frac{1}{2}$

Note : $\Delta\ P`R`Q`$ is a reduction of $\Delta\ PRQ$

In both figures :

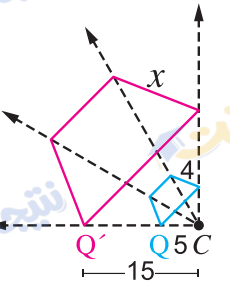
1. $\Delta\ PRQ \sim \Delta\ P`R`Q`$
2. Each side of $\Delta\ P`R`Q`$ is parallel to its corresponding side of $\Delta\ PRQ$

Example 1

In the given figure :

Determine whether the dilation with center C mapping Q to $Q`$ is an enlargement or a reduction.

Then find the dilation factor and the value of x



$\because\ Q \in \overline{CQ`}$

$\therefore$ The dilation mapping Q to $Q`$ is an enlargement with dilation factor k

$\therefore k = \frac{CQ`}{CQ} = \frac{15}{5} = 3$

$\therefore \frac{x}{4} = 3$ $\quad\quad$ $\therefore x = 4 \times 3 = 12$

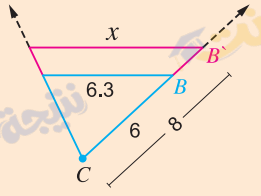
Note that

The dilation factor k equals each of :

- $\frac{\text{A side length in the image}}{\text{corresponding side length in the original}}$
- $\frac{\text{Distance of the image of a point from dilation center}}{\text{Distance of this point from the dilation center}}$

Self-Evaluation (1)

In the following figure :

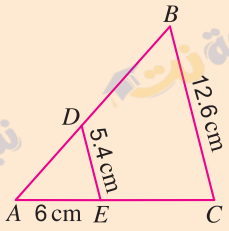


Determine whether the dilation with center C mapping B to $B`$ is an enlargement or a reduction. Then find the dilation factor and the value of x

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Self-Evaluation (2)

In the following figure:



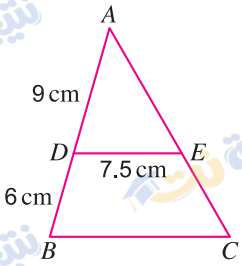
$\Delta\, ABC$ is the image of $\Delta\, ADE$ under a dilation with center A.

Find the dilation factor, and prove that : $\overline{DE} \,//\, \overline{BC}$, and then find the length of $\overline{AC}$

Example (2)

In the given figure:

$\Delta\, ADE$ is the image of $\Delta\, ABC$ under a dilation with center A. Find the dilation factor, and prove that : $\overline{DE} \,//\, \overline{BC}$, and then find the length of $\overline{BC}$



$\because \Delta\, ADE$ is the image of $\Delta\, ABC$ under a dilation with center A

$\therefore D$ is the image of B

$\because \frac{AD}{AB} = \frac{9}{15} = \frac{3}{5}$

$\therefore$ The dilation factor $k = \frac{3}{5}$

$\because \overline{DE}$ is the image of $\overline{BC}$ under dilation $\therefore \overline{DE} \,//\, \overline{BC}$

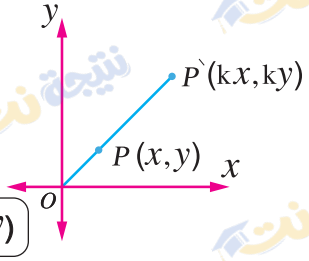
$\because \frac{DE}{BC} = k$ $\therefore \frac{7.5}{BC} = \frac{3}{5}$

$\therefore BC = \frac{5 \times 7.5}{3} = 12.5$ cm

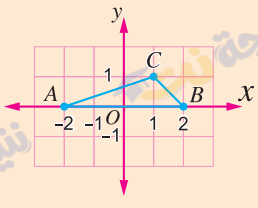
Dilation in the Coordinate Plane

For any point $P(x, y)$ in the coordinate plane, the image of P under a dilation with center at the origin $O(0, 0)$ and dilation factor k is the point $P'(kx, ky)$

The dilation rule : $(x, y) \rightarrow (kx, ky)$



Self-Evaluation (3)



Use the previous coordinate plane to find the image of $\Delta\, ABC$ under a dilation with center O and dilation factor 2

Example (3)

Draw $\Delta\, ABC$ with vertices $A(1, 3)$, $B(2, -1)$, $C(5, 0)$ in the coordinate plane, then draw its image under a dilation with center O and dilation factor 2

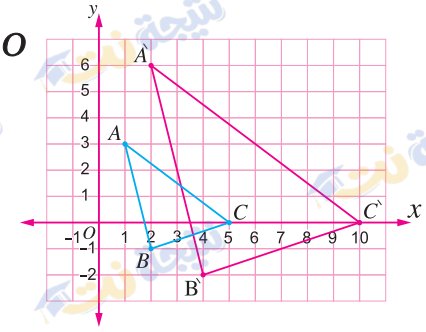
Use the dilation rule in the coordinate plane to find the image of each point of A, B, C under dilation with center O and dilation factor $k = 2$

$(x, y) \rightarrow (2x, 2y)$

$A(1, 3) \rightarrow A'(2, 6)$

$B(2, -1) \rightarrow B'(4, -2)$

$C(5, 0) \rightarrow C'(10, 0)$



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■ **Note from the previous example that**

$\Delta A`B`C`$ is the image of ΔABC under a dilation with dilation factor $= 2$ therefore :

1. $\Delta A`B`C` \sim \Delta ABC$
2. $k > 1$ so $\Delta A`B`C`$ is an enlargement of $\Delta ABC$, and the length of each side of $\Delta A`B`C`$ is twice the length of its corresponding side of ΔABC
3. Each side of $\Delta A`B`C`$ is parallel to its corresponding side of ΔABC

■ **Self-Evaluation (4)**

Draw in the coordinate plane ΔABC with vertices $A(-4, 4)$, $B(4, 0)$, $C(-2, -6)$, then draw its image under a dilation with center O and dilation factor $k = \frac{1}{2}$

Example 4

Draw in the coordinate plane polygon $ABCD$ with vertices $A(-3, 6)$, $B(0, 6)$, $C(3, 3)$, $D(-3, -3)$, then draw its image under a dilation with center O and dilation factor $k = \frac{1}{3}$

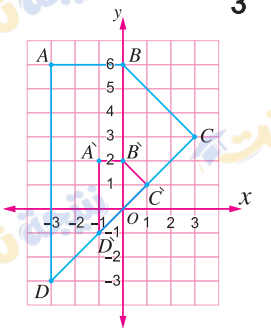
$(x, y) \longrightarrow (\frac{1}{3}x, \frac{1}{3}y)$

$A(-3, 6) \longrightarrow A`(-1, 2)$

$B(0, 6) \longrightarrow B`(0, 2)$

$C(3, 3) \longrightarrow C`(1, 1)$

$D(-3, -3) \longrightarrow D`(-1, -1)$



■ **Note from the previous example that**

$0 < k < 1$, so polygon $A`B`C`D`$ is a reduction of polygon $ABCD$

Lesson Assessment



First **Measuring Conceptual Understanding**

▸ **Choose the correct answer from the given ones:**

(1) In the following figure :

If $P`$ is the image of P under a dilation with center C, what is the dilation factor ?

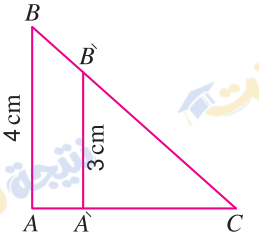


(a) $\frac{1}{3}$ (b) 3

(c) $\frac{1}{4}$ (d) 4

(2) In the following figure :

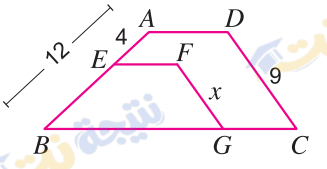
If $\overline{A`B`}$ is the image of $\overline{AB}$ under a dilation with center C, what is the dilation factor ?



(a) $\frac{3}{4}$ (b) $\frac{4}{3}$

(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

(3) If polygon $EBGF$ is the image of polygon $ABCD$ under a dilation with center B and dilation factor k, then .....



(a) $k = \frac{1}{3}, x = 3$ (b) $k = \frac{2}{3}, x = 6$

(c) $k = \frac{1}{3}, x = 6$ (d) $k = \frac{2}{3}, x = 3$

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4 What is the image of the point $P(-1, 3)$ under a dilation with center at the origin and dilation factor 4 ?

(a) $P'(3, 7)$ (b) $P'(-4, 7)$ (c) $P'(-4, 12)$ (d) $P'(-4, 3)$

5 What ratio expresses the dilation factor of a dilation with center O mapping the point $A(6, -2)$ to the point $B(3, -1)$?

(a) 2 (b) 3 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

6 If A' is the image of A under a dilation with center at the origin and dilation factor $\frac{3}{4}$, what is the dilation factor that makes A the image of A' with center at the origin?

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

7 If $A'(12, -6)$ is the image of A under a dilation with center at the origin and its rule is $(x, y) \longrightarrow (\frac{3}{2}x, \frac{3}{2}y)$, then the point A is .....

(a) $(8, -4)$ (b) $(18, -9)$ (c) $(-8, 4)$ (d) $(-18, 9)$

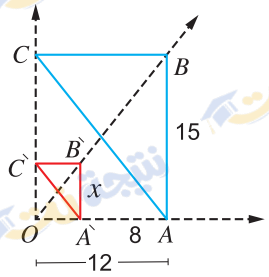
8 Which of the following ratios represents a dilation factor for reduction of a figure ?

(a) 120 % (b) $\frac{5}{4}$ (c) 2 (d) 50 %

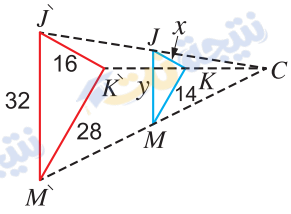
Second Applying Concepts

9 In each of the following figures, find the dilation factor, state whether it is an enlargement or a reduction, and find the value of each of x , y :

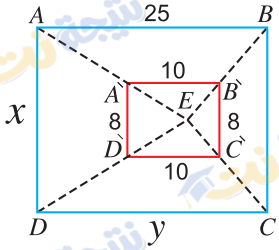
1 Dilation with center O makes $\Delta A'B'C'$ the image of ΔABC



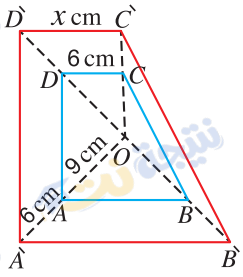
2 Dilation with center C makes $\Delta J'K'M'$ the image of ΔJKM



3 Dilation with center E makes polygon $A'B'C'D'$ the image of polygon $ABCD$



4 Dilation with center O makes polygon $A'B'C'D'$ the image of polygon $ABCD$



10 In each of the following, draw in the coordinate plane the polygon and its image under a dilation with center O and dilation factor k :

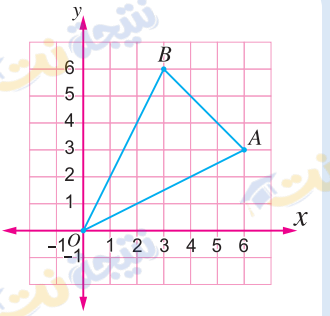
1 $A(2, -1)$, $B(0, 3)$, $C(-2, 1)$, $k = 2$

2 $A(-2, -4)$, $B(2, 0)$, $C(-4, 2)$, $D(-6, -2)$, $k = 0.5$

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(11) In the given figure :

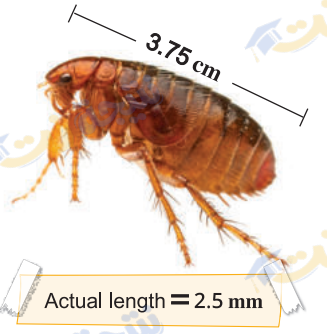
Draw the image of $\Delta\ OAB$ under a dilation with center O and rule $(x, y) \longrightarrow (\frac{1}{3}x, \frac{1}{3}y)$



Third Analysis and Subjects Integration

(12) **Science :**

Under a microscope, the length of an insect appears as shown in the picture. If the actual length of the insect is known, what is the dilation factor used ? Explain your reasoning.

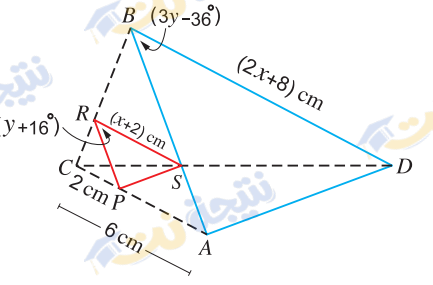


Creative Thinking

(13) A rectangle has dimensions x cm and y cm. What is the area of its image under a dilation with dilation factor $\frac{1}{3}$ in terms of x and y ?

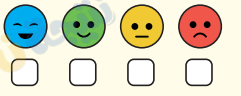
(14) In the given figure :

If $\Delta\ PSR$ is the image of $\Delta\ ADB$ under a dilation with center C, find the value of each of x and y



***Evaluate** your understanding!*

How well do you understand dilation?
Tick the right box



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Lesson

3 - 4

Trigonometric Ratios of an Acute Angle

Learning Outcomes

- Learn the trigonometric ratios of an acute angle in a right-angled triangle.
- Find the sine of an angle (sin).
- Find the cosine of an angle (cos).
- Find the tangent of an angle (tan).
- Use the trigonometric ratios of an acute angle to find the side lengths of a right-angled triangle.
- Use the trigonometric ratios of an acute angle to solve some real-life problems.

Vocabulary

- Sine
- Cosine
- Tangent

Notation in Mathematics

- Some Greek letters are used to symbolize angles such as : "θ" which is read as "theta" , "β" which is read as "beta"
- "Angle θ" is shorthand for "an angle whose measure is θ".

Note that

From the adjacent definition :

$$\sin\theta \div \cos\theta = \frac{a}{c} \div \frac{b}{c} = \frac{a}{c} \times \frac{c}{b} = \frac{a}{b} = \tan\theta$$

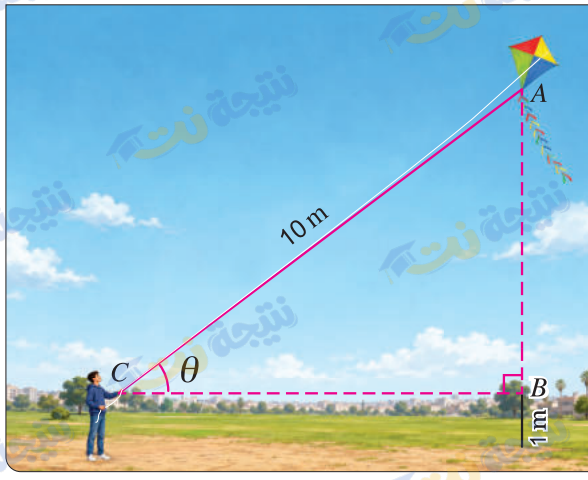
That is

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Get Ready!

The kite is one of the oldest inventions in China, used more than two thousand years ago for games and celebrations.

In the given figure: If $\frac{AB}{AC} = \frac{3}{5}$ and the string length is 10 meters, what is the height of the kite above the ground ?



In this lesson, you will learn the trigonometric ratios of an acute angle in a right-angled triangle and their use in calculating side lengths, which will enable you to solve such real-life problems.

