

Student's Book

MATHEMATICS

Second Preparatory - First Term

The Ministry of Education and Technical Education dedicates this book, with love, to the students, teachers, and families of the Arab Republic of Egypt



MATHEMATICS

Second Preparatory - First Term

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Class : .....

School : .....

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Introduction

In the framework of the continuous development of the education system, and as part of the development plan that began with the kindergarten stage and continues until the end of the secondary stage, according to Egypt's Vision 2030 and the Ministry of Education's development plan, this book has been prepared to complete and implement the development processes of educational curriculums for the preparatory stage to create a qualitative leap in the way Egypt's students are prepared to deal with future challenges.

We are pleased to present this mathematics textbook for the second preparatory grade for the first term, which includes a set of tasks and performances that help students solve mathematical problems. This aims to achieve learning outcomes that keep pace with the challenges of the 21st century, whose structure is reinforced by the rapid revolution in information and communication technology.

The current curriculum aims to bring about a qualitative leap in the teaching and learning of mathematics. It also aims to provide an appropriate means to achieve the general educational goals in an integrated manner that aligns with the following:

- Emphasizing the positive role of the student in the learning process.
- Included knowledge curriculum, skills, values, positive trends needed for education, good citizenship, productive work and active participation in sustainable development programs.
- Including of modern positive trends in curriculum building, such as critical thinking skills, problem-solving skills, self-learning skills, collaborative learning, and effective communication with knowledge sources.
- Developing performance skills by focusing on self-learning and collaborative work.
- Achieving integration between mathematics and other subjects across different educational stages.
- Providing students with the opportunity to choose activities that suit their abilities, tendencies, and needs.

Asking Almighty God to make this book beneficial and hope that it will be part of a great effort to elevate Egypt to the ranks of advanced countries, ensuring a great future for all students. We wish you a very successful academic year full of achievements and success.

May God grant success,
The Authors

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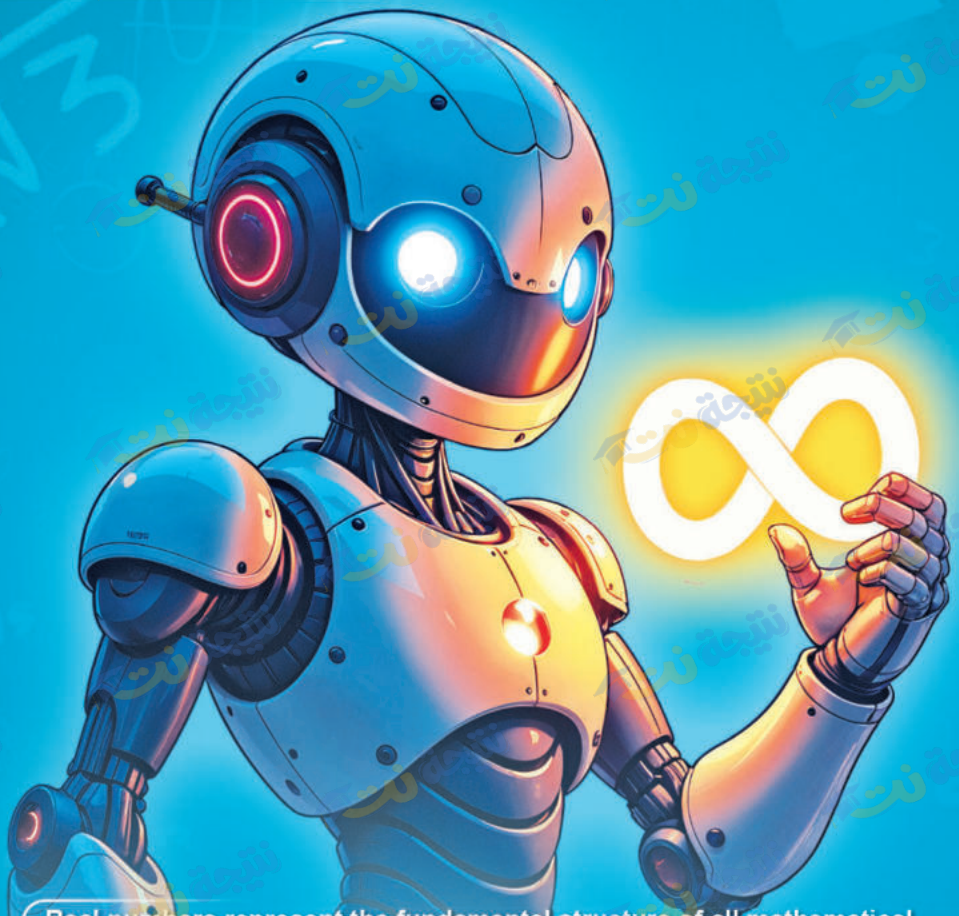
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UNIT 1

Numbers and Their Operations

Unit Lessons

1-1 Real Numbers
1-2 Intervals and Their Operations
1-3 Operations on Real Numbers
1-4 Laws of Square and Cube Roots
1-5 Laws of Exponents in Real Numbers



Real numbers represent the fundamental structure of all mathematical operations within artificial intelligence systems, and without them we cannot accurately represent the real world. Therefore, the relationship between artificial intelligence and real numbers is a fundamental integration between mathematics and technology.

▶ Can the role of real numbers be developed in building and training artificial intelligence algorithms?

Issues and Life Skills
- Critical Thinking
- Creative Thinking
- Sustainable Development
- Mathematical Communication

Values
- Respecting Others
- Environmental Awareness
- Responsibility
- Food Security Awareness

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Lesson

1 - 1

Real Numbers

Learning Outcomes

- Learn the concept of irrational number.
- Estimate the approximate value of irrational number and represent it on a number line.
- Learn the set of real numbers.
- Arrange a set of real numbers in descending or ascending order.
- Solve equations in the set of real numbers.

Vocabulary

- Irrational number
- Real numbers

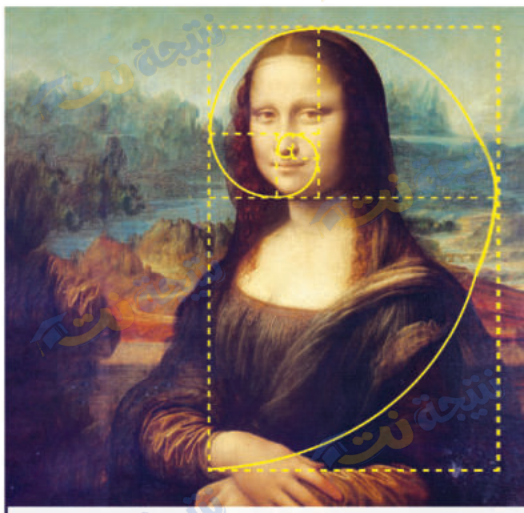
Previous Knowledge

A rational number is any number that can be written in the form of $\frac{a}{b}$ where a and b are integers, $b \neq 0$, and the set of rational numbers is denoted by the symbol Q.

- Give examples of rational numbers.

Get Ready!

The golden rectangle is a rectangle whose length is equal $\frac{1+\sqrt{5}}{2}$ to its width, and this ratio is called the 'golden ratio.' It is denoted by the symbol φ and appears in many aspects of nature. Artists have used it in their designs because it gives a sense of beauty and harmony. Leonardo da Vinci's Mona Lisa is one of the most famous artistic designs that applies this ratio. Why is this ratio not considered a rational number?



The Mona Lisa Portrait

In this lesson, you will learn the concept of irrational number and a new set of numbers called real numbers, which will enable you to answer questions like this.

Think & Discuss!

You have a set of numbers. Explain how you can arrange them from smallest to largest. Discuss with your classmates.

$\sqrt{40}$ $\quad$ 6 $\quad$ 4 $\quad$ $\sqrt{12}$

Learn!

Irrational numbers

An irrational number is a number that cannot be written in the form of $\frac{a}{b}$ where a and b are integers, $b \neq 0$, and the set of irrational numbers is denoted by the symbol Q'

Examples of irrational numbers:

- The square root of any number that is not a perfect square, such as: $\sqrt{2}$ where $\sqrt{2} \approx 1.414213562 \ldots$
- The cube root of any number that is not a perfect cube, such as : $\sqrt[3]{5}$ where $\sqrt[3]{5} \approx 1.709975947 \ldots$

Note That

Calculators give approximate values for irrational numbers such as : $\sqrt{2}$,$\sqrt[3]{5}$ where the decimal fraction is infinite and non-recurring, so such numbers cannot be written as rational numbers.

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■ Self-Evaluation (1)

Which of the following numbers are rationals, and which are irrationals?

$-\sqrt{4}$	17	$\sqrt{8}$
$0.\overline{2}$	0.3	$1\frac{1}{2}$
0	25%	$\sqrt[3]{25}$
$\sqrt{5}+1$	$\sqrt[3]{\lvert -64 \rvert}$	

■ Enriched Information

Not all irrational numbers are roots; there are other irrational numbers such as:

- The number π where:
  $\pi \approx 3.141592654...$
  It is equal to the ratio between the circumference of the circle and the length of its diameter.
- The golden ratio φ where:
  $\varphi \approx 1.618033989...$
  which is a distinctive ratio associated with beauty and harmony in design.

■ Note that

$9 < 11 < 16$
is an inequality of three sides which means that:
The number 11 is greater than the number 9
$(11 > 9)$
and the number 11 is less than the number 16
$(11 < 16)$
which means that: The number 11 lies between the two numbers 9 and 16.

Example 1

Which of the following numbers are rational numbers, and which are irrational? Give reasons.

1) -12 2) $\sqrt{25}$ 3) $\sqrt{13}$ 4) $\sqrt[3]{-27}$
5) $2\frac{2}{5}$ 6) $-\sqrt[3]{10}$ 7) 0.125 8) $3.\overline{6}$

■ Think

Can a number be both rational and irrational at the same time ?

▸ Each of the following numbers is a rational number

-12 , $\sqrt{25}$, $\sqrt[3]{-27}$, $2\frac{2}{5}$, 0.125 , $3.\overline{6}$

because each can be written in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$, as follows:

$-12 = \frac{-12}{1}$ | $\sqrt{25} = 5 = \frac{5}{1}$

$\sqrt[3]{-27} = -3 = \frac{-3}{1}$ | $2\frac{2}{5} = \frac{12}{5}$

$0.125 = \frac{125}{1000} = \frac{1}{8}$ | $3.\overline{6} = \frac{11}{3}$

■ Remember that

The number $3.\overline{6}$ is an infinite decimal (recurring) and it is a rational number where $3.\overline{6} = 3.6666... = \frac{11}{3}$

▸ Both numbers $\sqrt{13}$, $-\sqrt[3]{10}$ are irrational numbers.
Because: the number 13 is not a perfect square, the number 10 is not a perfect cube, and therefore neither can be written in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

Finding an approximate value for an irrational number

Any irrational number lies between two rational numbers and it can be represented on the number line by a point that lies between these two numbers.

Example 2

Find two consecutive integers between which each of the following numbers lies, and determine its approximate location on the number line.

1) $\sqrt{11}$ 2) $\sqrt[3]{20}$

(1) Find two consecutive integers, each of which is a perfect square, and 11 lies between them.

▸ The number 11 lies between the two numbers 9 and 16.

$\because 9 < 11 < 16 \quad \therefore \sqrt{9} < \sqrt{11} < \sqrt{16} \quad \therefore 3 < \sqrt{11} < 4$

$\therefore$ The number $\sqrt{11}$ lies between the two numbers 3 and 4 and since the number 11 is closer to the number 9 than to the number 16,

then the number $\sqrt{11}$ is closer to the number 3.



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■ **Self-Evaluation (2)**

Find the value of the integer n that satisfies: $n < \sqrt{17} < n+1$
Then find the approximate value of $\sqrt{17}$ to the nearest tenth and mark its approximate location on the number line.

(2) Find two consecutive integers, each of which is a perfect cube, and 20 lies between them.

▶ The number 20 lies between the two numbers 8 and 27

$\because 8 < 20 < 27 \quad \therefore \sqrt[3]{8} < \sqrt[3]{20} < \sqrt[3]{27} \quad \therefore 2 < \sqrt[3]{20} < 3$

$\therefore$ The number $\sqrt[3]{20}$ lies between the two numbers 2 and 3

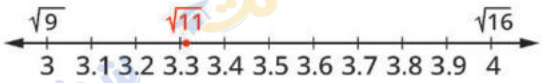
and since 20 is closer to 27 than to 8, so the number $\sqrt[3]{20}$ is closer to 3.



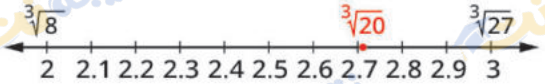
■ **Using Technology**

You can use a calculator to find a better approximation for an irrational number, for example:

- $\sqrt{11} \approx 3.3$



- $\sqrt[3]{20} \approx 2.7$

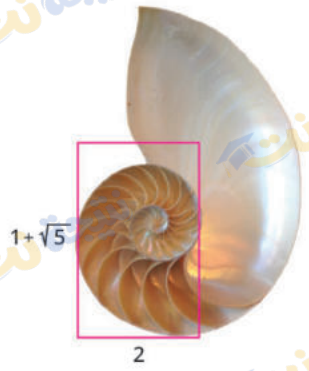


■ **Note that**

The ratio of the length of the golden rectangle to its width is approximately $\frac{1+\sqrt{5}}{2} \approx 1.6$ and in example (3) it is estimated to be approximately 1.5, that means the length of the golden rectangle is approximately one and a half times its width.

Example (3)

Nature: The golden rectangle is noticed in the shell of a marine organism, and the ratio of its length to its width is equal to $\frac{1+\sqrt{5}}{2}$.
Estimate this value.



First, estimate the value of $\sqrt{5}$ as follows:

$\because$ The number 5 lies between the two perfect square numbers 4 and 9

$\therefore 4 < 5 < 9 \quad \therefore \sqrt{4} < \sqrt{5} < \sqrt{9} \quad \therefore 2 < \sqrt{5} < 3$

$\because$ The number 5 is closer to the number 4 than to the number 9

$\therefore \sqrt{5}$ is closer to the number 2.

Use « $\sqrt{5} \approx 2$ » to estimate the value of $\frac{1+\sqrt{5}}{2}$

$\therefore \frac{1+\sqrt{5}}{2} \approx \frac{1+2}{2} \quad \therefore \frac{1+\sqrt{5}}{2} \approx 1.5$

■ **Self-Evaluation (3)**

A room has a square floor of area 15 m^2. Ahmed wants to surround the floor with a decorative strip. Estimate the length of this strip.

Real numbers

The set of real numbers is the set that consists of the union of the set of rational numbers and the set of irrational numbers, and is denoted by the symbol R

i.e. $R = Q \cup Q'$ **where** $Q \cap Q' = \varnothing$

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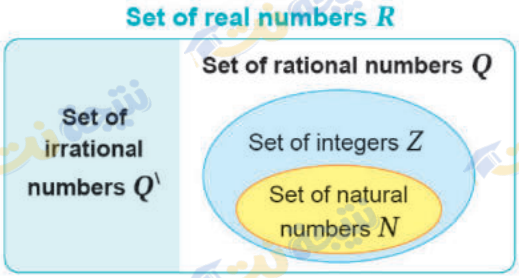
■ Note that

There is no square root for a negative real number.

The opposite Venn diagram shows the relationship between the sets of numbers, where:

$N \subset Z \subset Q \subset R$

$Q' \subset R$



■ Self-Evaluation (4)

Write all possible sets of the number sets (N, Z, Q, Q', R) to which each of the following numbers belongs:

1. $\sqrt{10}$
2. $\sqrt{100}$
3. $0.\overline{25}$
4. $-\sqrt[3]{8}$
5. $\sqrt[3]{9}$

Example 4

Put the appropriate symbol ∈ or ∉ in the empty box:

1. $0.\overline{63} \square Q'$
2. $\sqrt{5} \square R$
3. $|-4| \square N$
4. $\sqrt[3]{3\frac{3}{8}} \square Z$
5. $\sqrt[3]{3} \square Q$
6. $\sqrt{-1} \square R$

1. $\notin$ 2. $\in$ 3. $\in$ 4. $\notin$ 5. $\notin$ 6. $\notin$

Solving equations

Example 5

Find the solution set for each of the following equations:

1. $x^2 - 3 = 2$ where $x \in Q'$
2. $4x^3 = -32$ where $x \in Q'$
3. $\frac{1}{2}x^2 + 1 = -4$ where $x \in R$
4. $5x^3 - 1 = 4x^3 + 8$ where $x \in R$

■ Self-Evaluation (5)

Find the solution set for each of the following equations:

1. $\frac{1}{3}x^3 + 9 = 0$
2. $\frac{3}{4}x^2 = \frac{9}{2}$
3. $2(x^2 - 1) = -10$

If the substitution set is:
First: Q' Second: R

1. $\because x^2 - 3 = 2$
   $\therefore x^2 = 2 + 3 = 5$
   $\therefore x = \pm\sqrt{5} \in Q'$
   $\therefore$ The solution set $= \{\sqrt{5}, -\sqrt{5}\}$

2. $\because 4x^3 = -32$
   $\therefore x^3 = \frac{-32}{4} = -8$
   $\therefore x = \sqrt[3]{-8} = -2 \notin Q'$
   $\therefore$ The solution set $= \varnothing$

3. $\because \frac{1}{2}x^2 + 1 = -4$
   $\therefore \frac{1}{2}x^2 = -4 - 1 = -5$
   $\therefore x^2 = -5 \times 2 = -10$
   $\therefore x = \pm\sqrt{-10} \notin R$
   $\therefore$ The solution set $= \varnothing$

4. $\because 5x^3 - 1 = 4x^3 + 8$
   $\therefore 5x^3 - 4x^3 = 8 + 1$
   $\therefore x^3 = 9$
   $\therefore x = \sqrt[3]{9} \in R$
   $\therefore$ The solution set $= \{\sqrt[3]{9}\}$

Collaborative Activity

Use different sources (e.g., the Internet, your school library, etc.) and work with your classmates under your teacher's supervision to find out more about the golden ratio φ, its history, and its various uses in architecture and arts.

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Lesson Assessment



First Measuring Conceptual Understanding



▸ **Choose the correct answer from the given ones:**

(1) Which of the following numbers is an irrational number?

(a) $0.\overline{3}$ (b) $\sqrt{\frac{9}{25}}$ (c) $\sqrt{5}$ (d) $\sqrt[3]{-125}$

(2) If $X \in Z$, $X < \sqrt{29} < X + 1$, what is the value of X?

(a) 4 (b) 5 (c) 6 (d) 7

(3) Estimating $\sqrt{41}$ to the closest integer is .....

(a) 6 (b) 7 (c) 36 (d) 49

(4) Estimating $\sqrt[3]{25}$ to the closest integer is .....

(a) 2 (b) 3 (c) 5 (d) 12.5

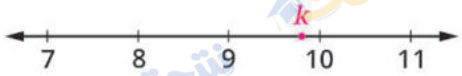
(5) Which of the following numbers is an irrational number that lies between 2 and 3?

(a) $2.\overline{5}$ (b) $\sqrt{10}$ (c) $\sqrt{6.25}$ (d) $\sqrt[3]{9}$

(6) If $1 < \sqrt{n} < 2$, which of the following is a possible value for the number n?

(a) 1 (b) 2 (c) 4 (d) 8

(7) Which of the following numbers is represented by k on the number line below?



(a) $\sqrt{86}$ (b) $\sqrt{96}$ (c) $\sqrt{81}$ (d) $\sqrt{78}$

(8) If X and Y are two consecutive integers, what is the value of $X + Y$?



(a) 11 (b) 13 (c) 15 (d) 17

▸ **Discuss:**

(9) Hamza and Aser answered the following question:

Find two consecutive integers between which the irrational number $-\sqrt{5}$ lies.

Hamza's answer:

$4 < 5 < 9$

$\sqrt{4} < \sqrt{5} < \sqrt{9}$

$2 < \sqrt{5} < 3$

$-3 < -\sqrt{5} < -2$

Aser's answer:

$-9 < -5 < -4$

$-\sqrt{9} < -\sqrt{5} < -\sqrt{4}$

$-3 < -\sqrt{5} < -2$

Which of them followed the correct method to solve the problem?

Second Applying Concepts



(10) **Classify each number in the opposite table as rational or irrational and explain why:**

0.323232...	$-\sqrt[3]{64}$	$1.\overline{6}$	$\sqrt{\frac{4}{9}}$	$\frac{12}{31}$	$\sqrt{6}$
$\sqrt[3]{16}$	17%	$2\frac{2}{3}$	$\sqrt[3]{-27}$	$-\sqrt{121}$	0

▸ **Find two consecutive integers between which each of the following numbers lies, and determine its approximate location on the number line:**

(11) $\sqrt{5}$ (12) $\sqrt{13}$ (13) $-\sqrt{11}$ (14) $\sqrt[3]{14}$

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▸ **Put one of the symbols <, >, or = in each box to get a true statement:**

(15) $\sqrt{96} \square 14$ (16) $-\sqrt{7} \square -2.6$ (17) $\sqrt[3]{25} \square \sqrt{9}$

(18) Arrange the numbers : $6.3\overline{6}$, $6\frac{4}{5}$, 7 , $\sqrt{40}$ from the smallest to the largest.

▸ **Find the solution set for each of the following equations in R:**

(19) $x^3 - 11 = 5$ (20) $2x^3 + 13 = 67$ (21) $3x^2 - 1 = -13$

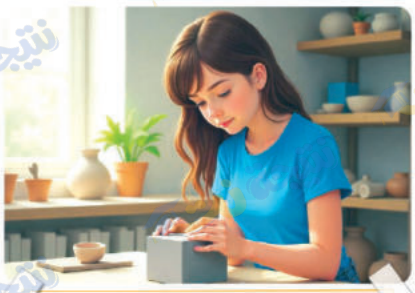
▸ **Find the solution set for each of the following equations in Q':**

(22) $2x^2 - 3 = 9$ (23) $6x^2 - 3 = 4x^2 + 7$ (24) $2(x^3 + 1) = 18$

Third Analysis and Subjects Integration

(25) **Geometry:** A square with an area of 29 cm^2, to which of the following sets of numbers does its side length belong? Select all possible sets.

☐ R ☐ Q ☐ Q' ☐ Z ☐ N



The arts help develop creativity and innovation in individuals.

(26) **Arts:** Samar formed a cube using clay with a total area of 300 cm^2. Find the length of its edge. Is the length of the edge a rational number?

(27) **Geometry:** Which is greater: the length of the side of a square with an area of 16 cm^2 or the length of the diagonal of a square with an area of 9 cm^2?

(28) **Agriculture:** A rectangular garden with an area of 80 m^2, its length is twice its width.
Find the length and width to the nearest meter.



Gardens purify the air and improve people's mental health.

(29) What is the smallest valid integer of variable n that makes both numbers $\sqrt{41+n}$ and $\sqrt{2n+20}$ be rational numbers?

Creative Thinking

(30) Find an irrational number lying between $0.\overline{3}$ and $0.\overline{6}$

(31) If $10 < \sqrt{x} < 11$, $12 < \sqrt{y} < 13$ where x and y are positive integers, find the largest value of $y - x$.

***Evaluate** your understanding!*

How well do you understand real numbers?
Tick the right box

☐ ☐ ☐ ☐

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Lesson

1 - 2

Intervals and Their Operations

Learning Outcomes

- Learn the concept of interval as a subset of the set of real numbers.
- Represent an interval on the number line.
- Perform intersection, union, difference, and complement operations on intervals.
- Use the concept of intervals to solve real-life problems.

Vocabulary

- Interval
- Bounded interval
- Unbounded interval
- Closed interval
- Open interval
- Intersection
- Union
- Difference
- Complement
- Universal set
- Infinity

Get Ready!

Hurghada is one of the world's most popular diving destinations,where tourists can enjoy diving in the Red Sea to see beautiful coral reefs and a variety of marine life.



Coral reefs and fish in the Red Sea

On one of the diving trips in Hurghada, the divers wanted to see two species of fish at the same time. If you know that:

- The first kind lives at depths from 15 m to 60 m.
- The second kind lives at depths from 10 m to 25 m. Magdy dived to a depth of 30 m, and Amal dived to a depth of 12 m., while Bassem dived to a depth of 20 m. Which of them can see both species at the same time?

In this lesson, you will learn the concept of intervals, types of intervals, and operations on intervals, which will enable you to solve such real-life problems.

Think & Discuss!

One day, Mustafa's university lectures were scheduled as follows:
From 9:00 a.m. to 12:00 p.m.
Then from 2:00 p.m. to 4:00 p.m.
If the university sports center is used to open daily from 10:00 a.m. to 1:00 p.m., what is the appropriate time for Mustafa to exercise at the university sports center on that day without affecting his lectures attendance? Discuss.



Learn!

Intervals

- **Between any two real numbers there are infinite real numbers.**
- **The set of real numbers and its subsets cannot be expressed by listing method, because it is impossible to list all its elements. Therefore, another method is used to express them, it is called 'intervals'.**

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First The bounded intervals

If a and b are real numbers such that $a < b$, then:

The set of real numbers greater than or equal to a and less than or equal to b

$\{X : X \in R , a \leq X \leq b\} = [a , b]$



The set of real numbers greater than a and less than b

$\{X : X \in R , a < X < b\} =]a , b[$



The set of real numbers greater than or equal to a and less than b

$\{X : X \in R , a \leq X < b\} = [a , b[$



The set of real numbers greater than a and less than or equal to b

$\{X : X \in R , a < X \leq b\} =]a , b]$



■ Note that

- $[a , b]$ is called a closed interval where $a \in [a , b]$, $b \in [a , b]$
- $]a , b[$ is called an open interval where $a \notin]a , b[$, $b \notin]a , b[$
- $[a , b[$ is called a half-open or half-closed interval where $a \in [a , b[$, $b \notin [a , b[$
- $]a , b]$ is called a half-open or half-closed interval where $a \notin]a , b]$, $b \in]a , b]$

■ Self-Evaluation (1)

Write in the form of an interval and represent on the number line each of the following sets:

1. $\{X : X \in R , -1 < X < 5\}$
2. $\{X : X \in R , 0.5 \leq X < \sqrt{9}\}$

Example 1

1 Write in the form of an interval and represent on the number line set A:

$A = \{X : X \in R , 1 < X \leq 3\}$

Is $\sqrt{2} \in A$?

2 Write in the form of an interval and represent on the number line set B:

$B = \{X : X \in R , -2 < X < 2\}$

Is $\sqrt[3]{-9} \in B$?

(1) $A =]1 , 3]$



By using a calculator :

$\sqrt{2} \approx 1.4$

$\therefore 1 < \sqrt{2} \leq 3$

$\therefore \sqrt{2} \in]1 , 3]$

(2) $B =]-2 , 2[$



By using a calculator :

$\therefore \sqrt[3]{-9} \approx -2.08$

$\therefore \sqrt[3]{-9} < -2$

$\therefore \sqrt[3]{-9} \notin]-2 , 2[$

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Second The unbounded intervals

- The set of real numbers extends along the number line in the positive direction to infinity (∞) and extends in the negative direction to infinity (– ∞)
- The symbols ∞ and – ∞ are used to express the unbounded sets as follows: If $a \in R$, then:

The set of real numbers greater than or equal to a $\{x : x \in R , x \geq a\} = [a , \infty[$ 	The set of real numbers less than or equal to a $\{x : x \in R , x \leq a\} =]-\infty , a]$ 
The set of real numbers greater than a $\{x : x \in R , x > a\} =]a , \infty[$ 	The set of real numbers less than a $\{x : x \in R , x < a\} =]-\infty , a[$ 

■ Note that

The interval $]-\infty , \infty[$ represents the set of real numbers R ,
$\infty \notin R$ and $-\infty \notin R$

■ History of mathematics

English scientist John Wallis was the first to use the symbol (∞).



■ Note that

- $R = R^+ \cup \{0\} \cup R^-$ where :
  R^+ is the set of positive real numbers,
  R^- is the set of negative real numbers.
- The set of non-negative real numbers $= R^+ \cup \{0\}$
- The set of non-positive real numbers $= R^- \cup \{0\}$

■ Self-Evaluation (2)

Write in the form of an interval and represent on the number line:

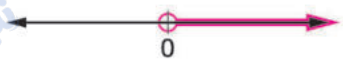
1. The set of negative real numbers R^-
2. The set of non-negative real numbers
3. $\{x : x \in R^+ , x < 3\}$

Example (2)

Write each of the following sets as an interval and represent it on the number line:

1. The set of positive real numbers
2. The set of non-positive real numbers
3. $\{x : x \in R , x \geq -2\}$
4. $\{x : x \in R , x < 1\}$

(1) $]0 , \infty[$



(2) $]-\infty , 0]$



(3) $[-2 , \infty[$



(4) $]-\infty , 1[$



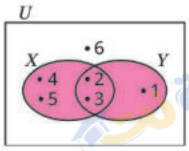
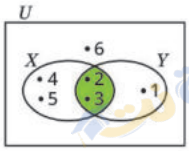
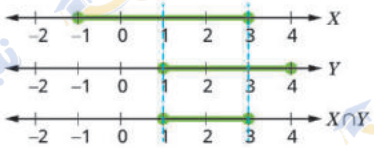
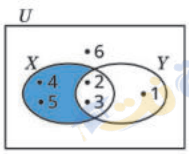
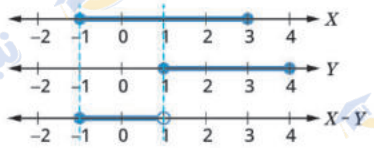
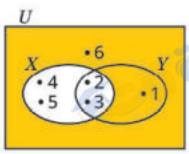
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Operations on intervals

Intervals are subsets of the set of real numbers,therefore, union, intersection, difference, and complement operations can be performed on them in the same way as on sets, as follows :

■ Note that

The universal set is a set that contains all of its subsets, and it is denoted by the symbol U.

	Sets	Intervals
	Assuming that: $U=\{1,2,3,4,5,6\}$ where U is the universal set $X=\{2,3,4,5\}$ $Y=\{1,2,3\}$	Assuming that: $U=\mathbb{R}$ where U is the universal set $X=[-1,3]$ $Y=[1,4]$
Union $(X \cup Y)$: The set of elements that belong to X or Y.	 $X \cup Y=\{1,2,3,4,5\}$	 $X \cup Y=[-1,4]$
Intersection $(X \cap Y)$: The set of elements that belong to both X and Y	 $X \cap Y=\{2,3\}$	 $X \cap Y=[1,3]$
Difference $(X-Y)$: The set of elements that belong to X but not to Y	 $X-Y=\{4,5\}$	 $X-Y=[-1,1[$
Complement X' : The set of elements that belong to U but not to X	 $X'=U-X=\{1,6\}$	 $X'=R-X$ $=]-\infty,-1[\cup]3,\infty[$

■ Think & Discuss

Is : $X-Y=Y-X$?

■ Self-Evaluation (3)

Find using the number line:

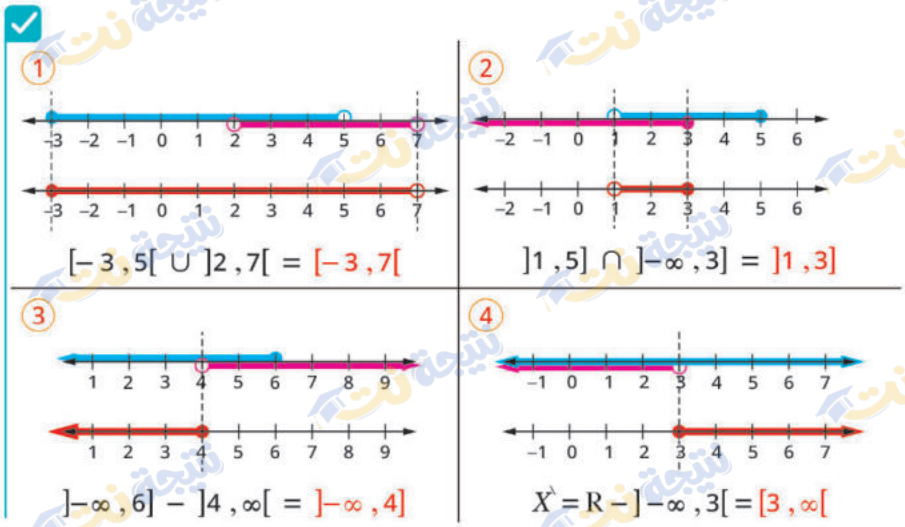
1. $[4,9] \cap [7,10[$
2. $]-3,2[\cup]0,\infty[$
3. $]0,5[-]3,7]$
4. X' where $X=[-5,\infty[$

Example 3

Find using the number line each of the following:

1. $[-3,5[\cup]2,7[$
2. $]1,5] \cap]-\infty,3]$
3. $]-\infty,6]-]4,\infty[$
4. X' where $X=]-\infty,3[$

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Lesson Assessment



First Measuring Conceptual Understanding



▸ **Choose the correct answer from the given ones:**

(1) If $a \in]2,5[$, which of the following can be the value of a?

(a) 1 (b) 2 (c) 4 (d) 5

(2) If $a \notin]-1,3]$, which of the following can be the value of a?

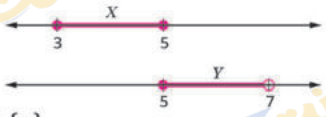
(a) -1 (b) 0 (c) 2 (d) 3

(3) If $X = [-5,7[$, which of the following is true?

(a) $7 \in X$ (b) $-5 \notin X$

(c) $0 \notin X$ (d) $-2 \in X$

(4) Which of the following represents $X \cap Y$?



(a) $\varnothing$ (b) $\{5\}$

(c) $[3,7[$ (d) $[3,5[$

(5) What is the interval resulting from $]2,5[\cup \{2\}$?

(a) $]2,5[$ (b) $[2,5]$

(c) $]2,5]$ (d) $[2,5[$

(6) Which of the following intervals does $-\sqrt{7}$ belong to?

(a) $[-2,-1]$ (b) $[-3,-2]$

(c) $[-4,-3]$ (d) $[-7,-6]$

Second Applying Concepts



(7) If $X =]-2,2[$, $Y = [1,4]$, find using the number line:

$X - Y$, $Y - X$, $X \cup Y$, $X \cap Y$

(8) If $A =]-5,\infty[$, $B =]-\infty,2]$, find using the number line:

$A \cup B$, $A \cap B$, $A - B$, $B - A$, A'

(9) If $A = [-1,3]$, $B =]-\infty,3[$, find each of the following:

$A - B$, $B - A$, $A \cup B$, $A \cap B$, B'

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10 Find each of the following:

① $[2,5[\cup]-2,3]$ ② $[2,7]-[3,8[$ ③ $]-1,3[\cap]-5,1]$

④ X' where $X=[4,8[$ ⑤ $[3,\infty[\cap]-\infty,7[$ ⑥ $]2,5] \cup]5,\infty[$

11 Find each of the following:

① $R \cap [2,5[$ ② $R^+ \cup]0,2]$ ③ $R^- - [-2,0[$ ④ $R^- \cup R^+$

12 Find each of the following:

① $N \cap [-3,0[$ ② $Z^+ \cap]-1,2[$ ③ $Z^- \cap]-2,3]$ ④ $\{3,5\} \cup]3,5[$

⑤ $\{3,7\} \cap [3,7[$ ⑥ $[2,5]-\{2,5\}$ ⑦ $[0,5]-\{5\}$

Third Analysis and Subjects Integration

13 Express each of the following situations using an appropriate inequality, then write it as an interval, and represent it on a number line.

① **Work :** You must be at least 25 years old and no more than 45 years old to apply for a job.

② **Geography :** The temperature in a city in North America ranges from – 2°C to 10°C.

14 Nutrition: Laila decided to keep two types of food in the refrigerator:

- The first type is fresh milk, the best temperature to keep it from 1°C to 4°C
- The second type is fresh meat, the best temperature to keep it from – 2°C to 2°C

What is the temperature range can the refrigerator be set to keep both types together at the same time without spoiling either of them?



Food preservation is vital to achieving sustainable development and enhancing food security.



Caring for and paying attention to parents is not a service but a duty and responsibility.

15 Social Responsibility: Amal and Iman are two sisters wanted to organize their Saturday time to take care of their elderly mother, so that at least one of them would be with their mother at any time of the day until their father returned back home.

- Amal can take care of her mother from 10:00 a.m. to 2:00 p.m.
- For Iman, she can take care of her mother from 12:00 p.m. to 4:00 p.m.

What is the time period that at least one of them would be with their mother?

Creative Thinking

16 If a, b, c, and d are real numbers, $a < c < b < d$, find each of the following:

$[a,b] \cap [c,d]$ ، $[a,b]-[c,d]$

17 If $[-2,a] \cap [b,7] = [2,4]$, what is the value of $\frac{b^a}{a^b}$?

***Evaluate** your understanding!*

How well do you understand intervals and their operations? Tick the right box



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Lesson

1 - 3 Operations on Real Numbers

Learning Outcomes

- Perform mathematical operations (addition, subtraction, multiplication, and division) on real numbers.
- Learn the properties of addition and multiplication of real numbers.
- Solve some real-life problems using mathematical operations on real numbers.

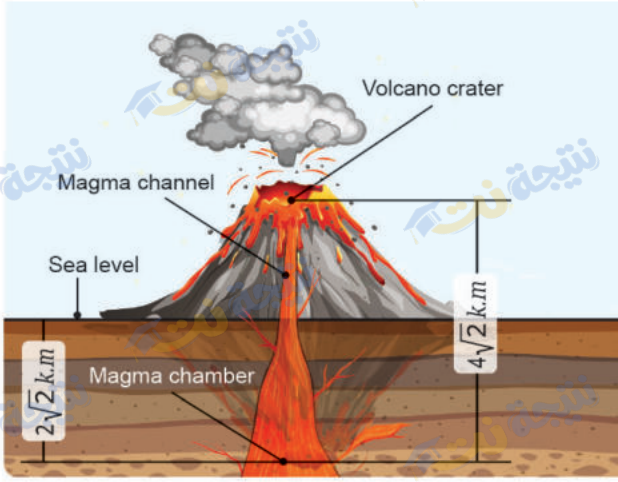
Vocabulary

- Closure
- Commutative
- Associative
- Additive identity
- Multiplicative identity
- Additive inverse
- Multiplicative inverse
- Radical term

Get Ready!

Lava comes out from a magma chamber deep under the Earth through a channel called a magma channel, until it reaches the crater of the volcano, where it explodes and rushes out in the form of lava, gases and volcanic ash.

In the figure in front of you, how far is the crater from the sea level?



In this lesson, you will learn how to perform mathematical operations on real numbers, which will enable you to solve such real-life problems.

Think & Discuss!

A rectangular carpet with dimensions $(\sqrt{5}+1)\ m$, $(\sqrt{5}-1)\ m$.

A square carpet with side length $\sqrt{5}\ m$.

Which of the two carpets is larger in area?

Which is larger in perimeter? Discuss.



Learn!

Operations on real numbers

Before studying how to perform mathematical operations (addition, subtraction, multiplication, and division) on real numbers, we will first introduce the concept of like radical terms.