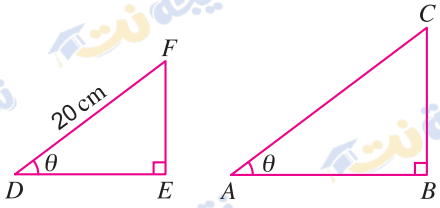
Think & Discuss!

In the given figure :

$AB : CB = 4 : 3$

Which of the following is the length of $\overline{FE}$?

9 cm 12 cm 16 cm



Learn!

Trigonometric Ratios

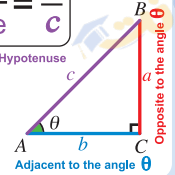
A trigonometric ratio is the ratio between the lengths of two sides of a right-angled triangle.

- Sine of an angle θ ($sin\,\theta$)
- Cosine of an angle θ ($cos\,\theta$)
- Tangent of an angle θ ($tan\,\theta$)

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

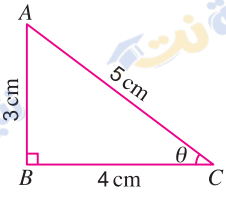
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$



For example : In the given figure :

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5} \quad , \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5}$$

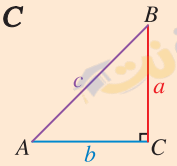
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$$



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Remember
Pythagoras Theorem

If ABC is a right-angled triangle at C

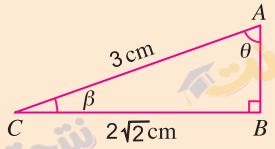


then $c^2 = a^2 + b^2$

Self-Evaluation (1)

In the following figure :

ABC is a right-angled triangle at B

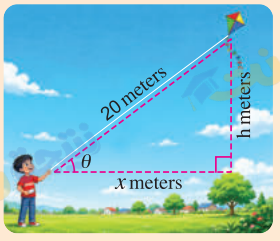


Find the trigonometric ratios for each of the two angles θ , β

Self-Evaluation (2)

In the following figure :

A kite is tied with a string 20 meters long, inclined at the horizontal with an angle θ such that $\sin\theta = \frac{3}{5}$

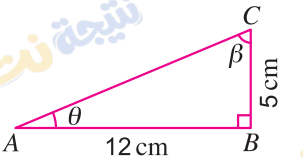


Find the value of each of h and x

Example (1)

In the given figure :

Find the trigonometric ratios for each of the two angles θ , β

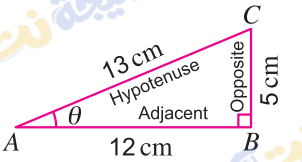


$\because m(\angle B) = 90°$

$\therefore (AC)^2 = (AB)^2 + (CB)^2$

$\therefore (AC)^2 = (12)^2 + (5)^2 = 169$

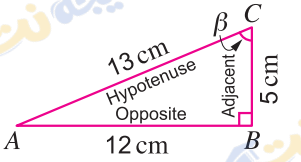
$\therefore AC = \sqrt{169} = 13$ cm.



$\therefore$ The trigonometric ratios for θ :

$\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$

and the trigonometric ratios for β :



$\sin\beta = \frac{12}{13}$, $\cos\beta = \frac{5}{13}$, $\tan\beta = \frac{12}{5}$

Note that

In Example (1) :

$\sin\theta = \cos\beta = \frac{5}{13}$, $\cos\theta = \sin\beta = \frac{12}{13}$

Generally :

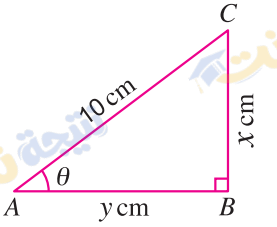
For any two complementary angles θ and β : $\sin\theta = \cos\beta$, $\cos\theta = \sin\beta$

Example (2)

In the given figure :

If $\sin\theta = \frac{3}{5}$, find the value of each of x and y

then find the value of : $\sin^2\theta + \cos^2\theta - \tan\theta$



$\because \sin\theta = \frac{3}{5}$, $\sin\theta = \frac{CB}{AC} = \frac{x}{10}$

$\therefore \frac{x}{10} = \frac{3}{5}$ $\therefore x = \frac{10 \times 3}{5} = 6$

Using Pythagoras' Theorem :

$AB = \sqrt{(AC)^2 - (BC)^2}$ $\therefore AB = \sqrt{10^2 - 6^2} = 8$ $\therefore y = 8$

$\because \cos\theta = \frac{AB}{AC}$ $\therefore \cos\theta = \frac{8}{10} = \frac{4}{5}$

$\because \tan\theta = \frac{BC}{BA}$ $\therefore \tan\theta = \frac{6}{8} = \frac{3}{4}$

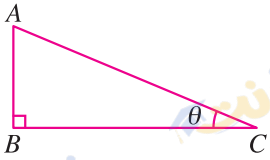
$\therefore \sin^2\theta + \cos^2\theta - \tan\theta = (\frac{3}{5})^2 + (\frac{4}{5})^2 - \frac{3}{4} = \frac{1}{4}$

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Finding the Trigonometric Ratios of an Acute Angle Given One of its trigonometric Ratios

Example 3

In the given figure :



If $\cos\theta = \frac{12}{13}$, find : $\sin\theta$, $\tan\theta$

If the length of $\overline{BC} = 24$ cm, then find the length of each of : $\overline{AB}$, $\overline{AC}$

$\because \cos\theta = \frac{12}{13}$ $\quad\therefore \frac{BC}{AC} = \frac{12}{13}$

Assuming :

$BC = 12k$, $AC = 13k$

Using Pythagoras' Theorem :

$\therefore AB = \sqrt{(AC)^2 - (BC)^2} = \sqrt{(13k)^2 - (12k)^2}$

$\therefore AB = \sqrt{169k^2 - 144k^2} = \sqrt{25k^2} = 5k$

$\therefore \sin\theta = \frac{AB}{AC} = \frac{5k}{13k} = \frac{5}{13}$, $\tan\theta = \frac{AB}{BC} = \frac{5k}{12k} = \frac{5}{12}$

At $BC = 24$ cm :

$\therefore \cos\theta = \frac{BC}{AC}$ $\quad\therefore \frac{24}{AC} = \frac{12}{13}$ $\quad\therefore AC = \frac{24 \times 13}{12} = 26$ cm

$\because \tan\theta = \frac{AB}{BC}$ $\quad\therefore \frac{AB}{24} = \frac{5}{12}$

$\therefore AB = \frac{24 \times 5}{12} = 10$ cm

Diversified Strategies

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$

Self-Evaluation (3)

In the following figure :



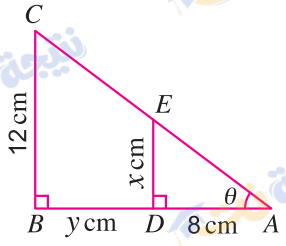
If $\tan\theta = \frac{24}{7}$,

find : $\sin\theta$, $\cos\theta$

If the length of $\overline{AC} = 12.5$ cm, then find the length of each of : $\overline{AB}$, $\overline{BC}$

Example 4

In the given figure :



If $\tan\theta = \frac{3}{4}$

, then find the value of each of : x , y

$\because$ In ΔADE : $m(\angle ADE) = 90°$

$\therefore \tan\theta = \frac{ED}{AD}$ $\quad, \because \tan\theta = \frac{3}{4}$

$\therefore \frac{x}{8} = \frac{3}{4}$ $\quad\therefore x = \frac{8 \times 3}{4} = 6$

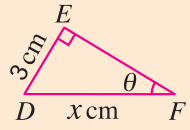
$\because$ In ΔABC : $m(\angle B) = 90°$

$\therefore \tan\theta = \frac{CB}{AB}$ $\quad\therefore \frac{12}{y+8} = \frac{3}{4}$

$\therefore y + 8 = \frac{4 \times 12}{3} = 16$ $\quad\therefore y = 16 - 8 = 8$

Self-Evaluation (4)

In the following figure:





If $\sin\theta = \frac{1}{2}$

, then find the value of each of : x , y

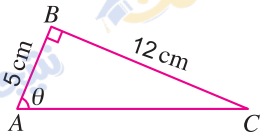
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Lesson Assessment

First — Measuring Conceptual Understanding

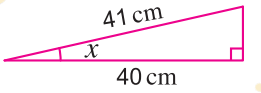
▸ **Choose the correct answer from the given ones:**

1. In the given figure : What is the value of $\sin\theta$?



(a) $\frac{5}{12}$ (b) $\frac{12}{13}$ (c) $\frac{5}{13}$ (d) $\frac{12}{5}$

2. In the given figure : What is the value of $\tan X$?



(a) $\frac{40}{41}$ (b) $\frac{9}{41}$ (c) $\frac{9}{40}$ (d) $\frac{40}{9}$

3. If $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$, what is the value of $\tan\theta$?

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

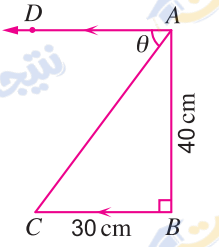
4. If A , B are the measures of two complementary angles and $\sin A = \frac{12}{13}$, what is the value of $\cos B$?

(a) $\frac{12}{13}$ (b) $\frac{5}{13}$ (c) $\frac{5}{12}$ (d) $\frac{13}{12}$

5. ABC is a right-angled triangle at B , if $26\,AB = 10\,AC$, what is the value of $\tan C$?

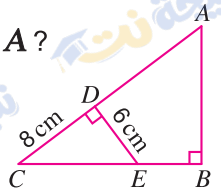
(a) $\frac{12}{5}$ (b) $\frac{5}{12}$ (c) $\frac{12}{13}$ (d) $\frac{5}{13}$

6. In the given figure : What is the value of $\cos\theta$?



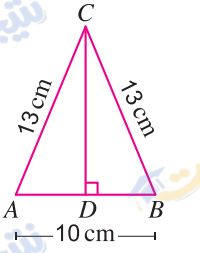
(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

7. In the given figure : What is the value of $\sin A$?



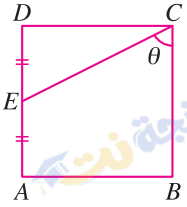
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5}$

8. In the given figure : What is the value of $\sin B$?



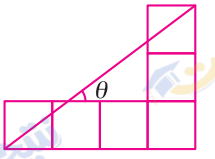
(a) $\frac{5}{13}$ (b) $\frac{12}{13}$ (c) $\frac{5}{12}$ (d) $\frac{12}{5}$

9. In the following figure : $ABCD$ is a square, E is the midpoint of $\overline{AD}$, what is the value of $\tan\theta$?



(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

10. In the following figure : 6 congruent squares are shown, what is the value of $\sin\theta$?



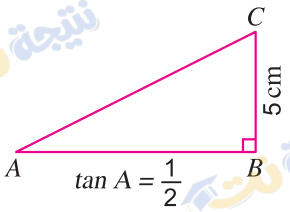
(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{3}{5}$

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▸ **Spot the mistake:**

(11) Wael and Sarah calculated the lengths of $\overline{AB}$ and $\overline{AC}$

Their answers are as follows. Which one is correct ?



Wael's answer:

$\because \tan A = \frac{CB}{AB}$ $\quad \therefore \frac{5}{AB} = \frac{1}{2}$

$\therefore AB = 2 \times 5 = 10$ cm

$\therefore AC = \sqrt{(5)^2 + (10)^2} = 5\sqrt{5}$ cm

Sarah's answer:

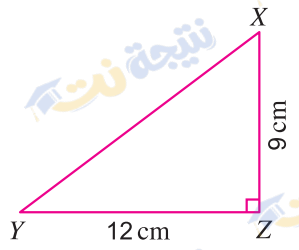
$\because \tan A = \frac{AB}{BC}$ $\quad \therefore \frac{AB}{5} = \frac{1}{2}$

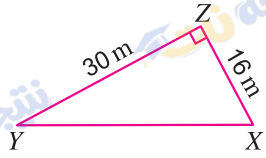
$\therefore AB = \frac{5}{2} = 2.5$ cm

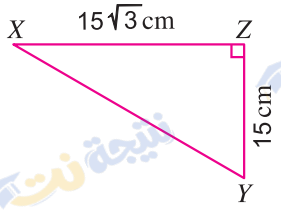
$\therefore AC = \sqrt{(5)^2 + (2.5)^2} = 2.5\sqrt{5}$ cm

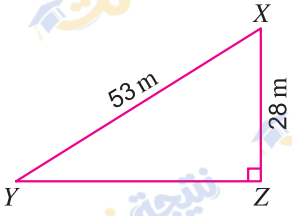
Second Applying Concepts

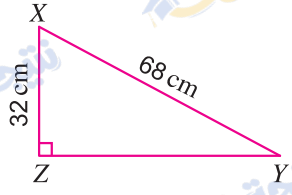
(12) In each of the following, find : $\sin X$, $\cos X$, $\tan X$, $\sin Y$, $\cos Y$, $\tan Y$:

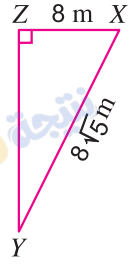
① 

② 

③ 

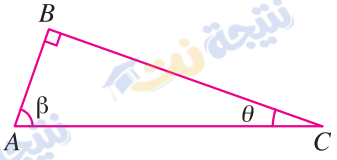
④ 

⑤ 

⑥ 

(13) In the given figure :

If $\sin \theta = \frac{1}{3}$, find :

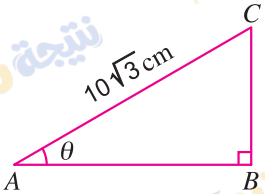


① $\cos \theta$ ② $\tan \theta$

③ $\cos \beta$ ④ $\tan \beta$

(14) In the given figure :

If $\tan \theta = \frac{1}{\sqrt{3}}$, find the length of each of $\overline{AB}$, $\overline{BC}$



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(15) In the given figure :

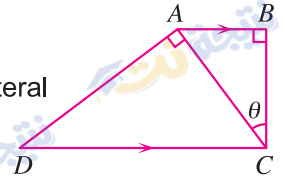
$ABCD$ is a quadrilateral

in which $\overline{AB} \parallel \overline{DC}$,

$m(\angle B) = m(\angle CAD) = 90°$,

the length of $\overline{AD} = 16$ cm.

If $\tan\theta = \frac{3}{4}$, find the length of $\overline{DC}$



(16) In the given figure :

ABC is a triangle in

which $m(\angle B) = 90°$,

the length of $\overline{AB} = 15$ cm

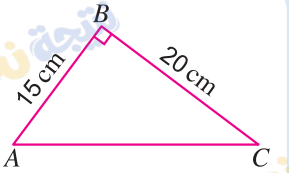
and the length of $\overline{BC} = 20$ cm.

Prove each of the following:

① $\tan A \times \tan C = 1$

② $\sin^2 A + \sin^2 C = 1$

③ $\cos A \cos C - \sin A \sin C = 0$



Third Analysis and Subjects Integration

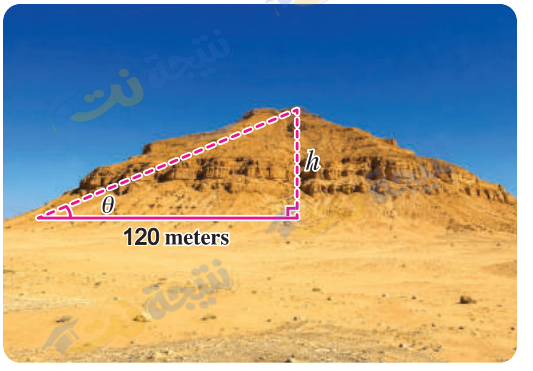
(17) In the following figure :

Find each of : $\sin\theta$, $\cos\theta$, $\tan\theta$



(18) In the following figure :

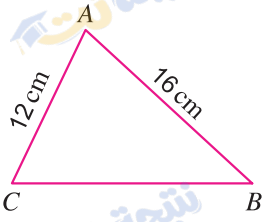
If $\sin\theta = \frac{5}{13}$, find the height of the mountain (h) in meters.