Like radical terms are terms that contain roots with the same index and the same radicand (the number below the root sign)



Examples of like radical terms:

- $4\sqrt{3}$, $2\sqrt{3}$
- $\sqrt[3]{5}$, $-3\sqrt[3]{5}$

Examples of unlike radical terms:

- $3\sqrt{2}$, $2\sqrt{3}$
- $2\sqrt[3]{5}$, $2\sqrt{5}$

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Example 1

If $x = -4\sqrt{3}$, $y = \sqrt{3}$, $z = \sqrt[3]{5}$,

find the value of each of the following:

1 $x + y$ 2 $x \times y$ 3 $\frac{x}{2y}$ 4 $2x - z^3$

1 $x + y = -4\sqrt{3} + \sqrt{3}$

$= (-4 + 1)\sqrt{3}$

$= -3\sqrt{3}$

2 $x \times y = -4\sqrt{3} \times \sqrt{3}$

$= (-4 \times 1) \times (\sqrt{3})^2$

$= -4 \times 3 = -12$

3 $\frac{x}{2y} = \frac{-4\sqrt{3}}{2 \times \sqrt{3}}$

$= \frac{-4\sqrt{3}}{2\sqrt{3}} = \frac{-4}{2} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= -2 \times 1 = -2$

4 $2x - z^3 = 2 \times (-4\sqrt{3}) - (\sqrt[3]{5})^3$

$= -8\sqrt{3} - 5$

■ Note that

When adding and subtracting like radical terms, we use the same method as for adding and subtracting like algebraic terms:
$-4x + x = (-4 + 1)x = -3x$

■ Note that

When multiplying radical terms, we use the same method as for multiplying algebraic terms:
$-4x \times x = (-4 \times 1)x^2 = -4x^2$

■ Note that

The radical term $-8\sqrt{3}$ is not like to the term 5 , so they can not be subtracted.

Example 2

Write each of the following numbers in the simplest form , so that the denominator is an integer:

1 $\frac{7}{\sqrt{3}}$ 2 $\frac{-15}{\sqrt{5}}$ 3 $\frac{12}{5\sqrt{2}}$

1 To make the denominator an integer, multiply both numerator and denominator by $\sqrt{3}$

$\therefore \frac{7}{\sqrt{3}} = \frac{7}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{3}}{3}$

2 To make the denominator an integer, multiply both numerator and denominator by $\sqrt{5}$

$\therefore \frac{-15}{\sqrt{5}} = \frac{-15}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{-15\sqrt{5}}{5} = -3\sqrt{5}$

3 To make the denominator an integer, multiply both numerator and denominator by $\sqrt{2}$

$\therefore \frac{12}{5\sqrt{2}} = \frac{12}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\overset{6}{\cancel{12}}\sqrt{2}}{5 \times \cancel{2}_1} = \frac{6\sqrt{2}}{5}$

■ Self-Evaluation (1)

Find the value of each of the following:

1 $3\sqrt{5} - 4\sqrt{5}$

2 $-5\sqrt{2} \times 3\sqrt{2}$

3 $\frac{(\sqrt{7})^2 \times 3\sqrt{3}}{7\sqrt{3}}$

■ Note that

• If $x \geq 0$, then :

$\sqrt{x} \times \sqrt{x} = (\sqrt{x})^2 = x$

• If $x \in R$, then :

$\sqrt[3]{x} \times \sqrt[3]{x} \times \sqrt[3]{x} = (\sqrt[3]{x})^3 = x$

Critical Thinking

1 Dina wrote:
$2\sqrt{3} + 3\sqrt{2} = 5\sqrt{5}$
Is what Dina wrote correct? Discuss.

2 Islam wrote:
$2\sqrt{5} + 5 = 7\sqrt{5}$
Is what Islam wrote correct? Discuss.

■ Self-Evaluation (2)

Write each of the following in the simplest form, so that the denominator is an integer:

1 $\frac{6}{\sqrt{3}}$ 2 $\frac{-5}{\sqrt{2}}$

3 $\frac{9}{5\sqrt{3}}$

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Properties of operations on real numbers

If a, b, and c are three real numbers, then:

Property	Addition operation	Multiplication operation
Closure	$a+b\in R$	$a\times b\in R$
Commutativity	$a+b=b+a$	$a\times b=b\times a$
Association (grouping)	$(a+b)+c=a+(b+c)$	$(a\times b)\times c=a\times(b\times c)$
Distribution of multiplication over addition	$a(b+c)=ab+ac$	
Identity	The additive identity is 0 $a+0=0+a=a$	The multiplicative identity is 1 $a\times 1=1\times a=a$
Inverse	The additive inverse of the number a is $(-a)$ where : $a+(-a)=0$	The multiplicative inverse of the number a is $\frac{1}{a}$ (where $a\neq 0$) where : $a\times\frac{1}{a}=1$

■ Think & Discuss

For any three real numbers a, b, and c:

1. Is $a-b=b-a$?
2. Is $\frac{a}{b}=\frac{b}{a}$?
3. Is $\frac{a}{b}\in R$?
4. Is $a(b-c)=ab-ac$?

■ Self-Evaluation (3)

If $x=(\sqrt{5}-2)$,

$y=(\sqrt{5}+2)$

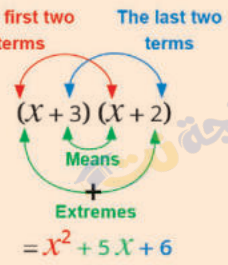
Find in its simplest form the value of the expression x^2-y^2

■ Previous Knowledge

- $(x+y)(x-y)=x^2-y^2$
- $(x+y)^2=x^2+2xy+y^2$
- $(x-y)^2=x^2-2xy+y^2$

■ Try it yourself

Try to find the product directly by inspection as follows:



$=x^2+5x+6$

Example 3

Find the result of each of the following in its simplest form :

1. $2\sqrt{3}(4+\sqrt{3})-4\sqrt{3}$
2. $(\sqrt{7}-1)(2\sqrt{7}+3)$
3. $(2\sqrt{2}+1)(2\sqrt{2}-1)$
4. $(2\sqrt{5}-5)^2$

(1) $2\sqrt{3}(4+\sqrt{3})-4\sqrt{3}=2\sqrt{3}\times 4+2\sqrt{3}\times\sqrt{3}-4\sqrt{3}$

$=8\sqrt{3}+2\times 3-4\sqrt{3}$

$=8\sqrt{3}+6-4\sqrt{3}$

$=8\sqrt{3}-4\sqrt{3}+6=4\sqrt{3}+6$

(2) $(\sqrt{7}-1)(2\sqrt{7}+3)=\sqrt{7}\times 2\sqrt{7}+\sqrt{7}\times 3-1\times 2\sqrt{7}-1\times 3$

$=14+3\sqrt{7}-2\sqrt{7}-3$

$=(14-3)+(3\sqrt{7}-2\sqrt{7})$

$=11+\sqrt{7}$

(3) $(2\sqrt{2}+1)(2\sqrt{2}-1)=(2\sqrt{2})^2-(1)^2=8-1=7$

(4) $(2\sqrt{5}-5)^2=(2\sqrt{5})^2+(2\times 2\sqrt{5}\times(-5))+(-5)^2$

$=20-20\sqrt{5}+25$

$=45-20\sqrt{5}$

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Example 4

Find in its simplest form the additive inverse and the multiplicative inverse of each of the following:

1) $3\sqrt{2}$ 2) $2-\sqrt{3}$

1) The additive inverse $= -3\sqrt{2}$

The multiplicative inverse $= \frac{1}{3\sqrt{2}}$

$\because \frac{1}{3\sqrt{2}} = \frac{1}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{6}$

$\therefore$ The multiplicative inverse in its simplest form $= \frac{\sqrt{2}}{6}$

2) The additive inverse $= -(2-\sqrt{3})$

$= -2+\sqrt{3}$

The multiplicative inverse $= \frac{1}{2-\sqrt{3}}$

$\because \frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{2+\sqrt{3}}{4-3} = \frac{2+\sqrt{3}}{1} = 2+\sqrt{3}$

$\therefore$ The multiplicative inverse in its simplest form $= 2+\sqrt{3}$

Note that

To simplify $\frac{1}{2-\sqrt{3}}$ multiply both the numerator and the denominator by $(2+\sqrt{3})$.

Self-Evaluation (4)

Find in its simplest form the additive inverse and the multiplicative inverse of each of the following:

1) $-4\sqrt{2}$ 2) $\frac{\sqrt{5}}{5}$

3) $3\sqrt{7}-4$

Example 5

In the opposite figure:

A covered glass fish tank in the shape of a cuboid. Find:

1) The volume of the glass tank.
2) The total area of the tank to the nearest m^2.



1) $\because$ The volume of the tank (V) = length × width × height

$\therefore V = 2\sqrt{2} \times 2 \times \sqrt{2}$

$= 2 \times 2 \times \sqrt{2} \times \sqrt{2} = 4 \times 2 = 8\ m^3$.

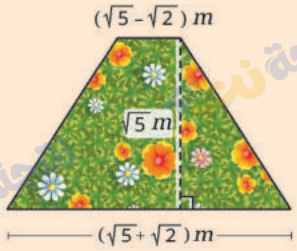
2) $\because$ Total area $(A) = 2$ (length × width + length × height + width × height)

$\therefore A = 2(2\sqrt{2} \times 2 + 2\sqrt{2} \times \sqrt{2} + 2 \times \sqrt{2})$

$= 2(4\sqrt{2} + 4 + 2\sqrt{2}) = 2(6\sqrt{2} + 4)$

$= 12\sqrt{2} + 8 \approx 25\ m^2$.

Self-Evaluation (5)



In the figure, a flower pot has a base in the shape of a trapezoid.
Find the area of the base of the pot.

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Lesson Assessment



First Measuring Conceptual Understanding



▸ Choose the correct answer from the given ones:

1. If $a+\sqrt{5}=0$, what is the value of a?
   (a) 0 (b) $\sqrt{5}$ (c) $-\sqrt{5}$ (d) $\frac{1}{\sqrt{5}}$

2. If $a\times\sqrt{2}=1$, what is the value of a ?
   (a) 1 (b) $\sqrt{2}$ (c) $-\sqrt{2}$ (d) $\frac{\sqrt{2}}{2}$

3. What is the value of $\sqrt{3}+\sqrt{3}$?
   (a) $\sqrt{3}$ (b) $2\sqrt{3}$
   (c) $\sqrt{6}$ (d) 3

4. If $a\sqrt{5}-4\sqrt{5}=3\sqrt{5}$, then :
   (a) $a=-1$ (b) $a=1$
   (c) $a=7$ (d) $a=10$

5. If $(2\sqrt{3})^{n}=12$, then:
   (a) $n=2$ (b) $n=3$
   (c) $n=4$ (d) $n=6$

6. If $\sqrt[3]{5}+3a=4\sqrt[3]{5}$, then :
   (a) $a=1$ (b) $a=\sqrt{5}$
   (c) $a=\sqrt[3]{5}$ (d) $a=5$

7. What is the additive inverse of the number $\frac{7}{\sqrt{7}}$ in its simplest form?
   (a) $\frac{\sqrt{7}}{7}$ (b) 7
   (c) $-\sqrt{7}$ (d) -7

8. What is the multiplicative inverse of the number $\sqrt{3}-2$?
   (a) $2-\sqrt{3}$ (b) $\sqrt{3}+2$
   (c) $-\sqrt{3}-2$ (d) $\sqrt{3}-2$

Second Applying Concepts



9 Make the denominator in each of the following an integer and write the number in its simplest form:

① $\frac{1}{\sqrt{15}}$ ② $\frac{30}{\sqrt{10}}$ ③ $\frac{10}{2\sqrt{5}}$ ④ $\frac{\sqrt{2}+3}{\sqrt{2}}$

⑤ $\frac{1}{2+\sqrt{6}}$ ⑥ $\frac{20}{5-\sqrt{15}}$ ⑦ $\frac{3}{\sqrt{5}+\sqrt{2}}$

10 Find the result of each of the following in its simplest form:

① $2\sqrt{7}-3\sqrt{3}+\sqrt{7}+2\sqrt{3}$

② $2\sqrt{5}-3\sqrt[3]{2}+4\sqrt{5}+\sqrt[3]{2}$

③ $(\sqrt[3]{5})^{3}\times 2\sqrt{3}$

④ $\sqrt{5}(2-\sqrt{5})-2(1+\sqrt{5})$

⑤ $(\sqrt{7}-1)^{2}+(\sqrt{7}+2)(\sqrt{7}-3)$

⑥ $(3+2\sqrt{2})^{2}-\sqrt{2}(1-\sqrt{2})$

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(11) If $x = \frac{1}{\sqrt{3}+2}$, $y = \sqrt{3}+2$, **find the value of each of the following in its simplest form:**

① $(x+y)^2$ ② xy ③ x^2+y^2

(12) If $\frac{x}{3\sqrt{2}} = \frac{y}{2\sqrt{3}} = 1$, **then find the value of** x^2+y^2

Third Analysis and Subjects Integration

(13) **Geometry :** Find the area of each of the following shapes in its simplest form :

① Square

$3\sqrt{5}$ cm

② Rectangle

$(2\sqrt{5}-\sqrt{3})$ cm

$(2\sqrt{5}+\sqrt{3})$ cm

③



$(\sqrt{5}+\sqrt{3})$ cm

(14) **Agriculture :**

Omar built two flower beds in his garden, one of which is rectangular with dimensions $(5+\sqrt{7})$ m, $(5-\sqrt{7})$ m, and the other is square with the same area as the first bed. Find the perimeter of each bed. Which one has the larger perimeter?



Growing roses helps purify the air, improves mood, and reduces stress.



Learning handicrafts develops fine motor skills and promotes creativity and innovation.

(15) **Handicrafts:**

Ahmed is working on a small project to produce box frames.

If he has a copper wire 90 cm long and wants to make two wire frames, one of them is a cube with a volume of 27 cm^3 and the other is a cuboid with dimensions $4\sqrt{3}$ cm, 4 cm, and $2\sqrt{3}$ cm. Is Ahmed's wire long enough to make both structures?

Creative Thinking

(16) When constructing a storage room in the shape of a cuboid, its dimensions were a, b, and c meters. If $ab = 3\sqrt{3}$, $bc = 5\sqrt{3}$, $ac = 5$, find the volume of the storage room.

***Evaluate** your understanding!*

How well do you understand operations on real numbers? Tick the right box



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Lesson

1 - 4

Laws of Square and Cube Roots

■ Learning Outcomes

- Learn the laws of multiplication and division of square roots.
- Learn the laws of multiplication and division of cube roots.
- Use the laws of square and cube roots to simplify numbers.
- Use the laws of roots to solve real-life problems.

Get Ready!



As a part of a school project, Osama and his classmates decided to create a small seedbed inside the school. It consists of two adjoining squares.

- The first square is for growing ornamental seedlings and has an area of 48 m^2.
- The second square is for growing aromatic plants and has an area of 12 m^2.

Osama and his friends wanted to build a fence around the entire seedbed to protect it. How long is this fence?

In this lesson, you will learn the laws of roots, which will enable you to solve such real-life problems.

■ Vocabulary

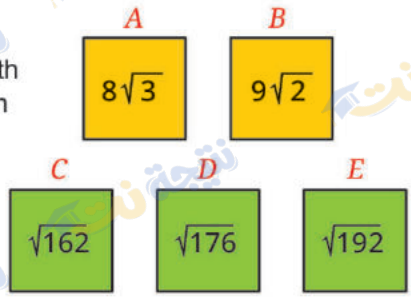
- Square root
- Cube root

Think & Discuss!

In a game where you choose two cards with two equal numbers, Sarah and Amal each chose a yellow card and a green card.

- Sarah chose cards A , C.
- Amal chose cards B , C.

Which one chose two equal numbers? Why?



■ Critical Thinking

- If a , b are two non-negative real numbers:
  Is: $\sqrt{a}+\sqrt{b}=\sqrt{a+b}$?
  Is: $\sqrt{a}-\sqrt{b}=\sqrt{a-b}$?
- If a , b are two real numbers:
  Is: $\sqrt[3]{a}+\sqrt[3]{b}=\sqrt[3]{a+b}$?
  Is: $\sqrt[3]{a}-\sqrt[3]{b}=\sqrt[3]{a-b}$?

Learn!

Multiplication and division of roots

Square roots	Cube roots
If a , b are two non-negative real numbers, then: $\sqrt{a}\times\sqrt{b}=\sqrt{a\times b}$ **Example :** $\sqrt{2}\times\sqrt{8}=\sqrt{2\times 8}=\sqrt{16}=4$	If a , b are two real numbers, then: $\sqrt[3]{a}\times\sqrt[3]{b}=\sqrt[3]{a\times b}$ **Example :** $\sqrt[3]{3}\times\sqrt[3]{-4}=\sqrt[3]{3\times(-4)}=\sqrt[3]{-12}$
$\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$ where $b\neq 0$ **Example :** $\frac{\sqrt{6}}{\sqrt{2}}=\sqrt{\frac{6}{2}}=\sqrt{3}$	$\frac{\sqrt[3]{a}}{\sqrt[3]{b}}\ \sqrt[3]{\frac{a}{b}}$ where $b\neq 0$ **Example :** $\frac{\sqrt[3]{16}}{\sqrt[3]{2}}=\sqrt[3]{\frac{16}{2}}=\sqrt[3]{8}=2$

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Example 1

Write each of the following numbers in its simplest form:

1) $\sqrt{32}$ 2) $\sqrt[3]{24}$ 3) $2\sqrt{\frac{5}{2}}$ 4) $3\sqrt[3]{\frac{-2}{3}}$

1) Factorize the number 32 into two factors, one of which is 16, which is the largest perfect square factor ($32 = 16 \times 2$)

$\therefore \sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

2) Factorize the number 24 into two factors, one of which is 8, which is the largest perfect cube factor ($24 = 8 \times 3$)

$\therefore \sqrt[3]{24} = \sqrt[3]{8 \times 3} = \sqrt[3]{8} \times \sqrt[3]{3} = 2\sqrt[3]{3}$

3) $2\sqrt{\frac{5}{2}} = 2 \times \frac{\sqrt{5}}{\sqrt{2}}$

$= 2 \times \frac{\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= 2 \times \frac{\sqrt{10}}{2} = \sqrt{10}$

4) $3\sqrt[3]{\frac{-2}{3}} = 3 \times \frac{\sqrt[3]{-2}}{\sqrt[3]{3}}$

$= 3 \times \frac{\sqrt[3]{-2}}{\sqrt[3]{3}} \times \frac{\sqrt[3]{3}}{\sqrt[3]{3}} \times \frac{\sqrt[3]{3}}{\sqrt[3]{3}}$

$= 3 \times \frac{\sqrt[3]{-2 \times 3 \times 3}}{3}$

$= 3 \times \frac{\sqrt[3]{-18}}{3} = -\sqrt[3]{18}$

■ Diversified strategies

$\because 2 = \sqrt{2} \times \sqrt{2}$

$\therefore 2\sqrt{\frac{5}{2}} = \sqrt{2} \times \sqrt{2} \times \frac{\sqrt{5}}{\sqrt{2}}$

$= \sqrt{2} \times \sqrt{5} = \sqrt{10}$

Example 2

Simplify each of the following to its simplest form:

1) $2\sqrt{3} + 5\sqrt{27} - \sqrt{48}$ 2) $\frac{\sqrt[3]{40} - 2\sqrt[3]{135}}{\sqrt[3]{5}}$

1) $2\sqrt{3} + 5\sqrt{27} - \sqrt{48} = 2\sqrt{3} + 5\sqrt{9 \times 3} - \sqrt{16 \times 3}$

$= 2\sqrt{3} + 5 \times \sqrt{9} \times \sqrt{3} - \sqrt{16} \times \sqrt{3}$

$= 2\sqrt{3} + 5 \times 3\sqrt{3} - 4\sqrt{3}$

$= 2\sqrt{3} + 15\sqrt{3} - 4\sqrt{3} = 13\sqrt{3}$

■ Self-Evaluation (1)

Write each of the following in its simplest form:

1) $\sqrt{50}$ 2) $4\sqrt{\frac{1}{2}}$

3) $2\sqrt[3]{16}$ 4) $2\sqrt[3]{\frac{3}{4}}$

■ Remember that

To make the denominator of an irrational number $\frac{\sqrt{a}}{\sqrt{b}}$ an integer, multiply both the numerator and the denominator by $\sqrt{b}$ as follows:

$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$

■ Remember that

To make the denominator of an irrational number $\frac{\sqrt[3]{a}}{\sqrt[3]{b}}$ an integer, multiply both the numerator and the denominator by $\sqrt[3]{b} \times \sqrt[3]{b}$ as follows:

$\frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}} \times \frac{\sqrt[3]{b}}{\sqrt[3]{b}} \times \frac{\sqrt[3]{b}}{\sqrt[3]{b}}$

$= \frac{\sqrt[3]{ab^2}}{b}$

■ Note that

When adding and subtracting unlike radical terms, simplify each term separately.

If you get like radical terms, you can add or subtract them.

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■ **Self-Evaluation (2)**

Simplify each of the following to its simplest form:

1. $2\sqrt{32}-\sqrt{18}+\sqrt{50}$
2. $2\sqrt[3]{16}+\sqrt[3]{54}-\sqrt[3]{128}$

(2) $\dfrac{\sqrt[3]{40}-2\sqrt[3]{135}}{\sqrt[3]{5}}=\dfrac{\sqrt[3]{8\times5}-2\sqrt[3]{27\times5}}{\sqrt[3]{5}}$

$=\dfrac{\sqrt[3]{8}\times\sqrt[3]{5}-2\times\sqrt[3]{27}\times\sqrt[3]{5}}{\sqrt[3]{5}}$

$=\dfrac{2\sqrt[3]{5}-2\times3\sqrt[3]{5}}{\sqrt[3]{5}}$

$=\dfrac{2\sqrt[3]{5}-6\sqrt[3]{5}}{\sqrt[3]{5}}$

$=\dfrac{-4\sqrt[3]{5}}{\sqrt[3]{5}}=-4$

■ **Diversified strategies**

$\dfrac{\sqrt[3]{40}-2\sqrt[3]{135}}{\sqrt[3]{5}}$

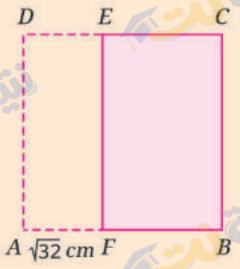
$=\dfrac{\sqrt[3]{40}}{\sqrt[3]{5}}-\dfrac{2\sqrt[3]{135}}{\sqrt[3]{5}}$

$=\sqrt[3]{\dfrac{40}{5}}-2\sqrt[3]{\dfrac{135}{5}}$

$=\sqrt[3]{8}-2\sqrt[3]{27}$

$=2-2\times3=-4$

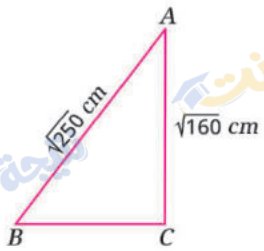
■ **Self-Evaluation (3)**



The area of the square $ABCD$ is equal to 200 cm^2. The rectangle $AFED$, which has a width of $\sqrt{32}$ cm. has been removed What is the area of the shape $FBCE$?

Example (3)

Geometry: In the opposite figure, ABC is a triangle with perimeter $\sqrt{1440}$ cm. Find the length of $\overline{BC}$.



∵ Perimeter of a triangle = sum of the lengths of its sides

$\therefore\sqrt{250}+\sqrt{160}+BC=\sqrt{1440}$

$\therefore\sqrt{25\times10}+\sqrt{16\times10}+BC=\sqrt{144\times10}$

$\therefore5\sqrt{10}+4\sqrt{10}+BC=12\sqrt{10}$

$\therefore9\sqrt{10}+BC=12\sqrt{10}$

$\therefore BC=12\sqrt{10}-9\sqrt{10}=3\sqrt{10}$ cm.

■ **Self-Evaluation (4)**

If $X=\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find in its simplest form the value of:

$$x+\frac{1}{x}$$

■ **Note that**

To simplify y, make the denominator a rational number by multiplying the numerator and denominator by $(\sqrt{5}-\sqrt{2})$.

Example (4)

If $X=\sqrt{5}+\sqrt{2}$, $y=\dfrac{3}{x}$, find in its simplest form:

1. $x-y$
2. $\dfrac{y}{x}$

$y=\dfrac{3}{x}=\dfrac{3}{\sqrt{5}+\sqrt{2}}\times\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\dfrac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}=\dfrac{3(\sqrt{5}-\sqrt{2})}{5-2}=\sqrt{5}-\sqrt{2}$

(1) $x-y=(\sqrt{5}+\sqrt{2})-(\sqrt{5}-\sqrt{2})=\sqrt{5}+\sqrt{2}-\sqrt{5}+\sqrt{2}=2\sqrt{2}$

(2) $\dfrac{y}{x}=\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}\times\dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\dfrac{(\sqrt{5}-\sqrt{2})^2}{5-2}=\dfrac{5-2\sqrt{10}+2}{3}$

$=\dfrac{7-2\sqrt{10}}{3}=\dfrac{7}{3}-\dfrac{2}{3}\sqrt{10}$

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Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

(1) Which of the following is correct ?

(a) $\sqrt{40} = 4\sqrt{10}$ (b) $\sqrt{32} = 8\sqrt{2}$

(c) $\sqrt{12} = 2\sqrt{3}$ (d) $\sqrt{48} = 3\sqrt{6}$

(2) What is the value of $\sqrt{18} + \sqrt{32}$?

(a) $\sqrt{50}$ (b) $7\sqrt{2}$

(c) $7\sqrt{4}$ (d) $5\sqrt{4}$

(3) Which of the following is equivalent to $\sqrt[3]{5} \times \sqrt[3]{25}$?

(a) $\sqrt[3]{30}$ (b) 5

(c) $5\sqrt[3]{5}$ (d) 125

(4) Which of the following is equivalent to $\frac{\sqrt{72}}{\sqrt{2}}$?

(a) $2\sqrt{6}$ (b) $6\sqrt{2}$

(c) 6 (d) $\sqrt{70}$

(5) Which of the following is equivalent to $\frac{\sqrt{25}}{\sqrt{15}}$?

(a) $\frac{5}{15}$ (b) $\frac{5}{3}$

(c) $\frac{\sqrt{5}}{3}$ (d) $\frac{\sqrt{15}}{3}$

(6) If $\sqrt{160} = x\sqrt{10}$, what is the value of x ?

(a) 8 (b) 6

(c) 4 (d) 3

(7) If $a\sqrt[3]{2} = \sqrt[3]{3} \times \sqrt[3]{10}$, what is the value of a ?

(a) 30 (b) $\sqrt[3]{30}$

(c) 15 (d) $\sqrt[3]{15}$

(8) If $2\sqrt{3} \times 4a = 8\sqrt{6}$, what is the value of a ?

(a) $\sqrt{2}$ (b) $\sqrt{3}$

(c) 2 (d) $\sqrt{6}$

(9) If $\frac{\sqrt{3}}{\sqrt{a}} = \frac{\sqrt{2}}{\sqrt{6}}$, what is the value of a ?

(a) $\sqrt{3}$ (b) $\sqrt{6}$

(c) 3 (d) 9

(10) If $x + \sqrt{28} = \sqrt{7}$, what is the value of x ?

(a) $\sqrt{21}$ (b) $-\sqrt{21}$

(c) $\sqrt{7}$ (d) $-\sqrt{7}$

Second Applying Concepts

(11) **Simplify each of the following to its simplest form:**

① $3\sqrt{\frac{1}{3}} - \sqrt{27}$

② $2\sqrt[3]{\frac{1}{4}} + \sqrt[3]{16}$

③ $\frac{2\sqrt[3]{5} \times \sqrt[3]{54}}{\sqrt[3]{10}}$

④ $\frac{\sqrt{98} + \sqrt{8}}{\sqrt{18}}$

⑤ $\frac{3\sqrt[3]{32} - 3\sqrt[3]{4}}{\sqrt[3]{4}}$

⑥ $3\sqrt{8} + \sqrt{18} - \sqrt{32}$

⑦ $4\sqrt{48} + \frac{4}{3}\sqrt{27} + \sqrt{75}$

⑧ $\sqrt[3]{192} - 3\sqrt[3]{375} + 2\sqrt[3]{24}$

⑨ $\frac{3}{2}\sqrt[3]{256} + \frac{12}{\sqrt[3]{16}} + \sqrt[3]{32}$

(12) **In each of the following, find the value of x :**

① $x\sqrt{32} = 2\sqrt{50} + \sqrt{72}$

② $x\sqrt[3]{10} = \sqrt[3]{80} - \sqrt[3]{270}$

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13 Answer the following questions:

① If $4\sqrt{5} = 2\sqrt{a}$ and $2\sqrt{24} = b\sqrt{6}$ find $a + b$.

② If $x = \sqrt{3} + \sqrt{2}$ and $y = \sqrt{3} - \sqrt{2}$, find in the simplest form each of the following:

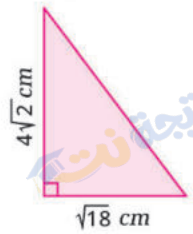
(1) $x \times y$ (2) $\frac{x}{y}$ (3) $(x - \frac{1}{x})^2$

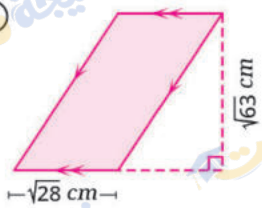
③ If $a = \sqrt[3]{5} + \sqrt[3]{2}$ and $b = \sqrt[3]{5} - \sqrt[3]{2}$, find in the simplest form each of the following:

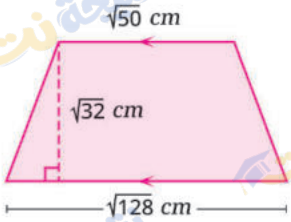
(1) $(a+b)^3$ (2) $a \times b$ (3) $a^2 - b^2$

Third Analysis and Subjects Integration

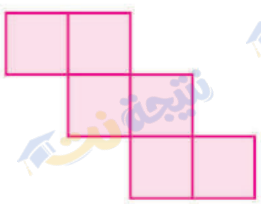
14 Geometry: Calculate the area of each of the following coloured shapes:

① 

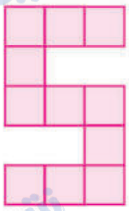
② 

③ 

15 Geometry: Each of the two opposite shapes consists of squares of equal area.
Find the perimeter of each shape
(the area of each shape is given below).



Area = 300 square centimeters

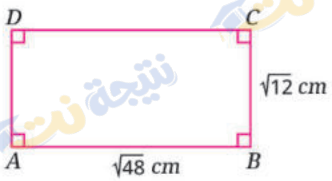


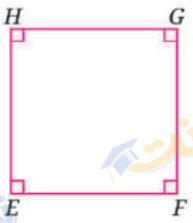
Area = 132 square centimeters



16 Construction: A piece of land in shape of a square with an area of 500 m^2. A square house with an area of 180 m^2 was built on it, and the rest of the area was left as a garden, as shown in the diagram. What is the length of the outer fence of the garden in meters?

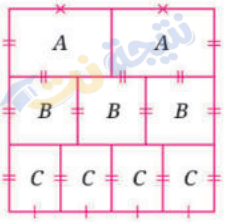
17 Geometry:
If the area of rectangle $ABCD$ is equal to the area of square $EFGH$, what is the length of $\overline{EF}$?





Creative Thinking

18 A square with an area of 432 cm^2 is divided into rectangles and squares as shown in the figure.
What is the difference between the perimeters of shapes A and B?



***Evaluate** your understanding!*

How well do you understand laws of square and cube roots? Tick the right box



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Lesson

1 - 5

Laws of Exponents in Real Numbers

Learning outcomes

- Learn repeated multiplication in the set of real numbers.
- Learn the laws of exponents in real numbers.
- Use the laws of exponents to simplify mathematical expressions.
- Use the laws of exponents to solve real-life problems.

Vocabulary

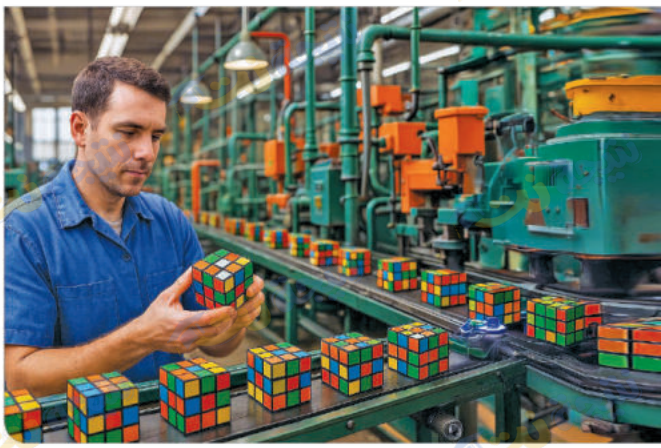
- Repeated multiplication
- Exponent
- Base
- Power

Note that

- $(-a)^n = a^n$ if n is an even integer.
- $(-a)^n = -a^n$ if n is an odd integer.

Get Ready!

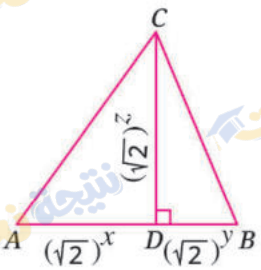
A company specializing in producing educational toys for children has decided to produce a toy in the form of a coloured cube made of safe plastic, with edge length $4\sqrt{3}\ cm$. What is the volume of this cube?



In this lesson, you will learn the laws of exponents in real numbers, which will enable you to solve such real-life problems.

Think & Discuss!

If $x + z = 4$, $y + z = 2$, what is the area of triangle ABC where $\overline{CD} \perp \overline{AB}$? Discuss.



Learn!

Repeated multiplication in the set of real numbers

The mathematical expression a^n is used to express the product of the real number a multiplied by itself n times.

$$\underbrace{a \times a \times a \times \cdots \times a}_{\text{real number repeated } n \text{ times}} = a^n$$

n ← Exponent

Base: a

Example:

$$\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} = (\sqrt{2})^5$$

Exponent = 5

Base = $\sqrt{2}$

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■ Self-Evaluation (1)

Find the value of each of the following:

1 $(-\sqrt{2})^5$ 2 $-(\sqrt{2})^5$

3 $(-\sqrt{2})^6$

Example (1)

Find the value of each of the following:

1 $(-\sqrt{5})^4$ 2 $-(\sqrt{5})^4$ 3 $(-\sqrt{5})^3$

1 $(-\sqrt{5})^4 = -\sqrt{5}\times-\sqrt{5}\times-\sqrt{5}\times-\sqrt{5} = 5\times5 = 5^2 = 25$

2 $-(\sqrt{5})^4 = -(\sqrt{5}\times\sqrt{5}\times\sqrt{5}\times\sqrt{5}) = -(5\times5) = -(5)^2 = -25$

3 $(-\sqrt{5})^3 = -\sqrt{5}\times-\sqrt{5}\times-\sqrt{5} = 5\times-\sqrt{5} = -5\sqrt{5}$

■ Think with your classmates

If a is a real number not equal to zero, and n is an integer, how can you prove each of the following:

1 $a^0 = 1$ 2 $a^{-n} = \frac{1}{a^n}$

Zero exponent and negative exponent

Any real number not equal to zero raised to the exponent zero is equal to 1

Thus : $a^0 = 1$ where a is a real number not equal to zero.

Examples: $5^0 = 1$, $(\sqrt{3})^0 = 1$, $(-\sqrt[3]{4})^0 = 1$

Any real number not equal to zero raised to the exponent $(-n)$ where n is an integer, is equal to the multiplicative inverse of the number itself raised to the exponent n

Thus: $a^{-n} = \frac{1}{a^n}$ where a is a real number not equal to zero.

Examples: $(\sqrt{3})^{-4} = (\frac{1}{\sqrt{3}})^4 = \frac{1}{(\sqrt{3})^4} = \frac{1}{9}$, $(\frac{1}{\sqrt[3]{4}})^{-3} = (\sqrt[3]{4})^3 = 4$

■ Note that

Division by zero has no meaning, and therefore, when there are symbols in the denominator, these symbols must not be equal to zero.

Laws of exponents in real numbers

First Laws of multiplication

When multiplying powers with the same base, keep the base and add the exponents.

$a^m \times a^n = a^{m+n}$ where a is a real number not equal to zero, m and n are integers.

Examples: $2^4 \times 2^6 = 2^{4+6} = 2^{10}$, $a^{-3} \times a^5 = a^{-3+5} = a^2$

When finding the power of the product of two numbers , the exponent is distributed over both numbers.

Thus: $(a\times b)^n = a^n \times b^n$ where a and b are real numbers not equal to zero, and n is an integer.

Example: $(3\sqrt{2})^2 = (3)^2 \times (\sqrt{2})^2 = 9\times2 = 18$

■ Note that

$(a+b)^n \neq a^n + b^n$

$(a-b)^n \neq a^n - b^n$

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Second Laws of division

When dividing powers with the same base, keep the base and subtract the exponents.

Thus : $\frac{a^m}{a^n} = a^{m-n}$ where a is a real number not equal to zero, m and n are integers.

Example: $\frac{(\sqrt[3]{7})^5}{(\sqrt[3]{7})^2} = (\sqrt[3]{7})^{5-2} = (\sqrt[3]{7})^3 = 7$

When finding the power of the quotient of two numbers, the exponent is distributed over both numbers.

Thus : $(\frac{a}{b})^n = \frac{a^n}{b^n}$ where a and b are real numbers not equal to zero, and n is an integer.