Creative Thinking

(19) In the given figure :

If ABC is an acute-angled triangle, find the value of : $\frac{\sin C}{\sin B}$



Evaluate *your understanding!*

How well do you understand trigonometric ratios of an acute angle?
Tick the right box



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Lesson

3 - 5

Trigonometric Ratios of Some Special Angles

Learning Outcomes

- Learn the trigonometric ratios of special angles **(30° , 60° , 45°)**.
- Use a calculator to find the trigonometric ratios of an angle given its measure.
- Find the measure of an angle if one of its trigonometric ratios is given using a calculator.
- Use trigonometric ratios to solve real-life problems.

Vocabulary

- Special angles

Get Ready!

Ancient Egyptian obelisks are tall stone towers decorating temples. Their square base supports a small pyramidal top. They were the first sundials, as the ancient Egyptians observed the shadow cast by the obelisk on the ground, which moves and changes length and direction, indicating parts of the day.

If the sun's rays make a **45°** angle with the ground, what is the length of the shadow of an obelisk **40** m high?

In this lesson, you will learn how to calculate trigonometric ratios, which will enable you to solve such real-life problems.



One of the Egyptian obelisks

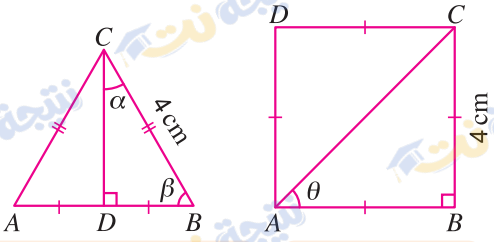
Think & Discuss!

In the given figure :

What is the value of θ in the square?

What are the values of α and β in the triangle?

Can you identify the trigonometric ratios of each of θ , β and α ?



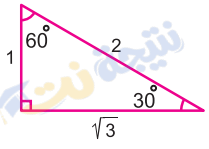
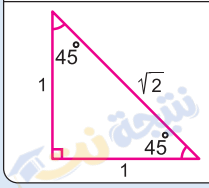
ABC is an equilateral triangle

$ABCD$ is a square

Learn!

Trigonometric Ratios of Some Special Angles

Use a right-angled triangle with angles **(30° , 60° , 90°)** , and another with angles **(45° ,45° ,90°)** to determine the trigonometric ratios of the special angles **30°** , **60°** , **45°** as follows :

Triangle	Sine (sin)	Cosine (cos)	Tangent (tan)
	$\sin 30° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$	$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
	$\sin 60° = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{1}{2}$	$\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$
	$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\tan 45° = \frac{1}{1} = 1$

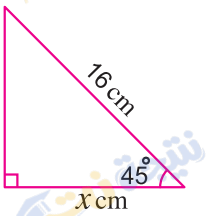
Think with your classmates

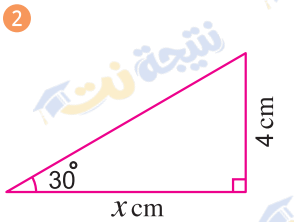
- In a **30°-60°-90°** triangle, if the length of the side opposite to **30°** is **1** cm, how can you determine the lengths of the hypotenuse and the side opposite to **60°** ?
- In a right-angled triangle with one acute angle **45°** , if the length of one leg is **1** cm, how can you determine the lengths of the other two sides?

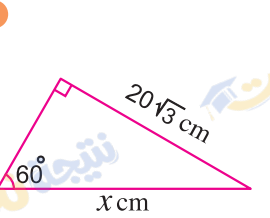
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Example 1

Find the value of x in each of the following figures :

1 

2 

3 

1 $\because \cos 45° = \frac{x}{16}$

$\therefore \frac{1}{\sqrt{2}} = \frac{x}{16}$

$\therefore x = \frac{16}{\sqrt{2}} = 8\sqrt{2}$

2 $\because \tan 30° = \frac{4}{x}$

$\therefore \frac{1}{\sqrt{3}} = \frac{4}{x}$

$\therefore x = 4\sqrt{3}$

3 $\because \sin 60° = \frac{20\sqrt{3}}{x}$

$\therefore \frac{\sqrt{3}}{2} = \frac{20\sqrt{3}}{x}$

$\therefore x = 40$

Using a Calculator to Find Trigonometric Ratios

Finding the trigonometric ratios of any angle , given its measure , using a calculator, with the keys :

sin , cos , tan

Example 2

Using a calculator, find each of the following, rounded to four decimal places :

1 $\sin 42°$ 2 $\cos 15° 17'$ 3 $\tan 74° 12' 36''$

1 sin 4 2 =

$\therefore \sin 42° \approx 0.6691$

2 cos 1 5 °’’’ 1 7 °’’’ =

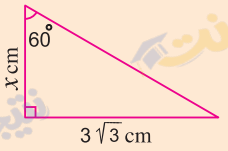
$\therefore \cos 15° 17' \approx 0.9646$

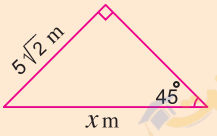
3 tan 7 4 °’’’ 1 2 °’’’ 3 6 °’’’ =

$\therefore \tan 74° 12' 36'' \approx 3.5363$

■ Self-Evaluation (1)

Find the value of x in each of the following figures:

1 

2 

■ Think with your classmates

When should you use *sin* , *cos* or *tan* to find an unknown side length in a right-angled triangle?

■ Remember that

Angle measurement units are: degrees, minutes, seconds , where :

- 1 degree = 60 minutes ($1° = 60'$)
- 1 minute = 60 seconds ($1' = 60''$)

■ Self-Evaluation (2)

Using a calculator, find each of the following, rounded to four decimal places:

1 $\tan 12°$

2 $\sin 32° 14'$

3 $\cos 47° 15' 39''$

■ Think

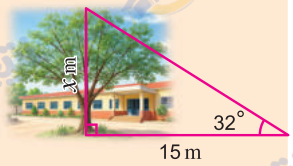
Can you use a calculator to find the trigonometric ratios of 0° and 90° ?

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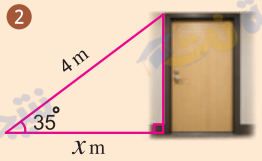
Self-Evaluation (3)

Find the value of x to the nearest two decimal places in each of the following :

1



2



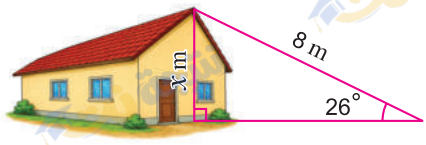
Example 3

Find the value of x to the nearest two decimal places in each of the following :

1



2



1 $\because \tan 54° = \frac{6}{x}$

$\therefore x = \frac{6}{\tan 54°} \approx 4.36$

2 $\because \sin 26° = \frac{x}{8}$

$\therefore x = 8 \sin 26° \approx 3.51$

Finding an Angle Measure Given One of its Trigonometric Ratios

If θ is an acute angle and :

- $\sin \theta = x$, then $\theta = \sin^{-1}(x)$
- $\cos \theta = x$, then $\theta = \cos^{-1}(x)$
- $\tan \theta = x$, then $\theta = \tan^{-1}(x)$

For example : If $\sin \theta = 0.834$, then θ is the angle whose sine is **0.834**

To find the measure of this angle, use the calculator keys in the following sequence :

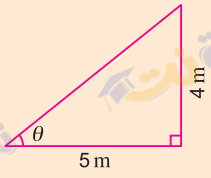


You find the measure of this angle approximately equal **56° 30′ 43″**

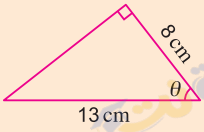
Self-Evaluation (4)

Find the value of θ in each of the following :

1



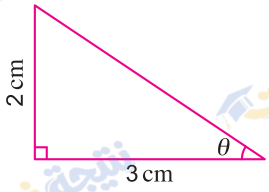
2



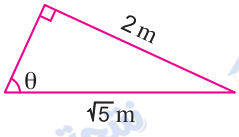
Example 4

Find the value of θ in each of the following :

1



2



1 $\because \tan \theta = \frac{2}{3}$

$\therefore \theta = \tan^{-1}\left(\frac{2}{3}\right)$

$\therefore \theta \approx 33° 41′ 24″$

2 $\because \sin \theta = \frac{2}{\sqrt{5}}$

$\therefore \theta = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

$\therefore \theta \approx 63° 26′ 6″$

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■ Self-Evaluation (5)

Find the value of θ if :

$\sqrt{3}\ sin(\theta - 16°) = sin\ 60°$

where $(\theta - 16°)$ is an acute angle.

■ Self-Evaluation (6)

Find the value of θ in each of the following, where θ is an acute angle.

1. $sin\ 75° = cos\ \theta$
2. $sin(\theta - 20°) = cos(\theta + 20°)$

■ Note that

For any two acute angles θ and β , if $sin\ \theta = cos\ \beta$, then $\theta + \beta = 90°$

Example (5)

Find the value of θ if :

$2\ cos(\theta + 20°) = tan\ 45°$, where $(\theta + 20°)$ is an acute angle.

$\because\ 2\ cos(\theta + 20°) = tan\ 45°$

$\therefore\ 2\ cos(\theta + 20°) = 1$ $\qquad$ $\therefore\ cos(\theta + 20°) = \frac{1}{2}$

$\therefore\ \theta + 20° = 60°$ $\qquad$ $\therefore\ \theta = 40°$

Example (6)

In ΔABC , if $m(\angle B) = 72°$, $sin\ A = cos\ 2A$, where $(2A)$ is an acute angle, **find :** $m(\angle C)$

$\because\ sin\ A = cos\ 2A$ $\qquad$ $\therefore\ A + 2A = 90°$

$\therefore\ 3A = 90°$ $\qquad$ $\therefore\ A = 30°$

$\therefore\ m(\angle C) = 180° - (72° + 30°) = 78°$

Lesson Assessment

First — Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

1. If $sin\ 40° = cos\ y$, where y is the measure of an acute angle, what is the value of y ?

   (a) 30° (b) 40° (c) 50° (d) 60°

2. What is the value of $(sin\ 30°)^2 + (cos\ 30°)^2$?

   (a) $sin\ 60°$ (b) $sin\ 45°$

   (c) $tan\ 45°$ (d) $cos\ 60°$

3. If ABC is an isosceles right-angled triangle at C , what is the value of $tan\ B$?

   (a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{3}}$

4. In an acute-angled triangle ABC , if $m(\angle A) = 70°$, $cos\ B = sin\ B$, what is m $(\angle C)$?

   (a) 65° (b) 70° (c) 40° (d) 30°

5. If θ is an acute angle and $sin(\theta - 10°) = \frac{1}{\sqrt{2}}$, what is the value of θ ?

   (a) 65° (b) 55° (c) 45° (d) 35°

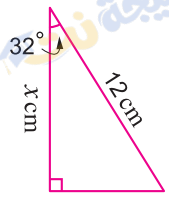
6. If $sin\ 4\theta = \frac{\sqrt{3}}{2}$, where 4θ is an acute angle, what is the value of θ ?

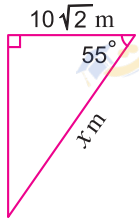
   (a) 15° (b) 30° (c) 45° (d) 60°

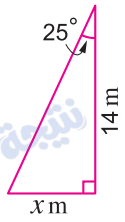
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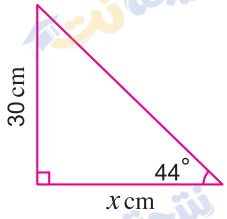
Second Applying Concepts

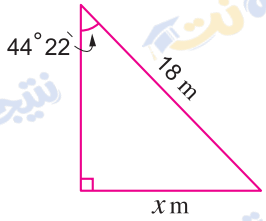
7 Find the value of x to the nearest two decimal places in each of the following :

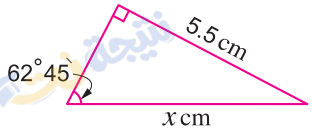
① 

② 

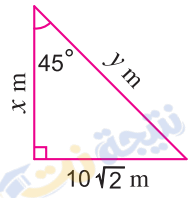
③ 

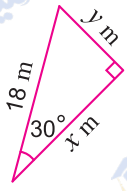
④ 

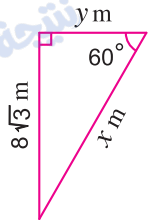
⑤ 

⑥ 

8 Find the value of each of x and y in each of the following :

① 

② 

③ 

9 Find the value of x in each of the following :

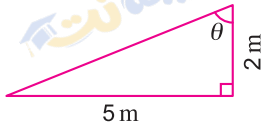
① $x \tan 45° \cos 30° = \tan 60°$ ② $x \sin 45° \cos 45° \tan 60° = (\tan 45°)^2 - (\cos 60°)^2$

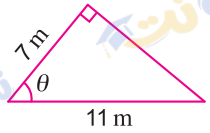
10 Find the value of θ in each of the following :

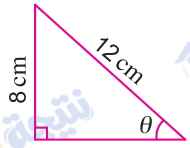
① $\cos 4\theta = \frac{1}{2}$, where 4θ is an acute angle. ② $\tan 3\theta = \sqrt{3}$, where 3θ is an acute angle.

③ $\frac{\cos 4\theta}{\sin 2\theta} = 1$, where 4θ is an acute angle.

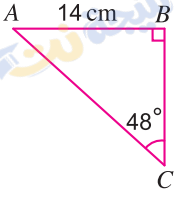
11 Find the value of θ in each of the following :

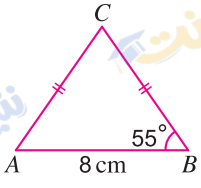
① 

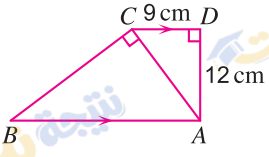
② 

③ 

12 Find the perimeter of ΔABC to the nearest two decimal places in each of the following :

① 

② 

③ 

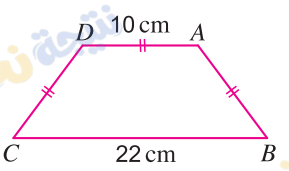
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(13) In the given figure :

$ABCD$ is a trapezium , $AB = AD = DC = 10$ cm, $BC = 22$ cm

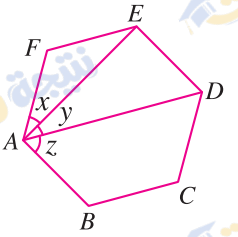
Find $m(\angle C)$ to the nearest minute,

then find the area of the trapezium $ABCD$



(14) In the given figure :

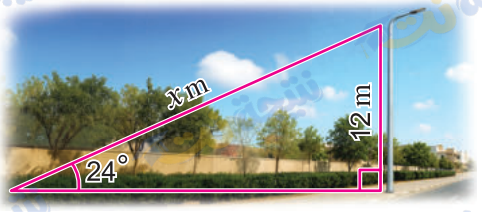
If $ABCDEF$ is a regular hexagon, find : $2 \sin x + 4 \sin y - 6 \cos z$



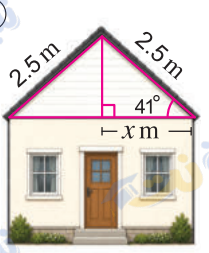
Third **Analysis and Subjects Integration**

(15) Find the value of x in each of the following to the nearest one decimal :

(1)



(2)



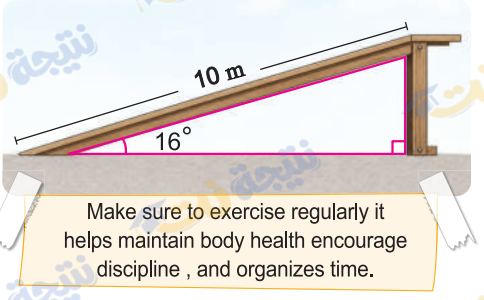
(3)



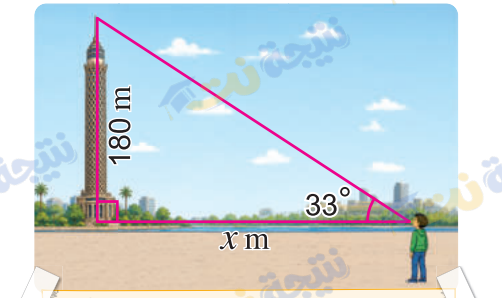
(16) **Sports :**

In the given figure :

When building a bike ramp, the inclination angle measures **16°** . If the ramp length is **10** m, what is its height?



Make sure to exercise regularly it helps maintain body health encourage discipline , and organizes time.



Cairo Tower is one of the major tourist landmarks, built from reinforced concrete in the shape of a lotus flower.

(17) **Tourism :**

In the given figure :

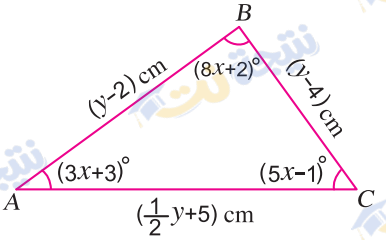
If the height of Cairo Tower is **180** m, and Ahmed looks at the top of the tower at an angle of measure **33°** , what is the distance between Ahmed and the base of the tower?

Creative Thinking

(18) In the given figure :

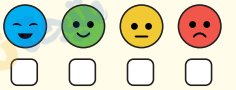
If $\sin 54° \approx 0.8$,

find the perimeter of ΔABC



***Evaluate** your understanding!*

How well do you understand the trigonometric ratios of some special angles? Tick the right box



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Lesson

3 - 6

The Relation between the Slopes of Two Parallel and Perpendicular Straight Lines

Learning Outcomes

- Deduce the slope of a straight line using the tangent of its inclination angle.
- Learn the relation between the slopes of parallel straight lines.
- Learn the relation between the slopes of perpendicular straight lines.
- Solve real-life problems using the slope of a straight line.

Vocabulary

- Slope of a straight line
- Two parallel straight lines
- Two perpendicular straight lines

Note that

The slope (m) of the straight line passing through the two points (x_1 , y_1) , (x_2 , y_2) is given by: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Get Ready!



On October 6th every year, Egypt celebrates the glorious October victories with spectacular airshows.
In one of the displays, airplanes fly along parallel paths.
If an airplane follows a path passing through the two points $(100\sqrt{3} , 201)$, $(300\sqrt{3} , 401)$ and another airplane flies along a path making a positive angle θ with the positive direction of x-axis, find the measure of θ

In this lesson, you will learn the relation between the slopes of two parallel straight lines, which will enable you to solve such real-life problems.

Think & Discuss!

If Magdy draws two straight lines ℓ_1 , ℓ_2 where ℓ_1 passes through $(7 , 5)$ and $(6 , -1)$ and ℓ_2 passes through $(-4 , 4)$ and $(2 , 3)$, which of the following statements is correct ?

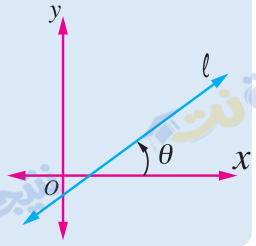
$\ell_1 // \ell_2$ | $\ell_1 \perp \ell_2$ | ℓ_1 , ℓ_2 intersect and they are not perpendicular

Learn!

Inclination Angle of a Straight Line

The inclination angle of a straight line is the angle that the straight line makes with the positive direction of x-axis, measured anti-clockwise from the positive direction of x-axis.

The inclination angle of the straight line $\ell = \theta$



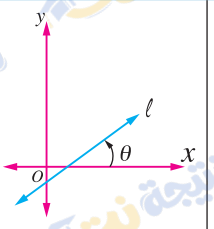
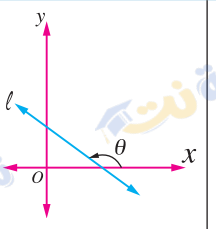
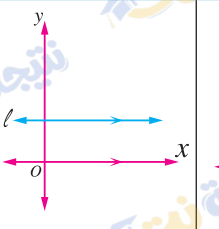
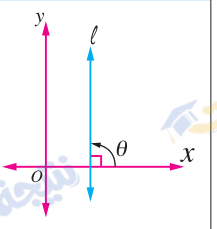
Slope of a Straight Line

The slope of a straight line is the tangent of the inclination angle of the straight line. $m = \tan \theta$

For example : The slope of a straight line with inclination angle of measure $45°$ is tan $45°$ That is $m = \tan 45° = 1$

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Note :

Slope is positive	Slope is negative	Slope is zero	Slope is undefined
			
$0° < \theta < 90°$ $\tan\theta > 0$	$90° < \theta < 180°$ $\tan\theta < 0$	$\theta = 0°$ $\tan\theta = 0$	$\theta = 90°$ $\tan\theta$ is undefined

For example:

- The slope of the straight line whose angle of inclination measures 60° is $\tan 60° = \sqrt{3}$
- The slope of the straight line whose angle of inclination measures 150° is $\tan 150° = \frac{-\sqrt{3}}{3}$

Example 1

Find the measure of the inclination angle of the straight line ℓ if :

1. The slope of the straight line ℓ is $\frac{3}{7}$
2. The straight line ℓ passes through the two points $(5, 6)$, $(4, 7)$

① $\because m = \frac{3}{7}$ $\quad \therefore \theta = \tan^{-1}\left(\frac{3}{7}\right)$

SHIFT tan 3 ÷ 7 = °,,,

$\therefore \theta \approx 23° 11' 55''$

$\therefore$ The measure of the inclination angle of the straight line $\ell \approx 23° 11' 55''$

② $\because m = \frac{6-7}{5-4} = -1$ $\quad \therefore \theta = 180° + \tan^{-1}(-1)$

1 8 0 + SHIFT tan − 1 =

$\therefore \theta = 135°$

$\therefore$ The measure of the inclination angle of the straight line $\ell = 135°$

Example 2

If the straight line passing through the two points $A(2k+1, -5)$, $B(-3, k)$ has an angle of inclination measuring $45°$, find the value of k

$\because$ The slope of the straight line $= \tan 45°$

$\therefore \frac{-5-k}{2k+1+3} = \tan 45°$ $\quad \therefore \frac{-5-k}{2k+4} = 1$ $\quad \therefore 2k+4 = -5-k$

$\therefore 2k+k = -5-4$ $\quad \therefore 3k = -9$ $\quad \therefore k = -3$

Note that

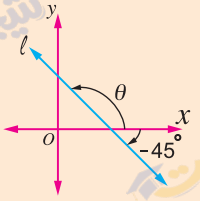
- The slope of any horizontal straight line (parallel to x-axis) is zero.
- The slope of any vertical straight line (parallel to y-axis) is undefined.

Self-Evaluation (1)

Find the measure of the inclination angle of the straight line ℓ if

1. The slope of the straight line ℓ is $\frac{5}{6}$
2. The straight line ℓ passes through the two points $(3, -\sqrt{3})$, $(-1, 3\sqrt{3})$

Note that



When using a calculator to find the measure for a negative slope, you find that : $\tan^{-1}(-1) = -45°$ which measures the angle clockwise from the positive direction of x-axis not the inclination angle so : $\theta = 180° + \tan^{-1}(-1) = 135°$

Self-Evaluation (2)

If the straight line passing through the two points $(1, k)$ and $(2k, 5)$ has an angle of inclination measuring $135°$, find the value of k.

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Relation between Slopes of Two Parallel Straight Lines

If ℓ_1 and ℓ_2 are non-vertical straight lines with slopes m_1 and m_2 respectively

And $\ell_1 // \ell_2$, then $m_1 = m_2$

That is: If two straight lines are parallel, then their slopes are equal.

And vice versa.

If $m_1 = m_2$, then $\ell_1 // \ell_2$

That is: If the slopes of two straight lines are equal, then the two straight lines are parallel.

Note that

Any two vertical straight lines are parallel.

Example 3

Prove that the straight line passing through the two points $A(2, 1)$, $B(4, -1)$ is parallel to the straight line with inclination angle of measure $135°$

Assuming that the slope of $\overleftrightarrow{AB} = m_1$ $\quad \therefore m_1 = \frac{1-(-1)}{2-4} = -1$

, the slope of the second straight line is $= m_2$

$\therefore m_2 = \tan 135° = -1$ $\quad \therefore m_1 = m_2$

$\therefore$ The two straight lines are parallel.

Self-Evaluation (3)

If the straight line ℓ_1 passes through the two points $(9, 4), (2, -1)$ and the straight line ℓ_2 passes through the two points $(-2, 2), (5, 7)$, prove that : $\ell_1 // \ell_2$

Example 4

Use slopes to prove that the points $A(4, 3)$, $B(2, 7)$, $C(8, -5)$ are collinear.

Assuming that the slope of $\overleftrightarrow{AB} = m_1$

$\therefore m_1 = \frac{7-3}{2-4} = -2$

Assuming that the slope of $\overleftrightarrow{AC} = m_2$

$\therefore m_2 = \frac{3+5}{4-8} = -2$ $\quad \therefore m_1 = m_2$

and the point A is a common point of $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$

$\therefore$ The points A, B, C are collinear.

Self-Evaluation (4)

If the points $A(1, -2)$, $B(4, k)$, $C(7, 3k-1)$ are collinear, find the value of k

Relation between Slopes of Two Perpendicular Straight Lines

If ℓ_1 and ℓ_2 are straight lines with slopes m_1 and m_2 respectively and neither is parallel to an axis

And $\ell_1 \perp \ell_2$, then $m_1 \times m_2 = -1$

That is: The product of the slopes of two perpendicular straight lines equals -1

And vice versa.

If $m_1 \times m_2 = -1$, then $\ell_1 \perp \ell_2$

That is: If the product of the slopes of two straight lines equals -1 , then the two straight lines are perpendicular.

Note that

- If one of two straight lines is vertical and the other is horizontal, then the straight lines are perpendicular.
- If the slope of the straight line ℓ_1 is $\frac{a}{b}$ (where $a \neq 0$) , then the slope of the straight line ℓ_2 which is perpendicular to it, is $\frac{-b}{a}$

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■ Self-Evaluation (5)

Prove that the straight line passing through $A(\sqrt{3}, -1)$, $B(3\sqrt{3}, 1)$ is perpendicular to the straight line with inclination angle of measure $120°$

Example 5

If the straight line ℓ_1 passes through $(-1, 6)$, $(5, 2)$, and the straight line ℓ_2 passes through $(6, 8)$, $(-2, -4)$, prove that : $\ell_1 \perp \ell_2$

$\because m_1 = \frac{6-2}{-1-5} = \frac{4}{-6} = \frac{-2}{3}$, $m_2 = \frac{8+4}{6+2} = \frac{12}{8} = \frac{3}{2}$

$\therefore m_1 \times m_2 = \frac{-2}{3} \times \frac{3}{2} = -1$ $\therefore \ell_1 \perp \ell_2$

■ Self-Evaluation (6)

If the points $A(6, 2)$, $B(4, 8)$, $C(-12, k)$ are the vertices of a right-angled triangle at A, find the value of k

Example 6

Prove that the points $A(0, -1)$, $B(-3, 2)$, $C(-1, 4)$, $D(2, 1)$ are vertices of a rectangle.

$\because$ The slope of $\overleftrightarrow{AB} = \frac{2-(-1)}{-3-0} = -1$

, the slope of $\overleftrightarrow{CD} = \frac{1-4}{2-(-1)} = -1$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = The slope of $\overleftrightarrow{CD}$

$\therefore \overline{AB} // \overline{CD}$ (1)

$\because$ The slope of $\overleftrightarrow{AD} = \frac{1-(-1)}{2-0} = 1$, the slope of $\overleftrightarrow{CB} = \frac{4-2}{-1-(-3)} = 1$

$\therefore$ The slope of $\overleftrightarrow{AD}$ = The slope of $\overleftrightarrow{CB}$

$\therefore \overline{AD} // \overline{CB}$ (2)

From (1) , (2) : The quadrilateral $ABCD$ is a parallelogram.

$\because$ The slope of $\overleftrightarrow{AB}$ × The slope of $\overleftrightarrow{AD} = -1 \times 1 = -1$

$\therefore \overline{AB} \perp \overline{AD}$

$\therefore$ The quadrilateral $ABCD$ is a rectangle.

■ Think with your classmates

How can you prove that a quadrilateral is a parallelogram, rectangle, rhombus, or square using slopes of straight lines ?

Lesson Assessment



First **Measuring Conceptual Understanding**



▸Choose the correct answer from the given ones :

(1) If $\overleftrightarrow{AB} // \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{-2}{3}$, what is the slope of $\overleftrightarrow{CD}$?

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$

(2) What is the slope of the straight line perpendicular to the straight line passing through $(6, 3)$, $(-2, -4)$?

(a) $\frac{-7}{8}$ (b) $\frac{7}{8}$ (c) $\frac{-8}{7}$ (d) $\frac{8}{7}$

(3) What is the slope of the straight line whose inclination angle measures $60°$?

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{3}$

(4) What is the slope of a straight line perpendicular to the X-axis ?

(a) 0 (b) 1

(c) –1 (d) Undefined

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(5) If the straight lines with slopes $\frac{-5}{3}$, $\frac{k}{6}$ are parallel, what is the value of k ?

(a) 10 (b) -10 (c) 5 (d) -5

(6) ABC is a right-angled triangle at B , if $A=(2,4)$, $B=(4,0)$, what is the slope of $\overleftrightarrow{BC}$?

(a) -2 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

(7) In the square $ABCD$, if the slope of $\overleftrightarrow{AB} = \sqrt{3}$, what is the slope of $\overleftrightarrow{BC}$?

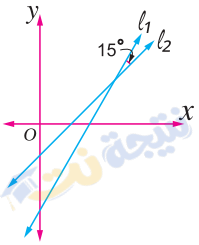
(a) $\frac{\sqrt{3}}{3}$ (b) $-\sqrt{3}$ (c) $\sqrt{3}$ (d) $-\frac{\sqrt{3}}{3}$

(8) In the rhombus $ABCD$, if the slope of $\overleftrightarrow{BD} = \frac{3}{4}$, what is the slope of $\overleftrightarrow{AC}$?

(a) $\frac{-4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{-3}{4}$ (d) $\frac{4}{3}$

(9) **In the given figure:**

If the slope of $\ell_1 = \sqrt{3}$, what is the slope of ℓ_2 ?



(a) $\tan 15^\circ$ (b) $\tan 30^\circ$

(c) $\tan 45^\circ$ (d) $\tan 60^\circ$

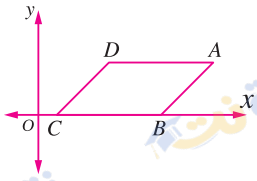
(10) **In the given figure :**

$ABCD$ is a parallelogram, where $A(6,2)$, $B(4,0)$

What is the measure of $\angle BCD$?