Example: $(\frac{6}{\sqrt{3}})^2 = \frac{6^2}{(\sqrt{3})^2} = \frac{36}{3} = 12$

Example 2

Simplify each of the following to its simplest form:

1. $\frac{(6a)^2}{2a}$
2. $\frac{(-\sqrt{3}ab)^4}{3a^4b^2}$
3. $\frac{(\sqrt{8})^4 \times (\sqrt{2})^3}{(2\sqrt{2})^5}$

1. $\frac{(6a)^2}{2a} = \frac{6^2 \times a^2}{2 \times a} = \frac{36a^2}{2a} = 18a^{2-1} = 18a$

2. $\frac{(-\sqrt{3}ab)^4}{3a^4b^2} = \frac{(-\sqrt{3})^4 \times a^4 \times b^4}{3 \times a^4 \times b^2}$

$= \frac{9}{3} \times a^{4-4} \times b^{4-2}$

$= 3 \times a^0 \times b^2$

$= 3 \times 1 \times b^2 = 3b^2$

3. $\frac{(\sqrt{8})^4 \times (\sqrt{2})^3}{(2\sqrt{2})^5} = \frac{(2\sqrt{2})^4 \times (\sqrt{2})^3}{(2\sqrt{2})^5} = \frac{2^4 \times (\sqrt{2})^4 \times (\sqrt{2})^3}{2^5 \times (\sqrt{2})^5}$

$= (2)^{4-5} \times (\sqrt{2})^{4+3-5}$

$= (2)^{-1} \times (\sqrt{2})^2 = \frac{1}{2} \times 2 = 1$

Self-Evaluation (2)

Simplify each of the following to its simplest form:

1. $\frac{(\sqrt{3})^2 \times (\sqrt{3})^4 \times (\sqrt{3})^{-1}}{(\sqrt{3})^5 \times (\sqrt{3})^{-2}}$
2. $\frac{(\sqrt{18})^5 \times (\sqrt{2})^3}{(\sqrt{12})^4}$

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Example 3

Simplify each of the following to its simplest form:

1. $(\sqrt{6})^{n} \times (\sqrt{3})^{n-1} \times (\sqrt{2})^{-n}$, then find the numerical value of the result when $n = 1$

2. $\dfrac{(10)^{2n} \times 0.001}{(10)^{-n+1} \times 100}$, then find the numerical value of the result when $n = 2$

✓ ① $(\sqrt{6})^{n} \times (\sqrt{3})^{n-1} \times (\sqrt{2})^{-n} = (\sqrt{3} \times \sqrt{2})^{n} \times (\sqrt{3})^{n-1} \times (\sqrt{2})^{-n}$

$= (\sqrt{3})^{n} \times (\sqrt{2})^{n} \times (\sqrt{3})^{n-1} \times (\sqrt{2})^{-n}$

$= (\sqrt{3})^{n+n-1} \times (\sqrt{2})^{n-n}$

$= (\sqrt{3})^{2n-1} \times (\sqrt{2})^{0}$

$= (\sqrt{3})^{2n-1} \times 1$

$= (\sqrt{3})^{2n-1}$

When $n = 1$:

$\therefore (\sqrt{3})^{2n-1} = (\sqrt{3})^{2\times1-1} = \sqrt{3}$

② $\dfrac{10^{2n} \times 0.001}{10^{-n+1} \times 100} = \dfrac{10^{2n} \times 10^{-3}}{10^{-n+1} \times 10^{2}}$

$= \dfrac{10^{2n-3}}{10^{-n+1+2}} = \dfrac{10^{2n-3}}{10^{-n+3}}$

$= 10^{(2n-3)-(-n+3)} = 10^{2n-3+n-3} = 10^{3n-6}$

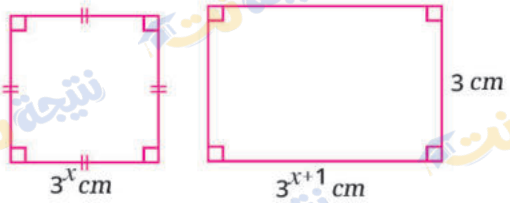
When $n = 2$:

$\therefore 10^{3n-6} = 10^{3\times2-6} = 10^{6-6} = 10^{0} = 1$

Example 4

In the opposite figure:

A square with a side length of 3^{x} cm and a rectangle with dimensions 3^{x+1} cm and 3 cm. If the perimeter of the square = 20 cm, find the area of the rectangle.



✓ ∵ Perimeter of the square = 20 cm

$\therefore 4 \times 3^{x} = 20$ $\quad\therefore 3^{x} = \dfrac{20}{4} = 5$

∵ Area of the rectangle A = length × width

$\therefore A = 3^{x+1} \times 3 = 3^{x} \times 3 \times 3 = 5 \times 9 = 45$

$\therefore$ Area of the rectangle = 45 cm^{2}.

■ Self-Evaluation (3)

Simplify to the simplest form:

1. $\dfrac{(\sqrt{5})^{n} \times (\sqrt{3})^{2-n}}{3 \times (\sqrt{15})^{-n}}$, then find the numerical value of the result when $n = -2$

2. $\dfrac{1000 \times 0.1^{n} \times 10^{n-1}}{10^{n+2}}$

■ Note that

$10 = 10^{1}$	$0.1 = 10^{-1}$
$100 = 10^{2}$	$0.01 = 10^{-2}$
$1000 = 10^{3}$	$0.001 = 10^{-3}$
$10000 = 10^{4}$	$0.0001 = 10^{-4}$
	And so on.

■ Self-Evaluation (4)

If $3^{x} = 4$,

find the numerical value of each of the following:

1. 3^{x+1}
2. 3^{x-1}
3. $3^{x+2} + 3^{x+3}$

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Lesson Assessment



First Measuring Conceptual Understanding



▸ **Choose the correct answer from the given ones:**

1. What is the result of $\frac{(a\,b)^3}{b^3}$?

(a) a^3 (b) $\frac{a}{b}$ (c) $\frac{a^3}{b^2}$ (d) $\frac{a^2}{b^3}$

2. What is the value of $(\frac{7}{\sqrt{7}})^0 - \frac{7}{(\sqrt{7})^0}$?

(a) 0 (b) 6 (c) 1 (d) -6

3. What is the result of $(\sqrt{3})^3 \times (\sqrt{3})^{-4}$?

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{3}$ (c) 3 (d) $\frac{1}{3}$

4. What is the result of $(\frac{\sqrt{5}}{10})^{-1}$?

(a) $2\sqrt{5}$ (b) $5\sqrt{2}$ (c) $\frac{5}{\sqrt{10}}$ (d) 2

5. What is the perimeter of a square with an area of $a\ cm^2$?

(a) $a^4\ cm$ (b) $(\sqrt{a})^4\ cm$ (c) $4\sqrt{a}\ cm$ (d) $4\,a\ cm$

6. What is the value of $4^2 + 4^2 + 4^2 + 4^2$?

(a) 4^2 (b) 4^3 (c) 4^8 (d) 4^{16}

7. If $2^{x-1} = 1$, what is the value of x?

(a) 0 (b) 1 (c) -1 (d) -2

8. If $\sqrt{6} \times (\sqrt{6})^x = 1$, what is the value of x?

(a) 0 (b) 1 (c) -1 (d) -2

9. What is the $\frac{1}{5}$ of the number $(\sqrt[3]{5})^6$?

(a) 5 (b) 25 (c) $\sqrt[3]{5}$ (d) $(\sqrt[3]{5})^8$

10. If $2^x = 3$, what is the value of 2^{x+1} ?

(a) 4 (b) 5 (c) 6 (d) 9

Second Applying Concepts



11. **Simplify each of the following to its simplest form:**

① $\frac{(-2x\,y)^{-3}}{x^{-4}\,y^{-3}}$

② $\frac{-3\,a^2\,b \times 4\,a^{-1}\,b^{-2}}{12\,(a\,b)^{-2}}$

③ $(\frac{2\sqrt{3}}{3\sqrt{2}})^4$

④ $\frac{(\sqrt{3})^3 \times 27}{(\sqrt{3})^5 \times (\sqrt{3})^4}$

⑤ $\frac{(\sqrt{18})^2 \times (\sqrt{2})^{-1}}{(\sqrt{8})^{-1}}$

⑥ $\frac{(2)^{2n+1} \times (3)^{2n+1}}{(6)^{2n}}$

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12 Simplify each of the following to its simplest form:

① $\dfrac{(\sqrt{6})^{n+2}\times(\sqrt{3})^{n}}{4\times(\sqrt{2})^{n-2}}$, then find the value of the result when $n=1$

② $\dfrac{10^{-n}\times 0.01}{10^{-n-6}\times 10^{n}}$, then find the value of the result when $n=3$

13 If $x=\sqrt{2}$, $y=\sqrt{6}$, what is the value of $\dfrac{y^{6}x^{-4}}{x^{2}}$?

14 If $x=2\sqrt{3}$, $y=3\sqrt{2}$, find the numerical value of the expression $(x^{2}-y^{2})^{-1}$

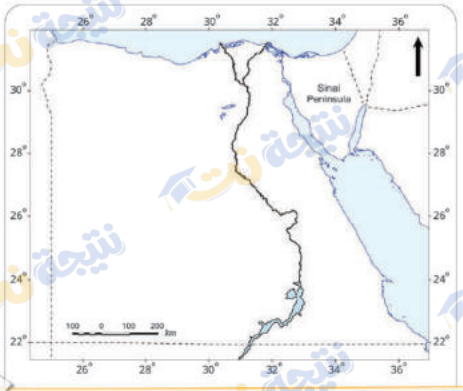
15 If $2^{x}=5$, find the value of each of the following:

① 2^{x+1} ② 2^{1-x} ③ $2^{x+2}+2^{x+3}$

16 If $x=\sqrt{5}$, $y=5$, find the value of $x^{-10}y^{10}$

Third Analysis and Subjects Integration

17 Geography: If you know that the area of the Sinai Peninsula is approximately 6% of the area of the Arab Republic of Egypt, which is approximately 10^{6} km^{2}, what is the approximate area of the Sinai Peninsula in square kilometers?



Sinai, the land of turquoise, and the cities of the canal are witnessing a real boom and achievements in all sectors of development, which is a source of pride.



Safety and security instructions must be followed in the workplace to ensure the safety of individuals.

18 Carpentry: A carpenter made a wooden box in the shape of a cuboid with a square base of side length $2\sqrt{7}$ feet, and a height of $3\sqrt{7}$ feet. What is the volume of the box?

Creative Thinking

19 Which of the following expresses the arithmetic mean of the two numbers 2^{x+1} and 2^{x} where $x\in Z$?

(a) 2 (b) 2^{2x+1}

(c) $2^{x-1}+2^{x}$ (d) $2^{x+1}+2^{x}$

***Evaluate** your understanding!*

How well do you understand laws of exponents in real numbers? Tick the right box

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Unit One Assessment

Evaluate your understanding!
How well do you understand Unit One? Tick the right box ☐ ☐ ☐ ☐

▸ **Choose the correct answer from the given ones:**

(1) If $a = 2\sqrt[3]{\frac{1}{4}} - \frac{1}{2}\sqrt[3]{16}$, what is the value of a ?

(a) -1 (b) 1

(c) $\sqrt[3]{2}$ (d) 0

(2) Which interval does represent $\bar{R} \cap]-2, 3]$?

(a) $]-2, 0[$ (b) $]-2, 0]$

(c) $]-2, 1]$ (d) $]0, 3]$

(3) If $(\sqrt{2})^{x-1} = \sqrt{3}$, what is the value of $(\sqrt{2})^{x}$?

(a) $\frac{1}{2}\sqrt{6}$ (b) $\frac{1}{2}\sqrt{3}$

(c) $\sqrt{6}$ (d) $\frac{1}{6}\sqrt{6}$

(4) Which of the following numbers is an irrational number?

(a) $\sqrt[3]{-27}$ (b) $-\sqrt{2\frac{1}{4}}$

(c) $0.\bar{5}$ (d) $3\sqrt{\frac{1}{3}}$

▸ **Complete each of the following with the correct answer:**

(5) The solution set of the equation $\frac{1}{2}x^2 + 2 = 0$ in R is .....

(6) If n is an integer, $n < \sqrt{29} < n + 1$ then $n =$ .....

(7) The multiplicative inverse of the number $\sqrt{10} - 3$ in its simplest form is .....

(8) If $X =]-1, \infty[$, then the interval that represents the complement of X is equal to .....

▸ **Answer the following questions:**

(9) Simplify the expression: $\frac{(\sqrt{15})^{n-2} \times (\sqrt{6})^{n}}{2 \times (\sqrt{10})^{n-2}}$ to its simplest form, then find the numerical value of the result when $n = 1$

(10) If $x = \sqrt{7} - \sqrt{6}$, then find in simplest form: $x - \frac{1}{x}$

(11) If $A =]-\infty, 1]$, $B =]-2, 2]$, then find in the form of an interval $B - A$

(12) Arrange the numbers in ascending order: $3\frac{1}{2}$, $3.\bar{5}$, $2\sqrt{3}$

Unit One Activity — The Golden Triangle



▸ **Activity Aim:** To learn new models and shapes that shows the golden ratio in mathematics.

▸ **Implementation Steps:**

In collaboration with one of your friends, try to do the following:

1. Use an artificial intelligence application to search for some examples of the golden ratio in mathematics.
2. List some shapes in which the golden ratio can be found.
3. Can the golden ratio be found in a triangle?
   Collect data on this case and mention the type of triangle that can be called (a golden triangle).
4. Explain the steps you made to answer the previous questions, showing some pictures of the shapes you came up with.

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UNIT 2

Algebra

Unit Lessons

2-1 Factorizing by Taking Out Greatest Common Factor GCF

2-2 Factorizing Trinomial

2-3 Factorizing Special Cases

2-4 Factorizing by Grouping



Factorization is an important mathematical operation used in certain algorithms of artificial intelligence, where each type of factorization integrates with a specific aspect of the design and training of intelligent models (applications of artificial intelligence).

▶ Can the role of factorization, in its various types, be developed in the construction and development of models and applications of artificial intelligence?

Issues and Life Skills

- Critical Thinking
- Creative Thinking
- Sustainable Development
- Mathematical Communication

Values

- Appreciation of Art and Beauty
- Environmental Awareness
- Respect
- Responsibility
- Cultural Awareness

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Lesson

2 - 1

Factorizing by Taking Out Greatest Common Factor GCF

Learning Outcomes

- Learn the concept of factorization by taking out the greatest common factor.
- Factorization a polynomial by taking out the greatest common factor.
- Use factorize by taking out the greatest common factor to solve problems.
- Solve equations using factorization by taking out the greatest common factor.

Vocabulary

- Factorization
- Polynomial
- Monomial
- Binomial
- Trinomial

Think with your classmates

Why is $-2x^3$ a polynomial while $3x^{-2}$ is not a polynomial, and also $3x^{\frac{1}{2}}+2$ is not a polynomial?

Get Ready!

Egypt occupies a world-leading position in squash, where Egyptian men and women players won many major championships in recent years. Suppose the area of the squash court is equal to $(4x^5+12x)\ m^2$, can its dimensions be $(4x^5)\ m$, $(12x)\ m$?

Or can they be $(x^4+3)\ m, 4x\ m$?



Egyptian champions in squash

In this lesson, you will learn how to factorize by taking out the greatest common factor, which will enable you to solve such problems.

Think & Discuss!

The expression $(12x^4+18x^3)$ can be written as a product of two factors in more than one way as follows:

$3x(4x^3+6x^2)$ $\quad 2x^2(6x^2+9x)$ $\quad 6x^3(2x+3)$

$6x(2x^3+3x^2)$ $\quad 3x^2(4x^2+6x)$

Which of the above products is a factorization of the expression $(12x^4+18x^3)$ by taking out the greatest common factor? Discuss.

Learn!

Factorizing a polynomial by taking out the greatest common factor (GCF)

A polynomial is a mathematical expression in which all the exponents of its variables are positive integers.

Examples of polynomials:

- polynomial (monomial): $-3xy$
- polynomial (binomial): $4xy^2+2x$
- polynomial (trinomial): $2x^2+3y+8$

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How to factorize a polynomial by taking out the greatest common factor

When factorizing the polynomial $6x^2 - 15x$ by taking out the GCF, follow the steps below:

1. Factorize each term as a product of primes and variables to the exponent of 1 as follows:

$$6x^2 = 2 \times 3 \times x \times x$$
$$15x = 3 \times 5 \times x$$

$\therefore$ The greatest common factor is $3x$

2. Divide $(6x^2 - 15x)$ by the greatest common factor $(3x)$, the quotient is $(2x - 5)$

3. Write the greatest common factor outside the brackets, write the quotient inside the brackets as in the given diagram.

Factorization

$6x^2 - 15x = 3x(2x - 5)$

Distribution

■ Previous Knowledge

- Multiplication can be distributed over addition, example :

$2x(x + 4) = 2x^2 + 8x$

- Multiplication can be distributed over subtraction, example :

$3x(2x - 5) = 6x^2 - 15x$

■ Self-Evaluation (1)

Factorize the following polynomials by taking out the greatest common factor:

1. $12x^2 + 16$
2. $6a^4 - 5a^2$
3. $4x^3 - 8x^2 + 12x$
4. $36ab - 24a^2b - 18ab^2$

■ Verifying the Solution

You can verify the solution by multiplying the factors to get the original polynomial.

Example 1

Factorize the following polynomials by taking out the greatest common factor:

1. $8x^2 - 16$
2. $15x^2y^2 - 30xy^3$
3. $2a^3b^2 + 8a^2b^3 - 16ab$
4. $-4x^4 + 12x^3 - 8x^2$

■ Note that

To find GCF between the terms of a polynomial:

1. Find GCF for numerical factors.
2. Take the repeated variable sharing in all terms with its smallest exponent.

(1) Since the GCF is 8

$\therefore 8x^2 - 16 = 8(x^2 - 2)$ (÷ 8)

(2) Since the GCF is $15xy^2$

$\therefore 15x^2y^2 - 30xy^3 = 15xy^2(x - 2y)$ (÷ $15xy^2$)

(3) $2a^3b^2 + 8a^2b^3 - 16ab = 2ab(a^2b + 4ab^2 - 8)$

(4) $-4x^4 + 12x^3 - 8x^2 = -4x^2(x^2 - 3x + 2)$

■ Note that

- If the coefficient of the first term in a polynomial is negative, it is preferable to take (–1) as a common factor from each term of the polynomial.
  For example: $-a + b - c = -(a - b + c)$
- A polynomial that cannot be factorized is called "prime polynomial".
  As: $2x^4 + 7$, $3x^2 + 4y^2$

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■ Self-Evaluation (2)

Find the numerical value of each of the following using factorization by taking out the greatest common factor:

1. $83 \times 57 + 83 \times 43$
2. $43 \times 87 - 43 \times 30 + (43)^2$
3. $13 \times 115 - 13 \times 16 + 13$

Example 2

Find the numerical value of each of the following using factorization by taking out the greatest common factor:

1. $19 \times 23 + 19 \times 27$
2. $37 \times 163 - 37 \times 26 - (37)^2$
3. $25 \times 158 - 25 \times 57 - 25$

1. $19 \times 23 + 19 \times 27 = 19\,(23 + 27) = 19 \times 50 = 950$
2. $37 \times 163 - 37 \times 26 - 37 \times 37 = 37\,(163 - 26 - 37)$
   $= 37 \times 100 = 3700$
3. $25 \times 158 - 25 \times 57 - 25 \times 1 = 25\,(158 - 57 - 1)$
   $= 25 \times 100 = 2500$

■ Self-Evaluation (3)

Factorize the following polynomials by taking out the greatest common factor:

1. $16\,a\,(a-b) + 24\,b\,(a-b)$
2. $(x+2)^2 + 3\,(x+2)$
3. $(x-5)^2 - x + 5$

Example 3

Factorize the following polynomials by taking out the greatest common factor:

1. $12\,x\,(x+y) - 18\,y\,(x+y)$
2. $(x-5)^2 - 2\,(x-5)$
3. $(x+3)^2 - x - 3$

■ Note that

The polynomial in brackets can be taken as a common factor.

1. Since the GCF is $6\,(x+y)$
   $\therefore 12\,x\,(x+y) - 18\,y\,(x+y) = 6\,(x+y)\,(2x - 3y)$
2. Since the GCF is $(x-5)$
   $\therefore (x-5)^2 - 2\,(x-5) = (x-5)\,(x-5-2) = (x-5)\,(x-7)$
3. $(x+3)^2 - x - 3 = (x+3)^2 - (x+3)$
   $= (x+3)\,(x+3-1)$
   $= (x+3)\,(x+2)$

■ Self-Evaluation (4)

If $(5\,a + 4\,b) = 65$,
$5\,a\,(2\,a - 3\,b) +$
$4\,b\,(2\,a - 3\,b) = 195$,
find the numerical value of the expression
$(2\,a - 3\,b)$

Example 4

If $x + 2y = 8$, $2x + y = 7$,
find the numerical value of the expression: $2\,x\,(x + 2\,y) + y\,(x + 2\,y)$

Factorization by taking out GCF can be used to simplify the given expression as follows:

$2\,x\,(x + 2\,y) + y\,(x + 2\,y) = (x + 2\,y)\,(2\,x + y)$
$= 8 \times 7 = 56$

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Solving equations using factorization by taking out the greatest common factor

Property:

If a, b are real numbers, and $a \times b = 0$, then :

Either $a = 0$ or $b = 0$ or both equal zero.