(a) 75° (b) 60°

(c) 45° (d) 30°



Second Applying Concepts

(11) Find the slope of a straight line with inclination angle of measure θ if θ equals :

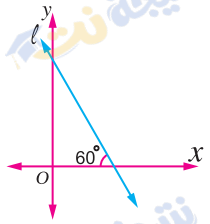
① 0° ② 30° ③ $75^\circ 4' 7''$ ④ 135°

(12) Find the measure of the inclination angle of a straight line with slope (m) if m equals :

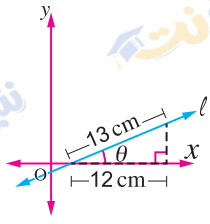
① 0.4 ② $\frac{-2}{3}$ ③ $\frac{4}{9}$

(13) In each of the following figures, find the slope of the straight line ℓ :

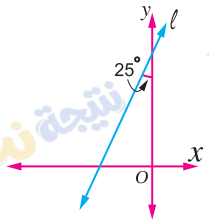
①



②



③



(14) If the straight line passing through $A(3k,1)$, $B(1,2)$ has inclination angle of measure θ, where $\tan\theta = \frac{1}{2}$, find the value of k

(15) If the straight line passing through $A(-4\sqrt{3}, k-6)$, $B(2\sqrt{3}, 2k+1)$ has inclination angle of measure 60°, find the value of k

(16) Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, perpendicular, or intersecting but not perpendicular :

① $A(-1,-4)$, $B(2,11)$, $C(1,1)$, $D(4,10)$

② $A(2,8)$, $B(-1,-2)$, $C(3,7)$, $D(0,-3)$

③ $A(-3,3)$, $B(0,2)$, $C(1,3)$, $D(-2,-6)$

④ $A(-1,3)$, $B(4,8)$, $C(-6,0)$, $D(2,8)$

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17 Prove that the straight line passing through $A(4,-3)$, $B(5,-4)$ is perpendicular to the straight line whose angle of inclination measures 45°

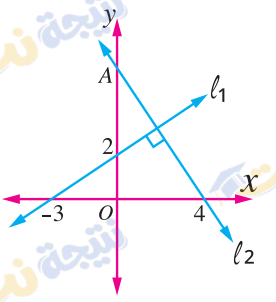
18 If the points $A(1,4)$, $B(5,5)$, $C(13,k)$ are collinear, find the value of k

19 Prove that the points $A(6,-5)$, $B(4,-7)$, $C(3,-3)$, $D(5,-1)$ are vertices of a parallelogram.

20 Prove that the points $A(3,-1)$, $B(5,4)$, $C(3,9)$, $D(1,4)$ are vertices of a rhombus.

21 In the given figure :

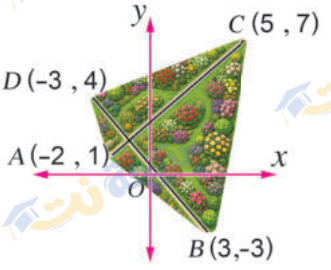
If $\ell_1 \perp \ell_2$, find the coordinates of the point A



Third Analysis and Subjects Integration

22 **Agriculture:** The vertices of the garden are $A(-2,1)$, $B(3,-3)$, $C(5,7)$, $D(-3,4)$, the two paths $\overline{AC}$ and $\overline{BD}$ pass through the garden.

Prove that the two paths are perpendicular.



Gardening improves air quality and provides a sense of comfort and enjoyment

23 **Excavations:** Paleontologists drew a dinosaur map using coordinate plane. One bone has the endpoints $(-5,8)$, $(10,-1)$ and another bone has the endpoints $(-10,-3)$, $(-5,-6)$

Are the two bones parallel?



24 In the given figure, the wooden pieces of a beach chair are represented by lines ℓ_1 , which passes through $A(4,k)$, $B(1,2)$ perpendicular to ℓ_2 , which passes through $C(1,4)$, $D(-4,1)$. Find the value of k

***Evaluate** your understanding!*

How well do you understand the relation between the slopes of two parallel and perpendicular straight lines? Tick the right box



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Lesson

3 - 7

Equation of the Straight Line

Learning Outcomes

- Find the equation of a straight line given its slope and y-intercept.
- Find the equation of a straight line given its slope and a point.
- Learn the standard form of a straight line equation.
- Use the slope of a straight line and y-intercept to graph the straight line.
- Use the equation of a straight line to solve some real-life problems.

Vocabulary

- Equation of straight line
- y-intercept

Note that

The equation of a straight line with slope m passing through the origin $(0\ ,0)$ is :

$$y = mx$$

Get Ready!



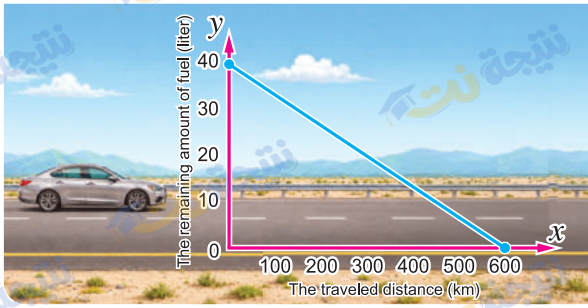
Souvenir shops

Luxor governorate is one of the world's top tourist destinations, with the Karnak, Luxor temples and the Valley of the Kings. Suppose a souvenir shop near to Luxor temple recorded annual sales as a linear relation between total sales (in thousand pounds) and the year of sale.
In **2020**, sales were **800,000** EGP and increase by **40,000** EGP each year due to number of tourists. Can you find the equation that represents the sales y in terms of the year of sale x
(where $x = 0$ represents **2020**, $x = 1$ represents **2021** , etc.)

In this lesson, you will learn the different forms of the equation of the straight line, which will enable you to solve such real-life problems.

Think & Discuss!

Sameh started a trip with a full **40** L tank. If the car consumes **1** L of fuel every **15** km, which of the following relations represents the relation between the remaining amount of fuel (y) and traveled distance (x) ?



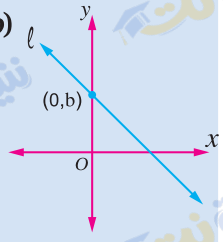
ⓐ $y = 40 - \frac{x}{15}$ ⓑ $y = 15x + 40$ ⓒ $y = x^2 - 40$

Learn!

Equation of the straight line

Equation of the straight line given its slope m and y-intercept

If the straight line ℓ intersects the y-axis at the point $(0\ ,b)$ then the y-intercept $= b$



The equation of the straight line ℓ given its slope m and y-intercept is written in the form: $y = mx + b$ (slope, y-intercept)

For example: If the slope of the straight line $= \frac{4}{5}$, and the y-intercept $= -3$, then the equation of this straight line is: $y = \frac{4}{5}x - 3$

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Example 1

Write the equation of the straight line in each of the following :

1. Its angle of inclination is of measure 135°, and the line passes through (0 , – 4)
2. The straight line passes through the origin, perpendicular to the straight line whose equation is $y = -2x + 3$

(1) ∵ The angle of inclination is of measure 135°

∴ $m = \tan 135° = -1$

∵ The straight line passes through (0 , – 4)

∴ The y-intercept = – 4

∴ The required straight line equation is $y = -x - 4$

(2) ∵ The straight line passes through the origin (0 , 0)

∴ The straight line equation is on the form: $y = mx$

∵ The slope of the straight line $y = -2x + 3$ equals – 2

∴ The slope of the perpendicular straight line $= \frac{1}{2}$

∴ The required straight line equation is $y = \frac{1}{2}x$

Example 2

Write the equation of the straight line whose slope is $\frac{3}{2}$ and intersects the y-axis at the point (0 , 1), then graph it using the slope and y-intercept.

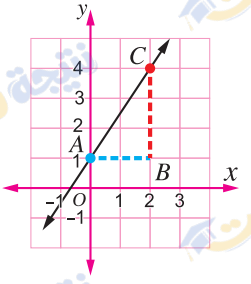
∵ The slope $= \frac{3}{2}$ and y-intercept = 1

∴ The straight line equation is : $y = \frac{3}{2}x + 1$

To graph the straight line :

Plot the point A (0 , 1)

∵ The slope $= \frac{3}{2} = \frac{\text{vertical change}}{\text{horizontal change}}$

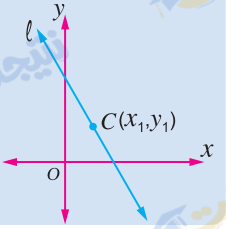


∴ From the point A , move horizontally 2 units to the right to reach the point B, then move vertically 3 units up to reach the point C, the straight line $\overleftrightarrow{AC}$ is the required graphical representation.

Equation of the straight line given its slope m and a point on it

If the straight line ℓ has slope m and passes through the point $C(x_1, y_1)$

, then its equation is $y - y_1 = m(x - x_1)$



For example : If the slope of the straight line $= \frac{2}{3}$ and passes through the point (3 , 4)

, then its equation is : $y - 4 = \frac{2}{3}(x - 3)$

Self-Evaluation (1)

Write the equation of the straight line in each of the following:

1. The straight line passes through (0 , – 3) and has inclination angle of tangent $\frac{5}{8}$
2. The straight line is parallel to the straight line whose equation is $y = \frac{1}{3}x$ and the y-intercept is – 4

Self-Evaluation (2)

Write the equation of the straight line whose slope is $\frac{-3}{4}$ and intersects the y-axis at the point (0 , 5) , then graph it using the slope and y-intercept.

Note that

1. The x-axis equation $y = 0$
2. The y-axis equation $x = 0$
3. The equation of the straight line passing through (x_1 , y_1) parallel to x-axis $y = y_1$
4. The equation of the straight line passing through (x_1 , y_1) parallel to y-axis $x = x_1$

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■ **Self-Evaluation** (3)

Write the equation of the straight line passing through the two points $(-3, -7)$, $(2, -2)$ in the form of slope and y-intercept.

Example 3

Write the equation of the straight line passing through the two points $(-4, 5)$, $(4, 1)$ in the form :

$y = mx + b$

$\because m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{-4-4} = \frac{-1}{2}$

$\because$ The equation of the straight line whose slope is m and passes through the point (x_1, y_1) is :

$y - y_1 = m(x - x_1)$

$\therefore$ The equation of the straight line is :

$y - 1 = \frac{-1}{2}(x - 4)$

$\therefore y - 1 = \frac{-1}{2}x - \frac{-1}{2} \times 4$

$\therefore y - 1 = \frac{-1}{2}x + 2$

$\therefore y = \frac{-1}{2}x + 3$

■ **Diversified strategies**

Assume the straight line equation is

$y = mx + b$

$\because m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{-4-4} = \frac{-1}{2}$

$\therefore y = \frac{-1}{2}x + b$

$\because$ The straight line passes through the point $(4, 1)$

$\therefore 1 = \frac{-1}{2}(4) + b$

$\therefore b = 3$

$\therefore$ The straight line equation is :

$y = \frac{-1}{2}x + 3$

■ **Note that**

You can use the point $(-4, 5)$ instead of the point $(4, 1)$ to form the equation and get the same form which is $y = \frac{-1}{2}x + 3$

Standard form of the straight line equation

$$Ax + By = C$$

where $A \geq 0$, A and B are not equal to zero together, A , B , C are integers whose greatest common factor is 1

■ **Note that**

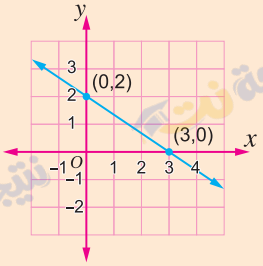
You can use the x-intercept and y-intercept to graph a straight line. For example,

$2x + 3y = 6$

x-intercept $= 3$,

y-intercept $= 2$

This means the straight line intersects the x-axis at $(3, 0)$, and the y-axis at $(0, 2)$



Note :

If $Ax + By = C$ is the standard form of a straight line equation, then :

- The slope of the straight line $= \frac{-x\text{-coefficient}}{y\text{-coefficient}} = \frac{-A}{B}$
- The y-intercept $= \frac{C}{B}$
- The x-intercept $= \frac{C}{A}$

For example : The straight line whose equation is $2x + 3y = 6$

- Its slope $= \frac{-2}{3}$
- The y-intercept $= \frac{6}{3} = 2$
- The x-intercept $= \frac{6}{2} = 3$

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Example 4

Write the standard form of the equation of the straight line passing through $(-4, 3)$, and perpendicular to the straight line whose equation is : $4x + y + 1 = 0$

The slope of the given straight line $= \frac{-x\text{-coefficient}}{y\text{-coefficient}} = \frac{-4}{1} = -4$

$\therefore$ The slope of the required straight line $= \frac{1}{4}$

$\therefore$ The required straight line equation is : $y - y_1 = m(x - x_1)$

$\therefore y - 3 = \frac{1}{4}(x - (-4))$ $\therefore 4(y - 3) = x + 4$

$\therefore 4y - 12 = x + 4$ $\therefore x - 4y = -16$

Example 5

If the straight line passing through $(7, -9)$, $(3, 3)$ is perpendicular to the straight line whose equation is : $2x + ky - 16 = 0$, find the value of k

Assume the slope of the straight line passing through the two points $(7, -9)$, $(3, 3)$ is m_1

$\therefore m_1 = \frac{-9 - 3}{7 - 3} = -3$

Assume the slope of the straight line $2x + ky - 16 = 0$ is m_2

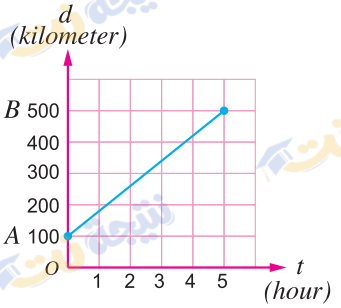
$\therefore m_2 = \frac{-2}{k}$

$\because$ The straight lines are perpendicular.

$\therefore m_1 \times m_2 = -1$ $\therefore -3 \times \frac{-2}{k} = -1$ $\therefore k = -6$

Example 6

The given figure represents the relation between the distance (d) in kilometers from a fixed point and the time (t) in hours for a car at a uniform speed between two cities A and B



Find:

1. The uniform speed of the car.
2. The car's distance from the origin at the beginning of motion.
3. The straight line equation representing the distance – time relation.
4. The distance between the car and the origin after 3 hours from the beginning of the motion.

Self-Evaluation 4

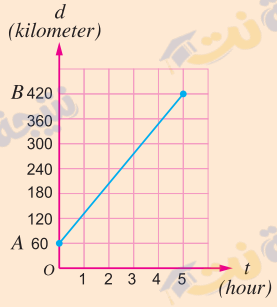
Write the standard form of the equation of the straight line passing through $(2, -5)$ and parallel to the straight line whose equation is : $3x - 4y + 12 = 0$

Self-Evaluation 5

Find the standard form of the equation of the straight line passing through $(2, 3)$ parallel to the straight line: $4x - 5y = 6$

Self-Evaluation 6

The following figure represents the relation between the distance (d) in kilometers and the time (t) in hours for a car at a uniform speed between two cities A and B.



Find:

1. The uniform speed of the car.
2. The car's distance from the origin at the beginning of motion.
3. The straight line equation representing the car's motion.

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(1) ∵ The straight line passes through the two points $(0, 100), (5, 500)$,
The slope of the straight line represents the car's uniform speed (V).

$\therefore V = \frac{500 - 100}{5 - 0} = 80$

$\therefore$ The uniform speed of the car $= 80$ km/h

(2) The car's distance from the origin at the beginning of motion $= 100$ km

(3) The straight line equation is : $d = m\,t + b$

$\therefore d = 80\,t + 100$

(4) At $t = 3$, then $d = 80\,(3) + 100 = 340$

$\therefore$ The distance between the car and the origin after 3 hours $= 340$ km.

Lesson Assessment



First Measuring Conceptual Understanding



▸ **Choose the correct answer from the given ones:**

(1) What is the y-intercept of the straight line $2x - y - 5 = 0$?

(a) -5 (b) $\frac{-5}{2}$ (c) $\frac{5}{2}$ (d) 5

(2) What is the slope of the straight line parallel to the straight line $3x - 4y = 7$?

(a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

(3) What is the slope of the straight line perpendicular to the straight line $3x - 4y = 7$?

(a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$

(c) $\frac{3}{4}$ (d) $\frac{4}{3}$

(4) What is the equation of the straight line passing through the two points $(-1, 4), (-4, 5)$?

(a) $y = -\frac{1}{3}x + \frac{11}{3}$ (b) $y = \frac{1}{3}x + \frac{13}{3}$

(c) $y = -\frac{1}{3}x + \frac{13}{3}$ (d) $y = \frac{1}{3}x - \frac{13}{3}$

(5) If the straight line whose equation is $3y - kx = 13$ is parallel to the straight line whose equation is $(k-1)x - 2y = 8$, what is the value of k ?

(a) 3 (b) 2 (c) 1 (d) -2

(6) If the straight line making an angle of measure $45°$ with the positive direction of x-axis is perpendicular to the straight line whose equation is $kx + 2y = 5$, what is the value of k ?

(a) -1 (b) 1 (c) 2 (d) -2

(7) What is the equation of the straight line passing through the origin and perpendicular to the straight line $y = \frac{1}{3}x - 7$?

(a) $y = 3x$ (b) $y = -3x$

(c) $y = \frac{1}{3}x$ (d) $y = -\frac{1}{3}x$

(8) What is the measure of the angle formed between the straight line $y - \sqrt{3}x + 5 = 0$ and the positive direction of x-axis ?

(a) $30°$ (b) $45°$ (c) $60°$ (d) $75°$

(9) What is the equation of the straight line parallel to the y-axis passing through $(-2, 1)$?

(a) $x = -2$ (b) $y = 1$

(c) $x = 1$ (d) $y = -2$

(10) What is the area of the triangle formed by the straight lines $5x - 6y - 30 = 0$, $x = 0$, $y = 0$?

(a) 7.5 square units (b) 15 square units

(c) 30 square units (d) 60 square units

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Second Applying Concepts

(11) Find the slope, x-intercept and y-intercept of each of the following straight lines :

① $-2x+5y=20$ ② $4x-2y+8=0$

③ $y=7x$ ④ $y=\frac{-2}{3}x-4$

(12) In each of the following , find the equation of the straight line in the form of slope and y-intercept :

① The slope $=-2$, y-intercept $=3$

② The slope $=\frac{5}{2}$, x-intercept $=-2$

③ The slope $=\frac{1}{2}$, the straight line passes through $(6,4)$

④ The straight line passes through $(6,1)$, $(8,-4)$

⑤ The straight line passes through $(0,-2)$, perpendicular to the straight line $y=x-2$

⑥ The straight line passes through $(-1,2)$, parallel to the straight line $2x+5y=10$

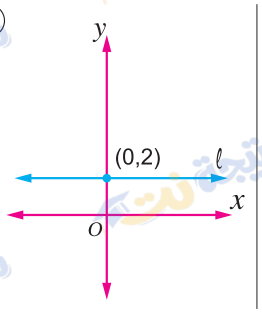
(13) Find the standard form of the equation of the straight line in each of the following :

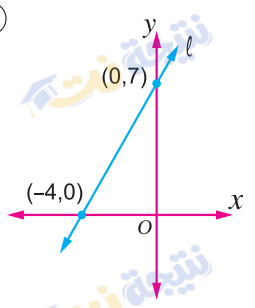
① The y-intercept $=4$, the x-intercept $=3$

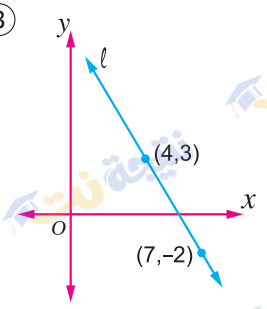
② The straight line passes through the origin parallel to the straight line passing through $(5,-3)$, $(2,4)$

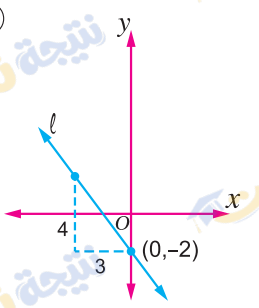
③ The straight line is perpendicular to $\overline{AB}$ from its midpoint where $A(2,5)$, $B(8,-3)$

(14) In each of the following , find the equation of the straight line ℓ :

① 

② 

③ 

④ 

(15) In each of the following , graph the straight line using the slope and y-intercept :

① $y=2x-3$ ② $y=\frac{-2}{5}x+2$ ③ $3x-4y-12=0$

(16) If ABC is a triangle in which $A(-6,-8)$, $B(6,4)$ and $C(-6,10)$, $\overleftrightarrow{AD} \perp \overline{BC}$, write the equation of $\overleftrightarrow{AD}$

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(17) A telecommunications company charges a fixed monthly subscription **25** EGP plus **1.5** EGP for each gigabyte used. Write an equation for the total cost according to the number of gigabyte used.



(18) **Sales:** A salesperson earns daily salary plus commission. When his sales reach 1,000 EGP in a day, his total earnings is **100** EGP; when his sales reach **1,400** EGP, his total earnings is **120** EGP , where the relation between his total earnings and sales is linear. Write an equation relating his sales x to his earnings y, and find what he earns when the sales reach **2,000** EGP in a day.

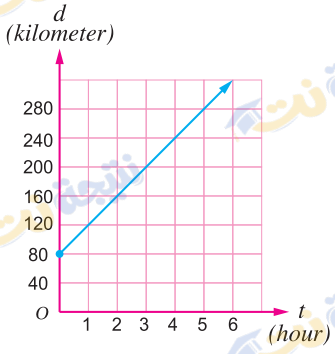
(19) The given figure represents the relation between the distance **(*d*)** in kilometers from a fixed point and the time **(*t*)** in hours for a car at a uniform speed. Find :

① The uniform speed of the car.

② The car's distance from the origin at the beginning of the motion.

③ The equation of the straight line representing the relation between the distance and the time.

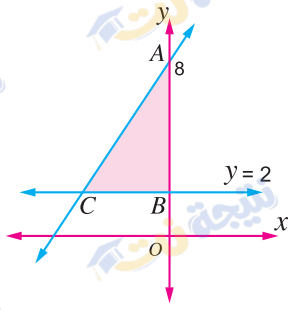
④ The distance between the car and the origin after **7** hours from the beginning of the motion.



Creative Thinking

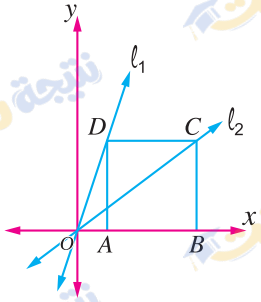
(20) Find the required under each figure :

① The area of ΔABC = **12** square units



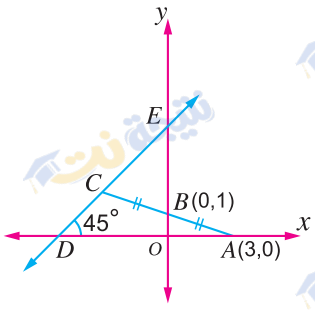
Find the equation of $\overleftrightarrow{AC}$

② $ABCD$ is a square, the equation of ℓ_1 is : $y = 3x$



Find the equation of the straight line ℓ_2

③ B is the midpoint of $\overline{AC}$



Find the equation of $\overleftrightarrow{DE}$

***Evaluate** your understanding!*

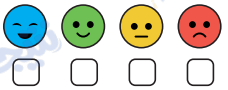
How well do you understand equation of the straight line? Tick the right box



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Unit Three Assessment

Evaluate your understanding!
How well do you understand Unit Three? Tick the right box



▸ Choose the correct answer from the given ones :

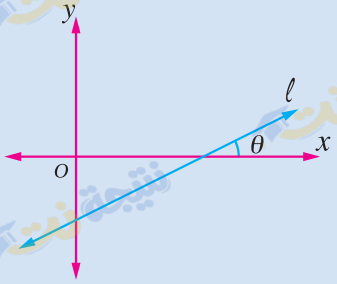
(1) If $\tan X \sin 30° = \cos 30°$, what is the value of X where X is the measure of an acute angle ?

(a) $30°$ (b) $45°$ (c) $60°$ (d) $75°$

(2) In the given figure : If the equation of the straight line ℓ is : $2y = x - 4$, which of the following ratios equals $\tan\theta$?

(a) 1 (b) $\frac{1}{2}$

(c) 2 (d) $-\frac{1}{2}$



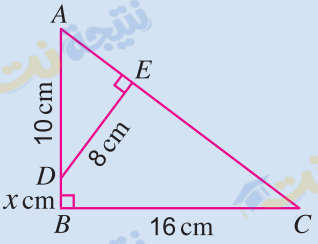
(3) What is the dilation factor of the dilation with center at the origin O and mapping $P(-3, 6)$ to $P'(-2, 4)$?

(a) $\frac{1}{3}$ (b) 2 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

(4) In the given figure :
What is the value of X ?

(a) 12 (b) 2

(c) 8 (d) 4

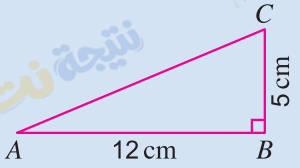


▸ Complete each of the following :

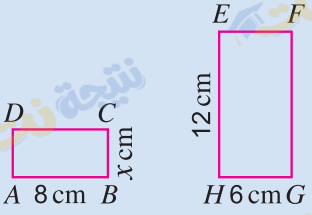
(5) The slope of the straight line whose equation is : $2x - 3y - 3 = 0$ equals .....

(6) If the point A is the image of the point $B(2, 4)$ under a dilation with center at the origin O and a dilation factor 2, then the point A is .....

(7) In the given figure :
$\sin A + \cos C =$ .....



(8) In the given figure :
If the rectangle $ABCD \sim$ the rectangle $EHGF$, then $X =$ .....



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▸ **Answer the following questions :**

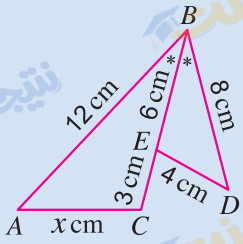
(9) ABC is an isosceles triangle, $AB = AC = 10\ cm$, $BC = 12\ cm$. Find : $m(\angle B)$, $\cos\frac{A}{2}$

(10) Find the standard form of the equation of the straight line passing through $A(-3, 5)$, $B(-1, 4)$

(11) In the given figure :

$\overrightarrow{BC}$ bisects $\angle ABD$

Prove that : $\Delta ABC \sim \Delta DBE$, then find the value of x



(12) If $A(0, 1)$, $B(3, 2)$, $C(2, -2)$, graph the triangle ABC and draw its image under a dilation with center at the origin and a dilation factor 2

Unit Three Activity — Using similarity of triangles to deduce Euclid's relations

▸ **Activity Aim:**

To prove Euclid's relations through similarity of triangles.



▸ **Implementation Steps :**

Using what you have learned about the concept of similarity of triangles, perform the following steps, then compare your conclusion with one of your classmates:

1. Draw ΔABC right-angled at C
2. Draw $\overline{CD}$ perpendicular to $\overline{AB}$ to intersect it at D
3. Observe the triangles ABC, ACD, CBD, then prove that : $\Delta ABC \sim \Delta ACD$, $\Delta ABC \sim \Delta CBD$
4. Using similarity of triangles, deduce: $(AC)^2 = AB \times AD$, $(BC)^2 = AB \times BD$, $(CD)^2 = AD \times BD$
5. Explain how to use these relations to find unknown lengths.

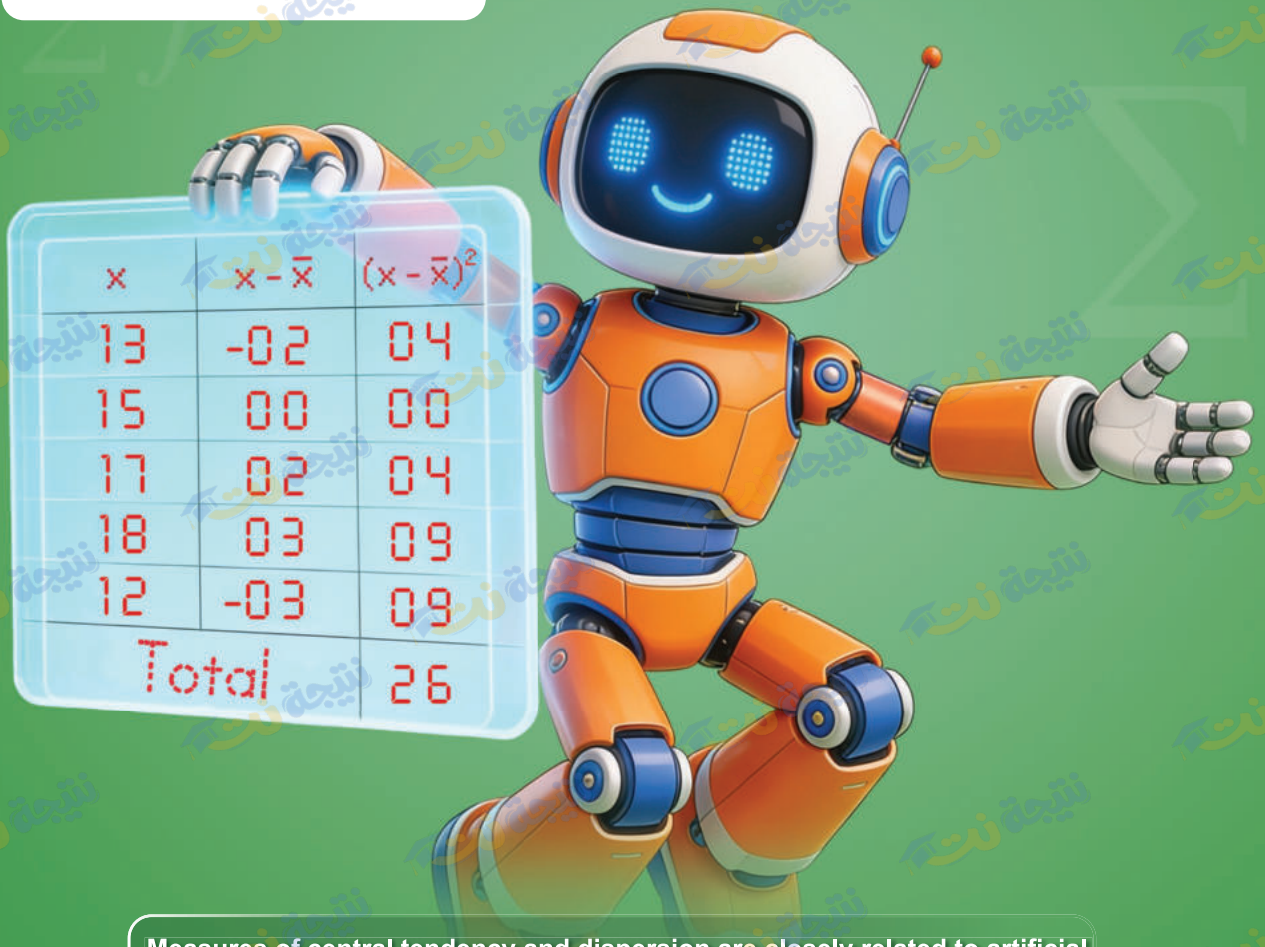
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UNIT 4

Statistics

Unit Lesson

4-1 Measures of Dispersion



Measures of central tendency and dispersion are closely related to artificial intelligence. They are used in prediction and analyzing results that AI algorithms depend on. This helps improve the quality of prediction and decision-making.