Example 5

Find the solution set of each of the following equations in R:

1) $3x^2 + 6x = 0$ 2) $10x^3 - 25x^2 = 0$ 3) $x^2 = 3x$

1) $\because 3x^2 + 6x = 0$ $\therefore 3x(x+2) = 0$

$\therefore$ Either $3x = 0$ hence $\boxed{x = 0}$

or $x + 2 = 0$ hence $\boxed{x = -2}$

$\therefore$ The solution set $= \{0, -2\}$

2) $\because 10x^3 - 25x^2 = 0$

$\therefore 5x^2(2x - 5) = 0$

$\therefore$ Either $5x^2 = 0$ hence $\boxed{x = 0}$

or $2x - 5 = 0$ hence $\boxed{x = \frac{5}{2}}$

$\therefore$ The solution set $= \{0, \frac{5}{2}\}$

3) $\because x^2 = 3x$ $\therefore x^2 - 3x = 0$

$\therefore x(x - 3) = 0$

$\therefore$ Either $\boxed{x = 0}$

or $x - 3 = 0$ hence $\boxed{x = 3}$

$\therefore$ The solution set $= \{0, 3\}$

■ Critical Thinking

When solving the equation $x^2 = 3x$, is it possible to divide both sides of the equation by the variable x?

■ Self-Evaluation (5)

Find the solution set of each of the following equations in R:

1) $x^2 + 2x = 0$

2) $2x^2 - 3x = 0$

3) $15x^3 + 12x^2 = 0$

■ Verifying the Solution

To verify the solution of the equation $3x^2 + 6x = 0$, substitute the value of x with both 0 and -2 in the original equation as follows:

▸ Substitute x with 0:

$3(0)^2 + 6(0) \stackrel{?}{=} 0$

$0 + 0 \stackrel{?}{=} 0$

$0 = 0$

So, 0 is a solution to the equation.

▸ Substitute x with -2:

$3(-2)^2 + 6(-2) \stackrel{?}{=} 0$

$12 + (-12) \stackrel{?}{=} 0$

$0 = 0$

So, -2 is a solution to the equation.

▸ Try to verify the solution to both equations 2 , 3 in example (5).

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Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

(1) What is the greatest common factor between the terms of the polynomial $12x^2 - 8x$?

(a) 2 (b) 4

(c) $2x$ (d) $4x$

(2) Which of the following represents the factorization of the polynomial $16x^2 - 24x^3$ by taking out GCF ?

(a) $8x(2x - 3x^2)$ (b) $4x^2(4 - 6x)$

(c) $8x^2(2 - 3x)$ (d) $8(2x^2 - 3x^3)$

(3) $6a - 8b =$ .....

(a) $6(a - 2b)$ (b) $4(2a - 4b)$

(c) $2(3a - 4b)$ (d) $14(a - b)$

(4) $x^2y - xy^2 =$ .....

(a) $xy(x - y)$ (b) $x^2y(1 - y)$

(c) $xy^2(x - 1)$ (d) $xy(y - x)$

(5) $x(a + b) + y(a + b) =$ .....

(a) $(x - y)(a - b)$ (b) $(x + y)(a + b)$

(c) $(x - y)(a + b)$ (d) $(x + y)(a - b)$

(6) If $x + y = 10$, $a - b = 7$, then :

$x(a - b) - y(b - a) =$ .....

(a) 3 (b) 17

(c) 51 (d) 70

▸ **Spot the mistake:**

(7) Sameh and Islam solved the equation $4x^2 = 8x$ as shown:

Which solution is correct? Discuss.

Sameh's solution:

$4x^2 = 8x$

$\frac{4x^2}{4x} = \frac{8x}{4x}$

$x = 2$

Islam's solution:

$4x^2 = 8x$

$4x^2 - 8x = 0$

$4x(x - 2) = 0$

Either $4x = 0$, hence $x = 0$

Or $x - 2 = 0$, hence $x = 2$

Second Applying Concepts

(8) **Factorize the following polynomials by taking out the greatest common factor (GCF):**

(1) $12y + 4$

(2) $4m - mn$

(3) $15a - 25b$

(4) $18a + 27ab$

(5) $-7k - 49m$

(6) $9a^2b - 6ab^2 + 3ab$

(7) $2a - 4ab - 6abc$

(8) $x^2y^3 + x^2y^2 + x^3y^2$

(9) $-4x^5 + 6x^4 - 8x^3 + 12x^2$

(10) $16a^6 - 4a^4 + 12a^2$

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9 **Factorize the following polynomials by taking out the greatest common factor (GCF):**

① $a(b+5)+2(b+5)$ ② $(4x-1)-y(4x-1)$ ③ $(2x-3)^2-2(2x-3)$

④ $(7x-5)^2-7x+5$ ⑤ $(x+2)^3-2(x+2)^2$ ⑥ $3x+6-(x+2)^2$

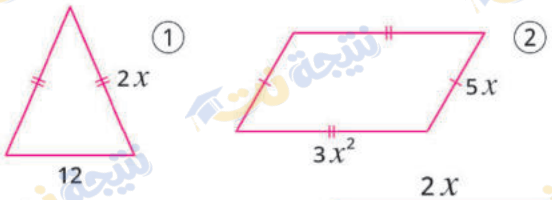
10 **Find the solution set of each of the following equations in R:**

① $x^2-x=0$ ② $4x^2+8x=0$ ③ $5x-x^2=0$

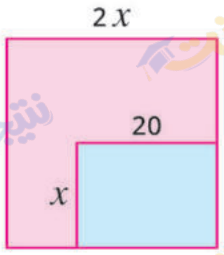
④ $x^2=5x$ ⑤ $2x^2-3x=0$ ⑥ $12x^2-4x=0$

Third Analysis and Subjects Integration

11 **Geometry :** Find the perimeter of each of the two given figures as two factors using factorization by taking out the greatest common factor.



12 **Engineering :** An engineer has a plot of land in form of a square with a side length of $2x$ m. He wants to build a house on it in form of a rectangle with dimensions of x m, 20 m and use the remaining area as a garden as shown in the figure, find the area of the garden and write the result as the product of two factors using factorization by taking out the greatest common factor.





The aquatic environment must be preserved from pollution to protect aquatic life

13 **Aquatic life:** A dolphin jumps up so that its height, in meters, after (t) seconds is equal to $(32t-16t^2)$. Write the height of the dolphin as the product of two factors using factorization by taking out the greatest common factor, then find its height at $t=1$

14 **Industry:** A school supplies factory produces two kinds of products, notebooks and pens, each notebook costs $(6x^2y)$ LE and each pen costs $(9xy^2)$ LE
If the factory produces 300 notebooks and 200 pens, write the mathematical expression that represents the total cost as the product of two factors using factorization by taking out the greatest common factor.



Small and medium sized enterprises create jobs and support the local economy.

Creative Thinking

15 If $6x+9=c(2x+d)$, $14x+10=a(7x+b)$,
find the numerical value of the expression d^c+b^a

Open Problem

16 Write a trinomial such that the greatest common factor of its terms is $3x^2$

***Evaluate** your understanding!*

How well do you understand factorizing by taking out greatest common factor GCF?
Tick the right box

☐ ☐ ☐ ☐

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Lesson

2 - 2

Factorizing Trinomial

Learning Outcomes

- Factorize the trinomial in the form: $x^2 + bx + c$
- Factorize the trinomial in the form: $ax^2 + bx + c$ (where $a \neq \pm 1$).
- Use factorization to solve quadratic equations.
- Use factorization to solve real-life problems.

Get Ready!



With the tremendous development in the smartphone industry, Global companies are competing to design screens with precise dimensions that provide ease of use and display quality.

To design a new smartphone, designers decided that the phone screen should be a rectangle, with the length of the screen 7 cm longer than its width, and the total area of the screen 120 cm^2. What are the length and width of the screen?

In this lesson, you will learn how to factorize trinomial , then use factorization to solve quadratic equations , which will enable you to solve such problems.

Vocabulary

- Factorizing
- Trinomial
- Quadratic equation

Think & Discuss!



Omar plans to build a rectangular swimming pool with a floor area of 40 m^2 and a perimeter of 26 m.

Can you find the dimensions of the floor of this swimming pool?

▸ Think about finding two numbers whose product is 40 and whose sum is 13 (half the perimeter of the floor of the swimming pool).

Learn!

First Factorizing the trinomial in the form $x^2 + bx + c$

When you multiply the binomials $(x + 2)$ and $(x + 3)$, you will find that the result is a trinomial as follows: $(x + 2)(x + 3) = x^2 + 5x + 6$.

Note that:

- The last term, 6, in the trinomial is the **product** of the last two terms in the binomials 2 , 3 .
- The coefficient of the middle term, 5, in the trinomial is the **sum** of the last two terms in the binomials 2 , 3 .

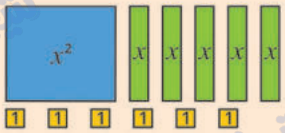
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■ Modelling

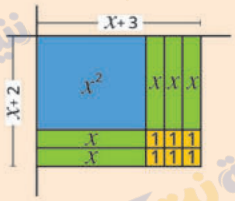
You can use card models to factorize

$x^2 + 5x + 6$

▸ **Use the following cards:**



▸ **Form a rectangle using all the cards as follows:**



$\therefore x^2 + 5x + 6$
$= (x + 2)(x + 3)$

■ Self-Evaluation (1)

Factorize each of the following:

1. $x^2 + 5x + 4$
2. $x^2 - 11x + 24$
3. $x^2 - 3x - 28$
4. $x^2 + x - 42$

■ Note that

When factorizing the trinomial $x^2 + bx + c$ *to be* in the form $(x + \ell)(x + m)$, then:

1. If c is positive, then ℓ and m have the same sign as b.
2. If c is negative, then ℓ and m have different signs, and the number with larger absolute value will have the same sign as b.

Factorizing a trinomial is the inverse operation of multiplication.

For example:

To factorize the trinomial $x^2 + 5x + 6$ look for two integers their product is 6 and their sum is 5. You will find that these numbers are 2 and 3,

where: $2 \times 3 = 6$, $2 + 3 = 5$

Thus : $x^2 + 5x + 6 = (x + 2)(x + 3)$

How to factorize a trinomial in the form $x^2 + bx + c$

To factorize a trinomial in the form: $x^2 + bx + c$ where b and c are integers, find two integers ℓ and m such that : $b = \ell + m$, $c = \ell m$, then write: $x^2 + bx + c = (x + \ell)(x + m)$

Example (1)

Factorize each of the following:

1. $x^2 + 7x + 12$
2. $x^2 - 6x + 8$
3. $x^2 + x - 12$
4. $x^2 - 7x - 18$

Solution

(1) To factorize the trinomial: $x^2 + 7x + 12$, look for two integers their product is (+12) and their sum is (+7).
∴ The two numbers are positive.

Possible products	Their sum	
(1) (12)	13	✗
(2) (6)	8	✗
(3) (4)	7	✓

∴ The two numbers are +4 , +3

∴ $x^2 + 7x + 12 = (x + 4)(x + 3)$

(2) To factorize the trinomial: $x^2 - 6x + 8$, look for two integers their product is (+8) and their sum is (–6).
∴ The two numbers are negative.

Possible products	Their sum	
(– 1) (– 8)	– 9	✗
(– 2) (– 4)	– 6	✓

∴ The two numbers are – 2 , – 4

∴ $x^2 - 6x + 8 = (x - 2)(x - 4)$

(3) To factorize the trinomial: $x^2 + x - 12$, look for two integers their product is (–12) and their sum is (+1).
∴ The two numbers have different signs, the number with larger absolute value will be positive.

Possible products	Their sum	
(– 1) (12)	+ 11	✗
(– 2) (6)	+ 4	✗
(– 3) (4)	+ 1	✓

∴ The two numbers are – 3 , + 4

∴ $x^2 + x - 12 = (x + 4)(x - 3)$

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④ To factorize the trinomial: $x^2 - 7x - 18$, look for two integers their product is (–18) and their sum is (–7).

∴ The two numbers have different signs, the number with larger absolute value will be negative.

∴ $x^2 - 7x - 18 = (x - 9)(x + 2)$

Possible products	Their sum	
(1) (– 18)	– 17	✗
(2) (– 9)	– 7	✓
(3) (– 6)	– 3	✗

∴ The two numbers are +2 , –9

■ Self-Evaluation (2)

Factorize each of the following completely:

1. $4x^2 - 8x - 60$
2. $-x^2 - 13x - 36$

Example 2

Factorize each of the following completely:

1. $3x^2 + 6x - 72$
2. $12x - 3x^3 - 9x^2$
3. $x^2 - 5xy + 6y^2$
4. $y(y + 1) - 30$
5. $x^6 - 10b^4 - 3x^3b^2$

■ Note that

The complete factorization is to write the polynomial in the form of the product of prime polynomials.

① $3x^2 + 6x - 72 = 3(x^2 + 2x - 24) = 3(x + 6)(x - 4)$

② $12x - 3x^3 - 9x^2 = -3x^3 - 9x^2 + 12x$

«Arrange the terms in descending order according to the exponents of x »

$= -3x(x^2 + 3x - 4)$ «Take out GCF»

$= -3x(x - 1)(x + 4)$ «Factorize the trinomial»

③ $x^2 - 5xy + 6y^2 = (x - 3y)(x - 2y)$

④ $y(y + 1) - 30 = y^2 + y - 30$ «Expand the brackets»

$= (y + 6)(y - 5)$

⑤ $x^6 - 10b^4 - 3x^3b^2 = x^6 - 3x^3b^2 - 10b^4$

«Arrange the terms in descending order according to the exponents of x »

$= (x^3 + 2b^2)(x^3 - 5b^2)$

Second Factorizing the trinomial in the form $ax^2 + bx + c$ where $(a \neq \pm 1)$

■ Self-Evaluation (3)

Factorize each of the following:

1. $3x^2 - 11x + 6$
2. $6x^2 + 7x + 2$
3. $2x^2 + x - 10$
4. $5x^2 - 28x - 12$

Example 3

Factorize each of the following:

1. $2x^2 + 7x + 6$
2. $4x^2 + x - 3$
3. $2x^4 + 9x^2 + 10$

① Factorize $2x^2$ into x , $2x$ ⟶ $(x$ $)(2x$ $)$

Factorize 6 into +1 , +6 or +2 , +3

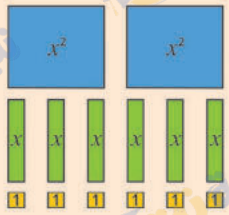
Ignore the negative factors because the coefficient of x is positive.

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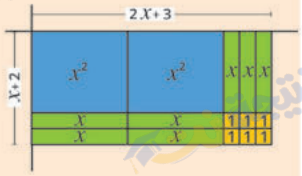
■ Modelling

Card models can be used to factorize $2x^2+7x+6$

► **Use the following cards:**



► **Form a rectangle using all the cards as follows:**



$\therefore 2x^2+7x+6$

$=(2x+3)(x+2)$

■ Think with your classmates

Can you factorize

$3x^4+11x+6$?

■ Self-Evaluation (4)

Find the solution set of each of the following equations in R:

1. $x^2-8x+15=0$
2. $x^2-3x=10$
3. $-x^2+24=5x$
4. $3x^2+11x-20=0$

• Determine the corresponding middle term as follows:

$\therefore 2x^2+7x+6=(x+2)(2x+3)$

Also the method of scissors can be used to perform these attempts as follows:

Possible factorization	corresponding middle term	
$(x+1)(2x+6)$	$6x+2x=8x$	✗
$(x+6)(2x+1)$	$x+12x=13x$	✗
$(x+2)(2x+3)$	$3x+4x=7x$	✓
$(x+3)(2x+2)$	$2x+6x=8x$	✗

$(2x+6)$, $(x+1)$	$(2x+1)$, $(x+6)$	$(2x+3)$, $(x+2)$	$(2x+2)$, $(x+3)$
$2x+6x=8x$ $\neq 7x$	$12x+x=13x$ $\neq 7x$	$4x+3x=7x$ (middle term)	$6x+2x=8x$ $\neq 7x$
(✗)	(✗)	(✓)	(✗)

$\therefore 2x^2+7x+6=(2x+3)(x+2)$

(2) Factorize $4x^2$ into $2x, 2x$ ⟶ $(2x\ \ )(2x\ \ )$

or $x, 4x$ ⟶ $(x\ \ )(4x\ \ )$

Factorize -3 into $-1, +3$ or $+1, -3$

Determine the corresponding middle term as follows:

$\therefore 4x^2+x-3=(x+1)(4x-3)$

Possible factorization	Corresponding middle term	
$(2x-1)(2x+3)$	$4x$	✗
$(2x+1)(2x-3)$	$-4x$	✗
$(x-1)(4x+3)$	$-x$	✗
$(x+1)(4x-3)$	x	✓
$(x-3)(4x+1)$	$-11x$	✗
$(x+3)(4x-1)$	$11x$	✗

(3) $2x^4+9x^2+10=(2x^2+5)(x^2+2)$

Solving equations using factorization

Example (4)

Find the solution set of each of the following equations in R:

(1) $x^2+5x-36=0$ (2) $x^2+30=-13x$ (3) $6x^2-13x+6=0$

(1) $\because x^2+5x-36=0$ $\therefore (x-4)(x+9)=0$

$\therefore$ Either $x-4=0$ hence $x=4$

or $x+9=0$ hence $x=-9$

$\therefore$ The solution set $=\{4, -9\}$

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■ **Verify the Solution**

You can verify the solution by substituting each value of x into the original equation to check that both sides of the equation are equal.

② $\because x^2 + 30 = -13x$ $\therefore x^2 + 13x + 30 = 0$

$\therefore (x + 3)(x + 10) = 0$

$\therefore$ Either $x + 3 = 0$ hence $\boxed{x = -3}$

or $x + 10 = 0$ hence $\boxed{x = -10}$

$\therefore$ The solution set $= \{-3, -10\}$

③ $\because 6x^2 - 13x + 6 = 0$ $\therefore (2x - 3)(3x - 2) = 0$

$\therefore$ Either $2x - 3 = 0$ hence $\boxed{x = \frac{3}{2}}$

or $3x - 2 = 0$ hence $\boxed{x = \frac{2}{3}}$

$\therefore$ The solution set $= \{\frac{3}{2}, \frac{2}{3}\}$

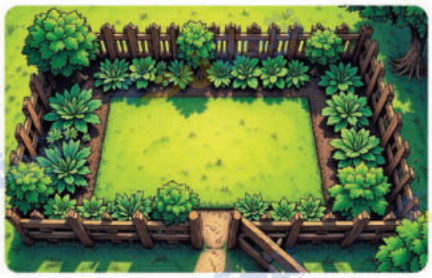
■ **Self-Evaluation** ⑤



One of the sponsors of the annual school party wants to increase the length and width of one of the pictures in their advertisement by the same amount in order to double the picture's area. If the original picture was 8 cm wide and 12 cm long, what are the new dimensions of the enlarged picture?

Example 5

Sarah has a rectangular garden with an area of $(3x^2 + 4x - 7)\ m^2$. Use factorization to find two possible dimensions for the garden. If Sarah wants to expand the garden so that each dimension is 3 m larger than it was, what is the area of the garden after it is expanded?



$\because$ The area of the garden $= (3x^2 + 4x - 7)\ m^2$

$= (3x + 7)(x - 1)$

$\therefore$ The width of the garden $= (x - 1)\ m$,

The length of the garden $= (3x + 7)\ m$.

After expanding the garden:

Its width $= (x - 1 + 3) = (x + 2)\ m$.

Its length $= (3x + 7 + 3) = (3x + 10)\ m$.