▶ Can we develop the methods of measures of central tendency and dispersion to help artificial intelligence get more accurate results?

Issues and Life Skills
- Mathematical Communication
- Critical Thinking
- Prediction
- Decision-making

Values
- Respecting Time
- Sports Awareness
- Environmental Awareness

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Lesson

4 - 1

Measures of Dispersion

■ Learning Outcomes

- Learn the concept of dispersion.
- Learn the measures of dispersion for a set of values.
- Learn the concept of range.
- Learn the concept of interquartile range.
- Learn the concept of standard deviation.
- Find the range and interquartile range for data shown in different forms.
- Find the standard deviation for a set of values.
- Find the standard deviation for a simple frequency distribution and for a frequency distribution with intervals.

■ Vocabulary

- Dispersion
- Measures of dispersion
- Range
- Interquartile range
- Standard deviation

■ Previous Knowledge

Some measures of central tendency are:

- The arithmetic mean = $\frac{\text{Sum of values}}{\text{Number of these values}}$
- The median is the middle value after arranging the values.
- The mode is the most common or frequent value.

Get Ready!



Sleep is one of the most important biological processes that the human body needs to restore energy and focus. Having enough regular hours of sleep every day is important for good daily performance. Ahmed recorded the number of hours he slept every day for two weeks. The results were shown as the following :

	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
The first week	8	7	7	6	7	6	8
The second week	8	5	8	7	6	5	10

Did Ahmed sleep at the same rate during the two weeks?
In which week were Ahmed's sleeping hours more consistant?

In this lesson, you will learn the measures of dispersion, which will enable you to solve such real-life problems.

Think & Discuss!

There are two groups of students. Each group has **5** students. Both groups took a test out of **50** marks. The marks of the students in each group are shown in front of you.

First group 33 , 33 , 33 , 27 , 24

Second group 43 , 35 , 33 , 33 , 6

Find the arithmetic mean, median, and mode for each group.
Which group has marks that are more consistant?
Which group has marks that are more spread out?
Are measures of central tendency sufficient to judge the level of each group?

Learn!

- You have already studied measures of central tendency, such as the arithmetic mean, median, and mode. These measures describe data by providing a value around which the other data values cluster.
- When comparing the marks of the two groups mentioned in "Think and Discuss", you find that:
  - The arithmetic mean is **30** for both groups.
  - The median is **33** for both groups. - The mode is **33** for both groups.
- Although both groups have the same measures of central tendency, there is a difference between the marks in the two groups.So, we need another type of measures to describe the degree of homogeneity (consistence) or dispersion (conversion) of the data values from each others.

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■ Note that

When comparing two or more sets of data, we need one measure of central tendency and one measure of dispersion for each set.

■ Think

What will the dispersion be if all the data values are equal?

• Since the marks of the first group are not less than **24** and not more than **33**, while the marks of the second group are not less than **6** and not more than **43**, the marks of the first group are more consistant than the marks of the second group. We can also say that the marks of the second group are more spread out than the marks of the first group.

Measures of Dispersion

Measures of dispersion describe the extent of similarity or differenece among a set of values or data. They measure how close or far these values from the center. In other words, they show the spread of data around the center. Examples of these measures of dispersion are range (R), interquartile range (IQR), standard deviation (σ)

1 Range

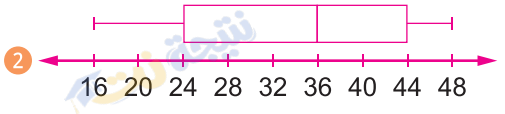
The range of a set of values (R) is the difference between the greatest value and the smallest value. The range is the simplest and easiest measure of dispersion. However, it depends only on two values, the smallest value and the greatest value. It ignores the other values, so it is affected by extreme values.

Example 1

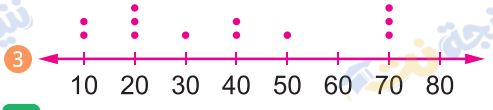
Find the range for each set of data in each of the following:

1) **48 , 61 , 65 , 52 , 38 , 64 , 66 , 64**

2)



3)



4)

Stems	Leaves
1	3 6 6 7
2	0 0 1 2 5 5 6 8
3	1 2

Key | 2|8 means 28

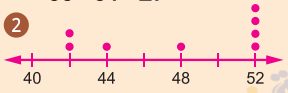
1) $R = 66 - 38 = 28$
2) $R = 48 - 16 = 32$
3) $R = 70 - 10 = 60$
4) $R = 32 - 13 = 19$

■ Self-Evaluation (1)

Find the range for each set of data in each of the following :

1) **23 , 53 , 43 , 67 , 41 , 27 , 35 , 31 , 27**

2)



3)

Stems	Leaves
3	2 5 7 9
4	1 3 3 4 5 5
5	4 7

key | 4 | 1 means 41

Example 2

Find the value of k that makes the range of the set of values **17 , 12 , 19 , k , 18 , 15 , 11** equal **13**

Notice that the smallest value is **11**, and the greatest value is **19**

- If $11 \leq k \leq 19$, then the range $= 8$ (refused)
- If $k > 19$, then $k - 11 = 13$

$\therefore \boxed{k = 24}$

- If $k < 11$, then $19 - k = 13$

$\therefore \boxed{k = 6}$

■ Self-Evaluation (2)

• Find the value of k that makes the range of the set of values : **18 , 15 , 11 , 20 , k , 8 , 14** equal **18**

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2 Interquartile Range

The interquartile range (IQR) is the difference between the upper quartile (third "Q_3") and the lower quartile (first "Q_1")

So that $IQR = Q_3 - Q_1$

The interquartile range is the range of the middle half of the data. It is a more accurate measure of spread when there are extreme values. However, one disadvantage is that it depends only on two values, Q_1 and Q_3, and ignores the other values

Example 3

Find the three quartiles for the following values, then calculate the interquartile range :

23 , 20 , 19 , 12 , 6 , 9 , 16 , 23 , 10 , 18 , 5

Arrange the values in ascending order :

5 , 6 , 9 , 10 , 12 , 16 , 18 , 19 , 20 , 23 , 23

Q_1 Q_2 Q_3

$\therefore Q_1 = 9 \quad , \quad Q_2 = 16 \quad , \quad Q_3 = 20$

$\therefore IQR = Q_3 - Q_1 = 20 - 9 = 11$

Self-Evaluation (3)

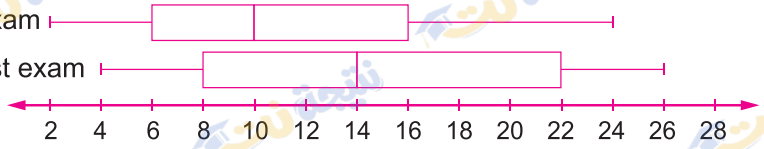
Calculate the interquartile range for each of the following:

1. 108 , 106 , 122 , 107 , 94 , 105
2. 58 , 71 , 73 , 64 , 66 , 57 , 67 , 68

Example 4

The following figure shows the distribution of marks in two exams for a group of students.

Find the interquartile range of the marks for each of the two exams.

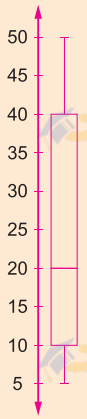


For the first exam : $Q_1 = 8 \quad , \quad Q_3 = 22 \quad \therefore IQR = 22 - 8 = 14$

For the second exam : $Q_1 = 6 \quad , \quad Q_3 = 16 \quad \therefore IQR = 16 - 6 = 10$

Self-Evaluation (4)

Find the three quartiles from the following box plot, then calculate the range and the interquartile range.



3 Standard Deviation

The standard deviation is the positive square root of the mean of the squares of deviations of the values from their arithmetic mean. It is denoted by σ, which is read as "sigma". The standard deviation is the most important accurate, and widely used measure of dispersion.

First Calculating the Standard Deviation for a Set of Values:

The standard deviation for a set of values is $\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$

where x is the value , n is the number of values

$\bar{x}$ is the arithmetic mean , $\sum$ is the summation operation

Note that

The smaller the value of the standard deviation, the closer the values are to each other. This means the values are more homogenious.

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Example 5

Find the standard deviation for the following values : 13 ,15,17,18,12

1 Find the arithmetic mean.

$$\overline{x} = \frac{\Sigma x}{n} = \frac{13+15+17+18+12}{5}$$

$$= \frac{75}{5} = 15$$

2 Form the table shown.

x	$x-\overline{x}$	$(x-\overline{x})^2$
13	– 2	4
15	0	0
17	2	4
18	3	9
12	– 3	9
Total		26

3 Calculate the standard deviation (σ) as follows :

$$\sigma = \sqrt{\frac{\Sigma (x-\overline{x})^2}{n}} = \sqrt{\frac{26}{5}} \approx 2.28$$

Self-Evaluation 5

Find the standard deviation for the following values :

7 , 6 , 13 , 11 , 8

Second Calculating the Standard Deviation for a Simple Frequency Distribution

The standard deviation for a simple frequency distribution is

$$\sigma = \sqrt{\frac{\Sigma f.(x-\overline{x})^2}{\Sigma f}}$$

where x is the value , f is the frequency of the value , Σf is the total of the frequencies , $\overline{x}$ is the arithmetic mean where $\overline{x} = \frac{\Sigma (f.x)}{\Sigma f}$

Example 6

The following frequency distribution shows the ages of **14** people sitting in a restaurant :

Age	2	5	10	12	16	34
Frequency	1	2	5	2	3	1

Find the standard deviation of the ages of the people in the restaurant.

Assuming age (x) , and frequency (f)

x	f	$f.x$	$x-\overline{x}$	$(x-\overline{x})^2$	$f.(x-\overline{x})^2$
2	1	2	– 10	100	100
5	2	10	– 7	49	98
10	5	50	– 2	4	20
12	2	24	0	0	0
16	3	48	4	16	48
34	1	34	22	484	484
Total	14	168			750

$$\overline{x} = \frac{\Sigma (f.x)}{\Sigma f} = \frac{168}{14} = 12$$

$$\sigma = \sqrt{\frac{\Sigma f.(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{750}{14}} \approx 7.32$$

Self-Evaluation 6

The following frequency distribution shows car sales in a showroom over several days:

Number of cars sold	Frequency
0	4
1	4
2	3
3	6
4	3

Find the standard deviation of the number of cars sold.

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Third Calculating the Standard Deviation for a Frequency Distribution with Intervals

The standard deviation for a frequency distribution with intervals is

$$\sigma = \sqrt{\frac{\Sigma f.(x_m - \bar{x})^2}{\Sigma f}}$$

where x_m is the center of the interval and equals

$$\frac{\text{the lower limit for the interval} + \text{the upper limit for it}}{2}$$

f is the frequency of the interval , Σf is the total of the frequencies

$\bar{x}$ is the arithmetic mean where $\bar{x} = \frac{\Sigma (f.x_m)}{\Sigma f}$

Example 7

The following table shows the masses of 50 people :

Intervals of masses in kilograms	60 –	70 –	80 –	90 –	100 –
Number of people	5	11	18	9	7

Find the standard deviation of the masses.

Intervals	Frequency f	Centers of the intervals x_m	$f.x_m$	$x_m - \bar{x}$	$(x_m - \bar{x})^2$	$f.(x_m - \bar{x})^2$
60 –	5	65	325	– 20.4	416.16	2080.8
70 –	11	75	825	– 10.4	108.16	1189.76
80 –	18	85	1530	– 0.4	0.16	2.88
90 –	9	95	855	9.6	92.16	829.44
100 –	7	105	735	19.6	384.16	2689.12
Total	50		4270			6792

$$\bar{x} = \frac{\Sigma (f.x_m)}{\Sigma f} = \frac{4270}{50} = 85.4$$

$$\sigma = \sqrt{\frac{\Sigma f.(x_m - \bar{x})^2}{\Sigma f}} = \sqrt{\frac{6792}{50}} \approx 11.66$$

Self-Evaluation 7

Find the standard deviation for the following frequency distribution :

Intervals	Frequency
5 –	4
15 –	5
25 –	8
35 –	7
45 –	6

Collaborative Activity



Work with your friends and use a weather application. Record, in a table, the highest and lowest temperatures in 10 Egyptian cities on one day. Use the data you collect to calculate the arithmetic mean and the standard deviation of the highest temperatures, the lowest temperatures.



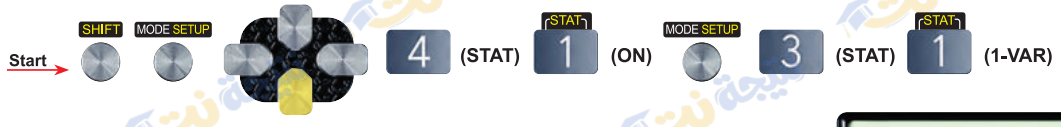
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Technology Activity Using the Calculator to Find the Standard Deviation

A scientific calculator can be used to find the standard deviation. The following steps explain how to solve the previous example using the calculator.

1

First, set the calculator mode by pressing the following keys from left to right.



The calculator screen shows :

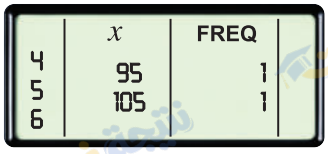


2

Enter the values of the interval centers (x_m) in the first column by pressing the following keys from left to right.



The calculator screen shows :



3

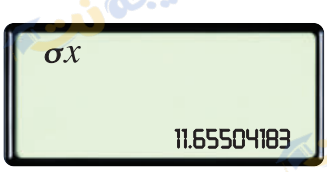
Use the arrow keys  to move to the second column $(FREQ)$, then enter the frequencies (f) by pressing the following keys from left to right,



then find the standard deviation by pressing the following keys from left to right.



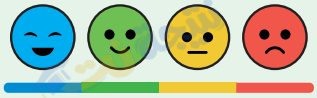
The calculator screen shows :



The value shown is the standard deviation. $\sigma \approx 11.66$

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Lesson Assessment



First Measuring Conceptual Understanding



▸ **Choose the correct answer from the given ones:**

(1) Which of the following is not a measure of dispersion?

(a) Range (b) Interquartile range

(c) Median (d) Standard deviation

(2) What is the range of the data set in the following figure?



(a) 50 (b) 220 (c) 245 (d) 270

(3) The staff of the Grand Egyptian Museum recorded the number of visitors, in thousands, over several days to make a statistical study, and the results are given in the following simple frequency table :

Days	First	Second	Third	Fourth	Fifth	Sixth	Seventh
Number of visitors	19	17	11	18	22	23	15

What is the range of the number of visitors during these days?

(a) 12 (b) 15 (c) 12000 (d) 15000

(4) If the interquartile range is 8.5 and the lower quartile is 29 , what is the upper quartile?