$\therefore$ The area of the garden after expansion $= (3x + 10)(x + 2)$

$= (3x^2 + 16x + 20)\ m^2$

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Lesson Assessment



First Measuring Conceptual Understanding



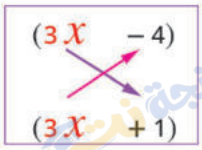
▸ **Choose the correct answer from the given ones:**

1. Which of the following represents the factorization of the polynomial $x^2 + x - 20$?

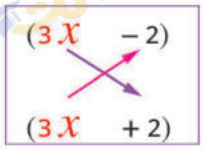
(a) $(x-4)(x+5)$ (b) $(x+4)(x+5)$

(c) $(x-4)(x-5)$ (d) $(x+4)(x-5)$

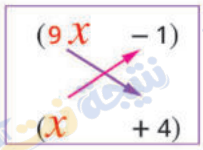
2. Which of the following represents the factorization of the polynomial $9x^2 - 9x - 4$?



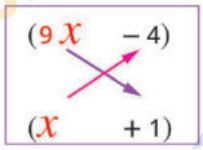
(a)



(b)



(c)



(d)

3. If $(x-3)(x+5) = 0$, which of the following is true?

(a) $x = 3$ or $x = 5$ (b) $x = 3$ or $x = -5$

(c) $x = -3$ or $x = 5$ (d) $x = -3$ or $x = -5$

4. If $x^2 + ax + 15 = (x+3)(x+b)$, what is the value of ab?

(a) 5 (b) 8 (c) 13 (d) 40

5. If $x^2 - 11x + 28 = (x-a)(x-b)$, what is the value of $a + b$?

(a) -16 (b) -11 (c) 11 (d) 16

6. If $(x-5)$ is one of the factors of the trinomial $(2x^2 - 7x - 15)$, what is the other factor?

(a) $x+3$ (b) $2x-3$ (c) $2x+3$ (d) $x-3$

7. If $3x - 2y = 6$, $2x + 3y = 5$, what is the numerical value of the polynomial $(6x^2 + 5xy - 6y^2)$?

(a) 10 (b) 11 (c) 12 (d) 30

▸ **Spot the mistake:**

8. Find the mistake in each of the following and correct it.

① $3x^2 + 7x - 6 = 0$

$(3x+2)(x-3) = 0$

$\therefore$ Either $3x + 2 = 0$

hence $x = \frac{-2}{3}$

or $x - 3 = 0$

hence $x = 3$

② $2x^2 - 5x - 3 = 9$

$(2x+1)(x-3) = 0$

$\therefore$ Either $2x + 1 = 0$

hence $x = \frac{-1}{2}$

or $x - 3 = 0$

hence $x = 3$

③ $x^2 + 5x = 24$

$x(x+5) = 24$

$\therefore$ Either $x = 24$

or $x + 5 = 24$

hence $x = 19$

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Second Applying Concepts

(9) Factorize each of the following completely:

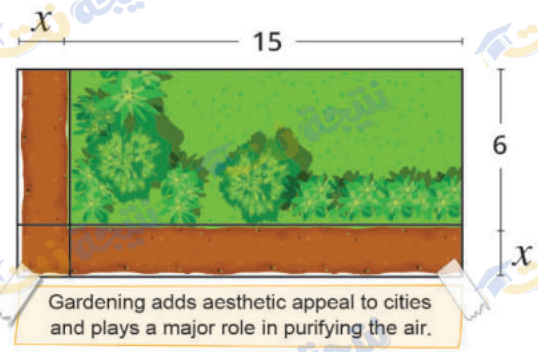
(1) $x^2 - 7x + 12$	(2) $y^2 + 42 + 13y$	(3) $-x^2 + 3x + 40$
(4) $14x - 32 + x^2$	(5) $3x^2 + 6x - 45$	(6) $5n^2 - 20n - 105$
(7) $3a^2 - 6a - 9$	(8) $2x^2 - 7x + 6$	(9) $3x^2 + 14x + 15$
(10) $6x^2 + 7xy - 3y^2$	(11) $5x^2 - 4y(7x + 3y)$	(12) $(x + 2)(x - 5) - 8$
(13) $6x^3 - 60x - 2x^2$	(14) $18 - 21x^2 - 9x^4$	(15) $42x^2y^2 - 30x^2y^4 + 12x^2y^3$

(10) Solve each of the following equations in R :

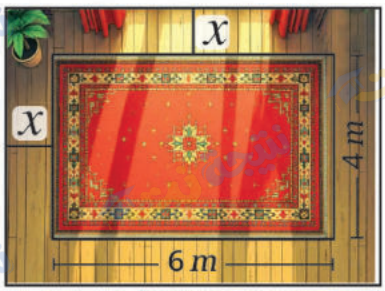
(1) $m^2 - 14m + 45 = 0$ (2) $x^2 - 21 = 4x$ (3) $3y^2 + 17y + 10 = 0$

Third Analysis and Subjects Integration

(11) Agriculture: Youssef has a rectangular garden of dimensions 6 m and 15 m , and he wants to increase its area by 46 m^2.
If the length and width of the garden are increased by the same amount, what are the new dimensions of the garden?



Gardening adds aesthetic appeal to cities and plays a major role in purifying the air.



Placing carpets in the right place with the right colours reflects good taste and an awareness of the importance of visual harmony.

(12) Decoration: A rectangular room has a floor area of 48 m^2. Fatma wanted to cover the floor with a carpet 6 m long and 4 m wide, leaving a strip of space between the carpet and the wall from each side of width x.
Find the width of this strip.

(13) Geometry: A triangle has an area of 35 cm^2. If the length of its base is 9 cm greater than its corresponding height, what is the height of the triangle?

Creative Thinking

(14) Find the positive integer values of b that make each of the following factorizable:

(1) $x^2 + bx + 12$ (2) $x^2 + bx - 14$ (3) $x^2 + 8x + b$

***Evaluate** your understanding!*

How well do you understand factorizing trinomial?
Tick the right box



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Lesson

2 - 3 Factorizing Special Cases

Learning Outcomes

- Factorize a trinomial that represents a perfect square.
- Factorize the difference between two squares.
- Factorize the difference between two cubes.
- Factorize the sum of two cubes.
- Use factorization to facilitate mathematical operations.
- Use factorization to solve some real-life problems.

Vocabulary

- Perfect square.
- Difference between two squares.
- Difference between two cubes.
- Sum of two cubes.

Get Ready!

The pyramids of Giza in Egypt are one of the greatest wonders of the ancient world. The base of the Great Pyramid is a square, and assuming that its area is $(x^2 + 6x + 9)\ m^2$, can you find its perimeter in terms of x?

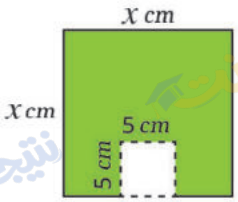


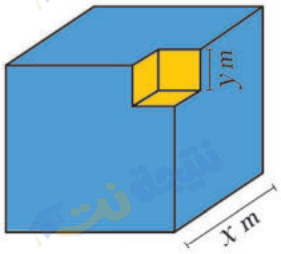
The Pyramids of Giza

In this lesson, you will learn how to factorize a perfect square trinomial and some other special cases, which will enable you to solve such problems.

Think & Discuss!

- If you have a square with a side length of x cm, and you decide to cut out a smaller square with a side length of 5 cm, as shown in the figure, can you express the area of the remaining part as the product of two factors? Discuss.





- If you have a cube with an edge length of x m, and you decide to cut out a smaller cube with an edge length of y m, as shown in the figure, can you express the volume of the remaining part as the product of two factors? Discuss.

Learn!

First Factorizing the trinomial that represents a perfect square

If the trinomial is arranged in ascending or descending order according to the exponents of one of its variables, it represents a perfect square if:

- The first term is a perfect square.
- The third term is a perfect square.
- The middle term is equal to $(\pm 2 \times \sqrt{\text{The first term}} \times \sqrt{\text{The third term}})$.

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• Factorizing the trinomial that represents a perfect square is to write it as the product of two equal factors
(i.e., the square of one of its equal factors).

That is:
$$a^2 + 2ab + b^2 = (a+b)^2$$
$$a^2 - 2ab + b^2 = (a-b)^2$$

■ Self-Evaluation (1)

Which of the following trinomials represent a perfect square and which do not ? Then factorize the trinomials that are in the form of a perfect square:

1. $16b^2 - 8b + 1$
2. $5x^2 + 20x + 4$
3. $x^2 + 2xy - y^2$
4. $9x^4 - 12x^2y + 4y^2$

Example (1)

Which of the following trinomials represent a perfect square and which do not? Then factorize the trinomials that are in the form of a perfect square:

1. $4x^2 + 20x + 25$
2. $x^2 - 6x - 9$
3. $x^2 + 9x + 81$
4. $25x^4 - 30x^2y + 9y^2$

(1) ∵ $4x^2 = (2x)^2$ «Perfect square» , $25 = (5)^2$ «Perfect square» , $2(2x)(5) = 20x$ «Middle term»

∴ The trinomial $(4x^2 + 20x + 25)$ represents a perfect square , then:

$$4x^2 + 20x + 25 = \left(\sqrt{4x^2} + \sqrt{25}\right)^2 = (2x+5)^2$$

the same sign as the middle term

(2) The trinomial $(x^2 - 6x - 9)$ is not a perfect square because the third term (-9) is negative, i.e. it is not a perfect square.

(3) ∵ $x^2 = (x)^2$ «Perfect square» , $81 = (9)^2$ «Perfect square» , $2(x)(9) = 18x$ «Not equal to the middle term»

∴ The trinomial $(x^2 + 9x + 81)$ is not a perfect square. Because the middle term is not equal to $(\pm 2 \times \sqrt{\text{The first term}} \times \sqrt{\text{The third term}})$.

(4) ∵ $25x^4 = (5x^2)^2$ «Perfect square» , $9y^2 = (3y)^2$ «Perfect square» , $-2(5x^2)(3y) = -30x^2y$ «Middle term»

∴ The trinomial $(25x^4 - 30x^2y + 9y^2)$ represents a perfect square , then:

$$25x^4 - 30x^2y + 9y^2 = \left(\sqrt{25x^4} - \sqrt{9y^2}\right)^2 = (5x^2 - 3y)^2$$

the same sign as the middle term

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Example 2

Find the value of a that makes each of the following a perfect square:

1) $x^2 + 14x + a$ 2) $x^2 - ax + 64$

1) For the trinomial to be a perfect square

$$a = \frac{(14x)^2}{4(x^2)} = \frac{196x^2}{4x^2} = 49$$

2) For the trinomial to be a perfect square

$$-ax = \pm 2(\sqrt{x^2})(\sqrt{64})$$
$$= \pm 16x$$
$$\therefore a = \pm 16$$

Note that

1) The middle term is equal to $\pm 2\sqrt{\text{The first term}} \times \sqrt{\text{The third term}}$

2) The first term is equal to $\frac{\text{the square of the middle term}}{4 \times (\text{the third term})}$

3) The third term is equal to $\frac{\text{the square of the middle term}}{4 \times (\text{the first term})}$

Second Factorizing the difference between two squares

$$a^2 - b^2 = (a - b)(a + b)$$

Self-Evaluation (3)

Factorize each of the following:

1) $x^2 - 1$
2) $16x^2 - 9$
3) $25x^2 - 36y^2$
4) $x^2y^2 - 4$

Example 3

Factorize each of the following:

1) $x^2 - 9$ 2) $9x^4 - 16y^2$ 3) $1 - 100x^2y^2$

1) $x^2 - 9 = (x - 3)(x + 3)$
2) $9x^4 - 16y^2 = (3x^2 - 4y)(3x^2 + 4y)$
3) $1 - 100x^2y^2 = (1 - 10xy)(1 + 10xy)$

Think

Can the sum of two squares $(x^2 + y^2)$ be factorized?

Third Factorizing the sum of two cubes and the difference between them

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Self-Evaluation (4)

Factorize each of the following:

1) $x^3 + 1$
2) $m^3 - 343n^3$
3) $8x^6 - 27y^3$
4) $27 - a^3b^3$

Example 4

Factorize each of the following:

1) $x^3 - 27$ 2) $a^6 - 8b^3$ 3) $1 + 512x^3y^3$

1) $x^3 - 27 = (x - 3)(x^2 + 3x + 9)$
2) $a^6 - 8b^3 = (a^2 - 2b)(a^4 + 2a^2b + 4b^2)$
3) $1 + 512x^3y^3 = (1 + 8xy)(1 - 8xy + 64x^2y^2)$

Self-Evaluation (2)

Find the value of a that makes each of the following a perfect square:

1) $x^2 - 10x + a$
2) $4x^2 + ax + 49$

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■ Self-Evaluation (5)

Factorize each of the following:

1. $y^2 - \frac{1}{9}$
2. $27x^3 - \frac{1}{64}$
3. $c^3 + \frac{1}{8}$
4. $x^2 + x + \frac{1}{4}$

Example 5

Factorize each of the following:

1. $\frac{1}{4}x^2 - \frac{1}{25}y^2$
2. $8y^3 - \frac{1}{27}$
3. $a^3b^3 + \frac{1}{216}$
4. $\frac{1}{4}x^2 + \frac{2}{3}x + \frac{4}{9}$

1. $\frac{1}{4}x^2 - \frac{1}{25}y^2 = (\frac{1}{2}x - \frac{1}{5}y)(\frac{1}{2}x + \frac{1}{5}y)$
2. $8y^3 - \frac{1}{27} = (2y - \frac{1}{3})(4y^2 + \frac{2}{3}y + \frac{1}{9})$
3. $a^3b^3 + \frac{1}{216} = (ab + \frac{1}{6})(a^2b^2 - \frac{1}{6}ab + \frac{1}{36})$
4. $\frac{1}{4}x^2 + \frac{2}{3}x + \frac{4}{9} = (\frac{1}{2}x + \frac{2}{3})^2$

■ Note that

It is possible to factorize polynomials containing rational coefficients and constants into a product of factors each factor is a polynomial also containing rational coefficients and constants.

■ Self-Evaluation (6)

Factorize each of the following completely:

1. $8x^3 - 18x$
2. $75x^3y + 60x^2y + 12xy$
3. $\frac{1}{3}x^2 - 3$
4. $32x^3(3a + b) - 50x(3a + b)$

Example 6

Factorize each of the following completely:

1. $5x^5 - 320x^2$
2. $16x^4 - y^4$
3. $12x - 9x^2 - 4$
4. $64 - (a + b)^2$

1. $5x^5 - 320x^2 = 5x^2(x^3 - 64)$
   $= 5x^2(x - 4)(x^2 + 4x + 16)$
2. $16x^4 - y^4 = (4x^2 - y^2)(4x^2 + y^2)$
   $= (2x - y)(2x + y)(4x^2 + y^2)$
3. First, arrange the polynomial terms in descending order according to the exponents of x as follows:
   $-9x^2 + 12x - 4 = -(9x^2 - 12x + 4)$
   $= -(3x - 2)^2$
4. $64 - (a + b)^2 = (8 + (a + b))(8 - (a + b))$
   $= (8 + a + b)(8 - a - b)$

■ Note that

The greatest common factor is taken out first, then the resulting bracket is factorized if possible.

■ Note that

The polynomial $(-9x^2 + 12x - 4)$ is not a perfect square trinomial, while the polynomial $(9x^2 - 12x + 4)$ is a perfect square trinomial.

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■ Self-Evaluation (7)

Use factorization to facilitate finding the value of each of the following:

1. $(49)^2 - 2(49)(39) + (39)^2$
2. $(87)^2 - (13)^2$

Example 7

Use factorization to facilitate finding the value of each of the following:

1. $(57)^2 + 2(57)(43) + (43)^2$
2. $(99)^2 - 1$

1. $(57)^2 + 2(57)(43) + (43)^2 = (57 + 43)^2 = (100)^2 = 10000$
2. $(99)^2 - 1 = (99 + 1)(99 - 1) = 100 \times 98 = 9800$

■ Self-Evaluation (8)

If $b - a = -3$, $a^2 - b^2 = 12$, find the numerical value of the expression: $a + b$

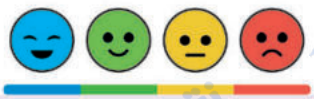
Example 8

If $4x^2 - 9y^2 = 115$, $2x - 3y = 5$,
find the numerical value of the expression: $2x + 3y$

$\because 4x^2 - 9y^2 = 115$ $\therefore (2x - 3y)(2x + 3y) = 115$

$\therefore 5(2x + 3y) = 115$ $\therefore 2x + 3y = 23$

Lesson Assessment



First **Measuring Conceptual Understanding**

▸ **Choose the correct answer from the given ones:**

1. If $x^2 - b^2 = (x - 3)(x + 3)$, what is the value of b ?
   (a) ± 1 (b) ± 3
   (c) ± 6 (d) ± 9

2. If $ax^2 - b = (3x - 2)(3x + 2)$, what is the value of $a + b$?
   (a) -13 (b) 13
   (c) -5 (d) 5

3. If $ax^3 - 27 = (2x - 3)(4x^2 + bx + 9)$, what is the value of ab ?
   (a) 6 (b) 12
   (c) 24 (d) 48

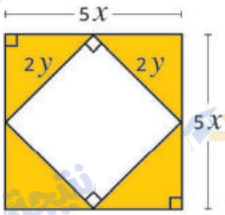
4. If the polynomial $(4x^2 + kx + 1)$ is a perfect square trinomial, what is the value of k ?
   (a) ± 1 (b) ± 2
   (c) ± 4 (d) ± 8

5. If the polynomial $(kx^2 - 12x + 9)$ is a perfect square trinomial, what is the value of k ?
   (a) 4 (b) ± 4
   (c) 2 (d) ± 2

6. If $x - y = -2$, $x^2 - y^2 = -16$, what is the arithmetic mean of the two numbers x , y ?
   (a) 2 (b) 4
   (c) 8 (d) 16

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(7) Which of the following statements represents the area of the coloured region in the given figure?



(a) $5x^2 - 2y^2$ (b) $25x - 4y$

(c) $(5x - 2y)^2$ (d) $(5x - 2y)(5x + 2y)$

▸ **Spot the mistake:**

(8) Omar and Khalid factorized $(x^3 - 8)$ as follows:

Omar's solution:

$x^3 - 8 = (x - 2)(x^2 - 2x + 4)$

Khalid's solution:

$x^3 - 8 = (x - 2)(x^2 - 4x + 4)$

Which of them was correct? Correct the mistake, if any.

Second Applying Concepts

(9) **Factorize each of the following:**

① $a^2 + 18a + 81$ ② $b^2 - 14b + 49$ ③ $9x^2 + 6x + 1$

④ $25 - 9x^2$ ⑤ $1 - 4a^2b^2$ ⑥ $k^2 - 36m^2$

⑦ $y^3 - 8$ ⑧ $x^3 + 27$ ⑨ $64x^3 - 125y^3$

(10) **Factorize each of the following completely:**

① $x^4 + 8x^3 + 16x^2$ ② $2xy^4 - 16yx^4$ ③ $121y - y^3$

④ $20x^3 - 45x$ ⑤ $-27x^2 + 75$ ⑥ $x^4 - 81$

⑦ $54x^3 + 16$ ⑧ $ax^4 - 343axb^3$ ⑨ $4x^4 + 4000x$

(11) **Factorize each of the following completely:**

① $x^2 - \frac{4}{81}$ ② $125x^3 + \frac{1}{8}$ ③ $\frac{1}{5}x^2 - 5$

④ $(x+3)^3 + 64$ ⑤ $(x+3)^2 - 16$ ⑥ $6a^2(x+2y) - 24b^2(x+2y)$

⑦ $x^4 - 13x^2 + 36$ ⑧ $x^6 - 7x^3 - 8$ ⑨ $m^6 - 64n^3$

⑩ $4x^{16} - 16y^4$ ⑪ $x^6 - 64y^6$

(12) **Solve each of the following equations in R:**

① $4x^2 + 1 = 4x$ ② $x^3 = 25x$ ③ $x^2 + 6x = -9$

(13) If $ab = 10$, $a - b = 3$,

find the numerical value of the expression: $a^3 - b^3$

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Third Analysis and Subjects Integration



Storing and organizing data facilitates its management and enhances information security.