(a) 12 (b) 21.5

(c) 37.5 (d) 41.5

(5) What is the interquartile range from the following box plot?



(a) 10 (b) 12 (c) 14 (d) 16

(6) If $\Sigma (x - \bar{x})^2 = 36$ for a set of 9 values, what is the value of σ ?

(a) 16 (b) 8

(c) 4 (d) 2

(7) What is the standard deviation of the values 8 , 8 , 8 , 8 , 8 ?

(a) 0 (b) 4

(c) 8 (d) 16

(8) In the given stem-and-leaf plot :

If the range is 4.6 , what is the value of $k + m$?

Stems	Leaves
35	2 3
36	4 6 7 8
m	8 9 9
38	0 1 5
39	4 k

Key	36 | 4 means 36.4

(a) 8 (b) 37

(c) 45 (d) 37.8

(9) If the range of the values: 55 , 61 , 56 , 59 , a , 54 is equal to 9 , which of the following could be a value of a ?

(a) 64 (b) 62 (c) 56 (d) 52

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▸ **Spot the mistake :**

(10) When finding the standard deviation of the values: **8 , 12 , 6 , 10 , 4** Wael and Hany gave the following solutions:

Wael's solution :

x	$x-\bar{x}$	$(x-\bar{x})^2$
8	0	0
12	4	16
6	−2	−4
10	2	4
4	−4	−16
Total		0

$\bar{x} = \frac{40}{5} = 8$

$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$= \sqrt{\frac{0}{5}} = 0$

Hany's solution :

x	$x-\bar{x}$	$(x-\bar{x})^2$
8	0	0
12	4	16
6	−2	4
10	2	4
4	−4	16
Total		40

$\bar{x} = \frac{40}{5} = 8$

$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$= \sqrt{\frac{40}{5}} = 2\sqrt{2}$

Which one followed the correct method? Discuss.

Second Applying Concepts

(11) Find the range for each of the following:

① **24 , 25 , 17 , 16 , 20**

② **48 , 76 , 61 , 49 , 59 , 41 , 52 , 47**

③

Stems	Leaves
0	4 5
1	2 3 3 4 9
2	3 4 7

Key: 2 | 7 means 27

(12) Find the interquartile range for each of the following:

① **34 , 25 , 46 , 17 , 29 , 16 , 32 , 33**

② **5.4 , 2.3 , 1.2 , 5.1 , 8.4 , 2.1 , 3.7**

③



(13) Find the standard deviation for each of the following sets of values:

① **14 , 16 , 13 , 18 , 12 , 17**

② **$a+5$, $a+2$, $a+7$, $a+8$, $a+3$**

③

Stems	Leaves
2	2 2 5 7
3	3 5 8
4	0 0 8

Key: 3 | 5 means 35

(14) Completing the table, find the standard deviation of the given set of values:

x	$x-\bar{x}$	$(x-\bar{x})^2$
12		
14		
15		
19		

(15) If the arithmetic mean of the values $x+1$, x , **8** , **4** , **5** is equal to x , find the standard deviation of these values.

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Third Analysis and Subjects Integration



Sports keep the body healthy, strengthen determination, and increase self-confidence.

16 **Sports :** The arithmetic mean height of a group of students in a basketball team at a secondary school is 187 cm and the standard deviation is 7 cm
The following table shows the heights of another group of students in the school handball team:

Student height in centimeters	184	186	188	189
Number of students	1	4	3	2

Find the arithmetic mean and standard deviation of the heights of the handball team students. Then compare them with the arithmetic mean and standard deviation of the basketball team students. Which team has more homogenious heights? Why?

17 **Internet :** The following table shows the internet consumption, in GB , of a family during one month:

Internet consumption in GB	10	16	18	19	23	24
Number of days	3	7	8	6	4	2

Find the arithmetic mean and standard deviation of the family's daily internet consumption.

18 **Wages :** The following table shows the daily wages of a group of workers in a factory:

Wage intervals in pounds	120 –	130 –	140 –	150 –	160 –	170 –
Number of workers	18	21	20	15	16	10

Find the arithmetic mean and standard deviation of the daily wages.

Creative Thinking

▸ **Choose the correct answer from the given ones :**

19 What is the standard deviation of five consecutive even numbers?

(a) $\sqrt{5}$ (b) $\sqrt{6}$ (c) $\sqrt{7}$ (d) $\sqrt{8}$

20 If the standard deviation of the data set: $7x$, $4x$, $5x$, $6x$, $3x$ where x is a positive integer, is equal to $4\sqrt{2}$, what is the value of x ?

(a) 1 (b) 2 (c) 3 (d) 4

21 If the standard deviation of seven consecutive positive numbers is equal to $\frac{\sqrt{k}}{3}$, what is the value of k ?

(a) 14 (b) 21 (c) 36 (d) 42

***Evaluate** your understanding!*

How well do you understand measures of dispersion?
Tick the right box



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Unit Four Assessment

Evaluate your understanding!
How well do you understand Unit Four? Tick the right box

▸ Choose the correct answer from the given ones :

(1) From the given stem-and-leaf plot,
what is the value of the range?

Stems	Leaves
26	1 4
27	4 6 7
28	7 8 9
29	2 3

Key | 27 \| 6 means 27.6

(a) 3.6 (b) 3.4
(c) 3.2 (d) 3

(2) If the range of the values $7-3k, 7+6k, 7-k, 7+k$ is equal to 18, where k is a positive integer, what is the value of k?
(a) 1 (b) 2 (c) 3 (d) 4

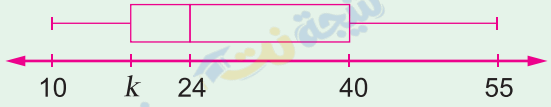
(3) What is the standard deviation of five consecutive integers if the smallest one is 3?
(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 2 (d) 1

(4) Which of the following sets of values has the greatest standard deviation?
(a) 10, 7, 9, 8 (b) 5, 5, 5, 5 (c) 6, 5, 4, 3 (d) 16, 20, 6, 2

▸ Complete each of the following :

(5) The range of the set of values is 23, 61, 43, 20, 17, 35, 29 is equal to .....

(6) From the given box plot :
If the interquartile range is 22, then the lower quartile is .....



(7) The standard deviation of the values : $17-b, 15-b, 12-b, 16-b$ is .....

(8) If $\Sigma(x-\bar{x})^2 = 60$, $n = 12$, then the standard deviation is .....

▸ Answer the following questions :

(9) The following frequency distribution shows the number of goals scored by 30 players from 5 penalty kicks each during one training:

Number of goals scored	0	1	2	3	4	5
Number of players	1	6	5	5	6	7

Find the arithmetic mean and standard deviation of the number of goals scored.

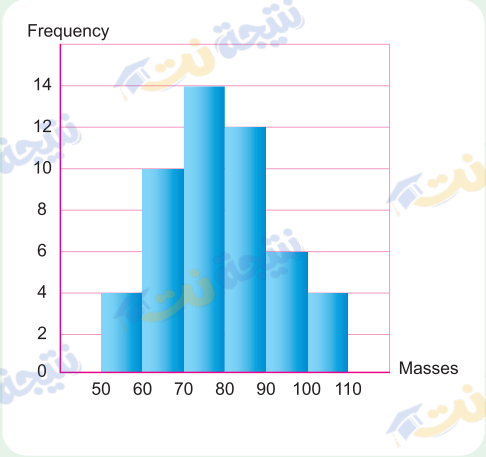
(10) If the range of the values: 21, $x+15$, 2, 5, 4, 7 is equal to 20, where x is a positive integer, find the value of x

(11) Find the interquartile range of the values: 15, 18, 22, 5, 29, 21, 17

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12 The given figure represents a histogram of the masses of a group of players in a sports club.

Find the arithmetic mean and standard deviation of the players' masses.



Unit Four Activity

Discovering Measures of Central Tendency and Dispersion in Our Daily Life

Activity Aim :

- To enrich students' application of measures of central tendency and dispersion.
- To provide students' skills in collecting and organizing data.
- To develop teamwork and analytical thinking skills.

Implementation Steps :



1. Divide the students into groups (of **2** to **4** students each as example).
2. Ask each group to choose a daily-life situation and collect data about it, such as:
   - Number of hours of using social media applications during a week.
   - Number of studying hours during **5** days.
   - Marks of a short quiz in one of the school subjects.
   - Number of hours students in the class spend practicing their favorite sport during a week.
3. Each group calculates the following:

   Arithmetic mean _ Median _ Mode _ Range _ Standard deviation.

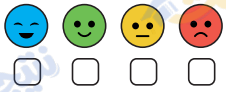
   Then the students answer some questions such as:
   - Which value represents the data best? Why?
   - Are the values close together or spread out?
4. Each group compares its results with the results of the other groups.

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Final Assessment

Evaluate your understanding!

How well do you understand the lessons of third preparatory? Tick the right box

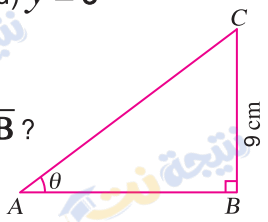


First Group

▸ **Choose the correct answer from the given ones:**

1. What is the equation of the axis of symmetry of the quadratic function f where $f(x) = (x-3)^2 + 1$?
   (a) $x = 1$ (b) $y = 1$
   (c) $x = 3$ (d) $y = 3$

2. In the given figure:
   If $\sin\theta = \frac{3}{5}$, what is the length of $\overline{AB}$?
   (a) 4 cm
   (b) 9 cm
   (c) 12 cm (d) 15 cm



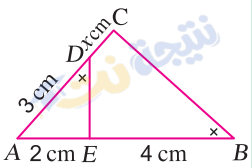
3. What is the image of the point $(-3, 6)$ under a dilation with center at the origin and a dilation factor 3?
   (a) $(-1, 2)$ (b) $(-9, 6)$
   (c) $(-3, 2)$ (d) $(-9, 18)$

4. What is the set of real zeros of the function f, where $f(x) = \frac{x^2-4}{x^2-5x+6}$?
   (a) $\{2, 3\}$ (b) $\{-2, 2\}$
   (c) $\{-2\}$ (d) $\{3\}$

5. What is the standard form of the equation of the straight line whose angle of inclination is of measure 45°, and its y-intercept is 3?
   (a) $x - y = -3$ (b) $x + y = -3$
   (c) $x - y = 3$ (d) $x + y = 3$

6. If $2\sin\theta = \tan\theta$, where θ is the measure of an acute angle, what is the value of θ?
   (a) 60° (b) 45° (c) 30° (d) 15°

7. In the given figure:
   If $m(\angle ADE) = m(\angle B)$, what is the value of x?
   (a) 1 (b) 2
   (c) 3 (d) 4



8. What is the slope of the perpendicular to the straight line whose equation is $x = 3y + 1$?
   (a) -3 (b) 3 (c) $\frac{-1}{3}$ (d) $\frac{1}{3}$

9. What is the interquartile range of the data represented by the given stem-and-leaf plot?
   (a) 36 (b) 3.6
   (c) 4.7 (d) 2

Stems	Leaves
3	1 2 3
4	5 5 7 8
5	1 3 4
6	7
Key	3 | 1 means 3.1

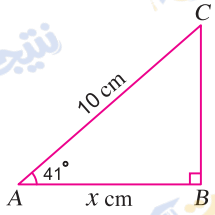
Second Group

▸ **Answer the following questions:**

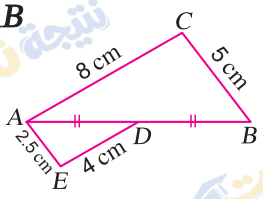
1. Find, in standard form, the equation of the straight line passing through the two points $(0, 1)$ and $(-4, 3)$

2. Without using a calculator, prove that:
   $\tan 60° = \frac{2\tan 30°}{1 - \tan^2 30°}$

3. In the given figure :
   Find the value of x, rounding to one decimal place.



4. In the given figure :
   If D is the midpoint of $\overline{AB}$, prove that:
   (1) $\Delta ABC \sim \Delta DAE$
   (2) $\overline{AE} \parallel \overline{BC}$



5. Simplify $f(x)$ to its simplest form, stating the domain of the function f, where
   $f(x) = \frac{x^2+x}{x^2-1} - \frac{5-x}{x^2-6x+5}$

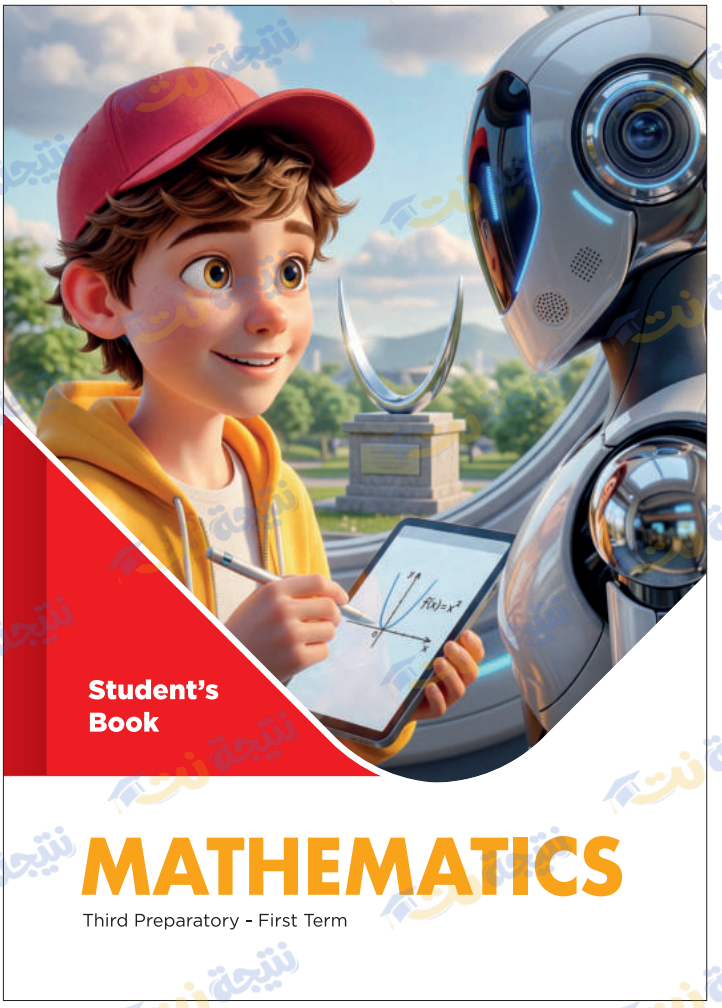
6. Graph the quadratic function f where $f(x) = (x-2)^2 + 3$, indicating on the graph the axis of symmetry, and find the minimum or maximum value of the function f

7. The following frequency distribution shows the ages of 10 children in years.

Age in years	7	10	11	12	14	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation of the ages in years.

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MATHEMATICS

Third Preparatory - First Term

تأليف وإعداد: شركة GPS للطبع والنشر والتوزيع

مقاس الكتاب	ورق المتن	ورق الغلاف	ألوان الكتاب	عدد الصفحات بالغلاف	التجليد	تاريخ الطبعة	كود الكتاب
19 x27 سم	٧٠ جرامًا	١٨٠ جرامًا	٤ ألوان	116 صفحة	دبوس	2027 - 2026	

رقم الإيداع:

الترقيم الدولى:

يحظر طبع أو نشر أو تصوير أو تخزين أو توزيع أي جزء من هذا الكتاب بأية وسيلة إلكترونية أو ميكانيكية أو بالتصوير أو خلاف ذلك،

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