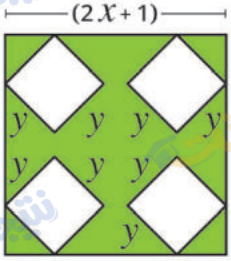
(14) **Technology :** Data is stored in storage units in the form of cubic units.
The capacity of the first-generation storage unit is $216\,x^3$ data unit, and the capacity of the second-generation storage unit is $343\,y^3$ data unit. Find the total capacity of both units as the product of two factors.



Travel contributes to promoting understanding and tolerance between different cultures and deepens awareness of the value of cultural diversity.

(15) **Science :** The pressure difference P above and below the wing of an aircraft is described by the formula $P=\frac{1}{2}\,d\,v^2-\frac{1}{2}\,d\,u^2$ where d is the air density, v is the speed of the air passing over the wing, and u is the speed of the air passing under the wing.
Write this formula as the product of prime polynomials.

(16) **Investment :** A construction company divided a square plot of land with side length $(2x+1)$ m into 4 squares, each with side length $y\ m$, as shown in the figure. The rest of the land was left as green space. Write the area of the coloured region as the product of some factors.



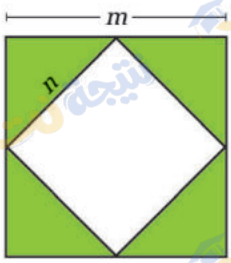
(17) **Geometry :** If the volume of a cuboid is (x^4-625) cubic units, by using factorization, how can you find its possible dimensions?



(18) **Agriculture:** A garden consists of two adjacent plots of land, the larger one is a square with an area of $(25x^2+10x+1)\ m^2$, and the smaller one is a square with an area of $(4x^2+4x+1)\ m^2$.
A fence is to be built around the garden.
Find the length of the fence in terms of x, then find the perimeter of the garden if x is equal to 10 m.

Creative Thinking

(19) In the given figure, the sum of the perimeters of the two squares is 76 cm, and the area of the coloured region is 57 cm^2.
What is the value of $(m-n)$?



***Evaluate** your understanding!*

How well do you understand factorizing special cases?
Tick the right box



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Lesson

2 - 4

Factorizing by Grouping

Learning Outcomes

- Factorize a polynomial by grouping.
- Learn how to complete the square.
- Use completing the square to factorize and solve equations.
- Use factorization by grouping to solve real-life problems.

Vocabulary

- Factorizing by grouping
- Completing the square

Get Ready!

The Central Library at Cairo University is one of the most important educational and cultural facilities within the university, and it is one of the largest university libraries in the Arab world.

It can accommodate more than a thousand readers at the same time. If the area of the rectangular plot of land on which it is built is $(3x^3 - 6x^2 + 4x - 8)\ m^2$, how can you determine the dimensions of this plot?

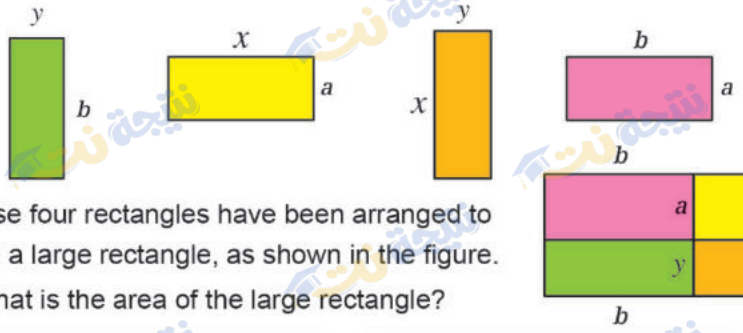


Cairo University

In this lesson, you will learn how to factorize by grouping, which will enable you to solve such problems.

Think & Discuss!

The following figures are four rectangles with the given dimensions:



These four rectangles have been arranged to form a large rectangle, as shown in the figure.

- What is the area of the large rectangle?

Bassem's answer:
The area of the rectangle is equal to $ab + xy + ax + by$

Maryam's answer:
The area of the rectangle is equal to $(b + x)(a + y)$

Which one of them is correct ? Discuss, what do you deduce ?

Learn!

Factorizing by grouping

A four-term polynomial can be factorized using one of the following two methods:

The first method

Group the polynomial into two binomials so that you can factorize each binomial, then take out the common binomial as a common factor.

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■ Self-Evaluation (1)

Factorize each of the following completely:

1. $x^2+4x+bx+4b$
2. $xy+12-3x-4y$
3. $2x^2y^2+24xy-6xy^2-8x^2y$
4. $x^3+6x^2-4x-24$

Example 1

Factorize each of the following completely:

1. $ax+by+ay+bx$
2. $3x^2-3y+xy-9x$
3. $3x^3+2x^2-3x-2$
4. x^2-5x-y^2+5y
5. $2x^3y-6x^2y-18xy+54y$

1. $ax+by+ay+bx$
$=(ax+ay)+(by+bx)$
$=a(x+y)+b(y+x)$
$=(x+y)(a+b)$

■ Another Solution

$ax+by+ay+bx$
$=(ax+bx)+(by+ay)$
$=x(a+b)+y(b+a)$
$=(a+b)(x+y)$

2. $3x^2-3y+xy-9x$
$=(3x^2-9x)+(xy-3y)$
$=3x(x-3)+y(x-3)$
$=(x-3)(3x+y)$

■ Note that

In number (2) if the grouping has been done this way:

$3x^2-3y+xy-9x$
$=(3x^2-3y)+(xy-9x)$
$=3(x^2-y)+x(y-9)$

There is no common factor so this grouping fails and you should try to regroup in a different way.

3. $3x^3+2x^2-3x-2$
$=(3x^3+2x^2)+(-3x-2)$
$=x^2(3x+2)-(3x+2)$
$=(3x+2)(x^2-1)$
$=(3x+2)(x-1)(x+1)$

4. $x^2-5x-y^2+5y=(x^2-y^2)+(5y-5x)$
$=(x-y)(x+y)+5(y-x)$
$=(x-y)(x+y)-5(x-y)$
$=(x-y)(x+y-5)$

■ Note that

$y-x=-(x-y)$

5. $2x^3y-6x^2y-18xy+54y=2y(x^3-3x^2-9x+27)$
$=2y[(x^3-3x^2)+(-9x+27)]$
$=2y[x^2(x-3)-9(x-3)]$
$=2y(x-3)(x^2-9)$
$=2y(x-3)(x-3)(x+3)$
$=2y(x-3)^2(x+3)$

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Note

The trinomial in the form $a x^2 + b x + c$, where $(a \neq 1)$ can be factorized by using grouping as follows:

1. Find two factors whose product is (ac) and whose sum is (b).
2. Rewrite $(b x)$ as the sum of these two factors.
3. Use factorization by grouping.

Example 2

Factorize $3x^2 + 8x + 4$ using factorizing by grouping.

$\because a = 3, b = 8, c = 4$

Find two numbers whose product $(a c)$ is 12 and whose sum (b) is 8

$\therefore$ These two numbers are 6, 2

Rewrite $8x$ as the sum of $(6x + 2x)$ and then use factorizing by grouping as follows:

$$3x^2 + 8x + 4 = 3x^2 + 6x + 2x + 4 = (3x^2 + 6x) + (2x + 4)$$
$$= 3x(x + 2) + 2(x + 2)$$
$$= (x + 2)(3x + 2)$$

The second method

Group the polynomial into a trinomial that represents a perfect square, and the fourth term must also be a perfect square, so that the original polynomial is factorized as a difference between two squares.

Example 3

Factorize each of the following completely:

1. $x^2 + 16y^2 - 8xy - 49$
2. $4m^2 - n^2 + 10nk - 25k^2$

1) $x^2 + 16y^2 - 8xy - 49 = (x^2 - 8xy + 16y^2) - 49$
$$= (x - 4y)^2 - (7)^2$$
$$= (x - 4y - 7)(x - 4y + 7)$$

2) $4m^2 - n^2 + 10nk - 25k^2 = 4m^2 - (n^2 - 10nk + 25k^2)$
$$= 4m^2 - (n - 5k)^2$$
$$= [2m - (n - 5k)][2m + (n - 5k)]$$
$$= (2m - n + 5k)(2m + n - 5k)$$

■ Self-Evaluation (2)

Use factorizing by grouping to factorize each of the following trinomials:

1. $2x^2 - x - 28$
2. $6x^2 + 19x + 15$

■ Self-Evaluation (3)

Factorize each of the following completely:

1. $x^2 - 12xy + 36y^2 - 4$
2. $x^2 + y^2 - 64 + 2xy$

■ Think

When factorizing the following polynomials by grouping:

1. $x^2 + 4x - y^2 + 4$
2. $x^2 + 4x - y^2 + 4y$

discuss how each one is grouped.

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Completing the square

Completing the square is the operation of adding a term to a binomial so that it becomes a trinomial in the form of a perfect square.

Completing the square can be done in one of the following two ways:

1. Either add (the middle term) $= (\pm 2 \times \sqrt{\text{the first term}} \times \sqrt{\text{the third term}})$.

   For example: $(x^2 + 49)$ and add $\pm 2\sqrt{x^2}\sqrt{49} = \pm 14x$

   Then $(x^2 \pm 14x + 49)$ is a perfect square, i.e. it is written as $(x \pm 7)^2$

2. OR add (the third term) $= \dfrac{\text{The square of the middle term}}{4 \times \text{The first term}}$

   For example: $(x^2 + 10x)$ and add $\dfrac{(10x)^2}{4x^2} = 25$

   Then $(x^2 + 10x + 25)$ is a perfect square, i.e. it is written as $(x + 5)^2$

and the following examples illustrate this.

Example 4

Factorize each of the following:

1. $81x^4 + 4$
2. $x^4 + 15x^2y^2 + 64y^4$

✓

(1)

$81x^4 + 4 = 81x^4 + 4 + 36x^2 - 36x^2$

«Add and subtract $2\sqrt{81x^4}\sqrt{4} = 36x^2$

$= (81x^4 + 36x^2 + 4) - (36x^2)$

$= (9x^2 + 2)^2 - (6x)^2$

$= (9x^2 + 2 - 6x)(9x^2 + 2 + 6x)$

$= (9x^2 - 6x + 2)(9x^2 + 6x + 2)$

(2)

$x^4 + 15x^2y^2 + 64y^4 = x^4 + 15x^2y^2 + 64y^4 + 16x^2y^2 - 16x^2y^2$

«Add and subtract $2\sqrt{x^4}\sqrt{64y^4} = 16x^2y^2$

$= (x^4 + 16x^2y^2 + 64y^4) + 15x^2y^2 - 16x^2y^2$

$= (x^2 + 8y^2)^2 - x^2y^2$

$= (x^2 + 8y^2 - xy)(x^2 + 8y^2 + xy)$

■ Self-Evaluation (4)

Factorize each of the following:

1. $x^4 + 64$
2. $4x^4 + 11x^2 + 9$

■ Note that

To factorize polynomials that include at least two terms each is a perfect square, and the exponents of their variables are 4 or multiples of 4, use the method of completing the square by adding and subtracting (twice the product of the square roots of the two square terms) in order not to affect the value of the polynomial, then factorize by grouping.

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■ Self-Evaluation (5)

Find the solution set of the equation:

$x^2 - 2x - 4 = 0$ in R

Example 5

Find the solution set of the equation : $x^2 - 8x + 3 = 0$ in R

$\because x^2 - 8x + 3 = 0$

$\therefore x^2 - 8x = -3$

By adding $\frac{(-8x)^2}{4x^2} = 16$ to both sides of the equation

$\therefore x^2 - 8x + 16 = -3 + 16$

$\therefore (x-4)^2 = 13$ $\quad \therefore x - 4 = \pm\sqrt{13}$

$\therefore x = 4 \pm \sqrt{13}$

$\therefore$ The solution set $= \{4 - \sqrt{13}, 4 + \sqrt{13}\}$

■ Note that

We cannot solve this example using trinomial factorization, but it can be solved by using completing the square.

■ Self-Evaluation (6)

If $3b - 2a = 1$, $a^2 - 2b = 10$, find the numerical value of the expression:

$2a^3 - 3a^2 b - 4ab + 6b^2$

Example 6

If $x^2 + 3y = 31$, $2x - y = 8$,

what is the numerical value of the expression

$2x^3 - x^2 y + 6xy - 3y^2$?

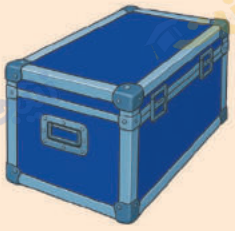
$2x^3 - x^2 y + 6xy - 3y^2 = (2x^3 - x^2 y) + (6xy - 3y^2)$

$= x^2(2x - y) + 3y(2x - y)$

$= (2x - y)(x^2 + 3y) = 8 \times 31 = 248$

■ Self-Evaluation (7)

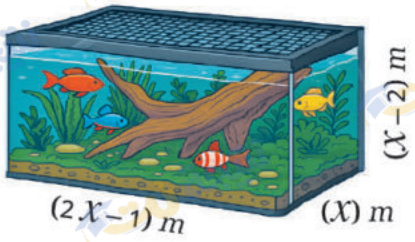
A company manufactures storage units for photography equipments for trips. The unit is in the shape of a cuboid with a square base whose side length is $(x - 1)$ feet and its height is x feet.



If the volume of the unit is 2 cubic feet, find the dimensions of the storage unit.

Example 7

A cuboid glass box used for decoration. Its dimensions, in meters, are shown in the figure. If the volume of the box is 5 m^3, find its dimensions in meters.



The volume of the box $= 5\ m^3$

$\therefore x(x-2)(2x-1) = 5$

$\therefore x(2x^2 - 5x + 2) = 5$ $\quad \therefore 2x^3 - 5x^2 + 2x - 5 = 0$

$\therefore (2x^3 - 5x^2) + (2x - 5) = 0$ $\quad \therefore x^2(2x - 5) + (2x - 5) = 0$

$\therefore (2x - 5)(x^2 + 1) = 0$

$\therefore$ Either $2x - 5 = 0$ hence $\boxed{x = \frac{5}{2}}$

or $x^2 + 1 = 0$ hence $x^2 = -1$ (refused)

$\therefore$ The dimensions of the glass box are 0.5 m, 2.5 m, and 4 m.

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Lesson Assessment



First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

1. $x^2 - ax + bx - ab =$ .....

(a) $(x - a)(x - b)$ (b) $(x - a)(x + b)$ (c) $(x + a)(x - b)$ (d) $(x + a)(x + b)$

2. If $xy + 5x + 7y + 35 = (x + a)(y + b)$, what is the value of $a - b$?

(a) 2 (b) -2 (c) 12 (d) -12

3. If $x^3 + 5x^2 - 4x - 20 = (x + a)(x + b)(x + c)$, what is the value of $a + b + c$?

(a) 2 (b) 5 (c) 7 (d) 9

4. If $x^2 + a = 12$, $x + b = 5$, what is the numerical value of the polynomial $(x^3 + bx^2 + ax + ab)$?

(a) 7 (b) 17 (c) 30 (d) 60

5. When completing the square for: $(x^2 + 9)$ add .....

(a) $\pm 3x$ (b) $\pm 6x$ (c) $\pm 9x$ (d) $\pm 12x$

6. When completing the square for: $(x^2 + 12x)$ add .....

(a) ± 9 (b) ± 36 (c) 9 (d) 36

7. If $(x^2 + 2x + 2)$ is one of the two factors of $(x^4 + 4)$, what is the other factor?

(a) $x^2 - 2x + 2$ (b) $x^2 - 2x - 2$ (c) $x^2 + x + 2$ (d) $x^2 - x + 2$

Second Applying Concepts

8. **Factorize each of the following completely:**

(1) $ax + bx + b + a$ (2) $12y + 2x - 8xy - 3$ (3) $6m - n + 3mn - 2$

(4) $x^2 - 2x - xy + 2y$ (5) $11x^3 - 6x^2 + 11x - 6$ (6) $45x^3 + 20x^2 + 9x + 4$

9. **Factorize each of the following trinomials using factorizing by grouping:**

(1) $2x^2 + 5x + 3$ (2) $4x^2 - 7x - 2$ (3) $2x^2 + 13x + 15$

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(10) Factorize each of the following completely:

① $x^2 - y^2 - 6x - 6y$

② $x^2 - a^2 - 2x + 1$

③ $x^4 + 2x^3 - 9x^2 - 18x$

④ $4x^2 - 12xy + 9y^2 - 49$

⑤ $5a^2b - 10a^2b^2 + 4b^2 - 8b^3$

⑥ $m^3 + n^3 - m - n$

⑦ $4k^4 - 49k^2 + 14k - 1$

⑧ $16x^2 - 25a^2 + y^2 + 8xy$

(11) Factorize each of the following completely:

① $x^4 + 4y^4$

② $4m^4 + 1$

③ $64x^4 + y^4$

④ $a^4 + a^2b^4 + b^8$

⑤ $y^4 - 23y^2 + 1$

⑥ $x^4 - 11x^2 + 1$

⑦ $16a^4 + 20a^2b^2 + 9b^4$

⑧ $k^5 + 5k^3 + 9k$

⑨ $4x^2(4x^2 - 7y^2) + y^4$

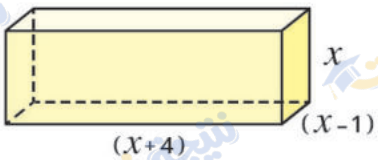
(12) Find the solution set of each of the following equations in R:

① $x^3 + x^2 - 4x = 4$

② $x^2 + 12x + 15 = 0$

Third Analysis and Subjects Integration

(13) Geometry: If the volume of the box is 12 cubic units, find its dimensions.



(14) Geometry: If the volume of a cuboid is $(x^4 - 2x^3 + 3x^2 - 6x)$ cubic feet, find possible dimensions of the cuboid in terms of x, then find the numerical values of the dimensions when $x = 4$



(15) Decoration: A picture is surrounded by a frame with a thickness of x inches, as shown in the figure. If the area of the picture is $(4x^3 + 6x^2 + 2x + 3)$ square inches, find possible dimensions of the framed picture in terms of x.

Creative Thinking

(16) If $a + b = 3$, find the value of $a^3 + 3a^2b + 3ab^2 + b^3$

(17) Factorize completely: $x^8 - 16y^8$

***Evaluate** your understanding!*

How well do you understand factorizing by grouping?
Tick the right box

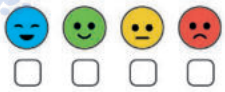


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Unit Two Assessment

Evaluate your understanding!

How well do you understand Unit Two? Tick the right box



▸ **Choose the correct answer from the given ones:**

(1) If the trinomial $(9x^2 + kx + 49)$, is a perfect square, what is the value of k?

(a) ± 7 (b) ± 21

(c) ± 42 (d) ± 72

(2) If $x^2 - 14x + 24 = (x - a)(x - b)$, what is the value of $a + b$?

(a) -10 (b) -14

(c) 10 (d) 14

(3) If $x + y = 7, m - n = 5$, then $x(m - n) - y(n - m) =$ .....

(a) 2 (b) 12

(c) 35 (d) 70

(4) $x^3 + y^3 = ($ ..... $)(x^2 - xy + y^2)$

(a) $(x - y)$ (b) $(x + y)$

(c) $(x^2 - y^2)$ (d) $(x^2 + y^2)$

▸ **Complete each of the following with the correct answer:**

(5) The greatest common factor of the polynomial $16x^3 + 8x^2 - 32x$ is .....

(6) When completing the square for $(x^2 + 24x)$, you should add .....

(7) If $(m^2 + 4m + 8)$ is one of the factors of $(m^4 + 64)$, then the other factor is .....

(8) If $x^3 - 3x^2 - x + 3 = (x + a)(x + b)(x + c)$, then $(a \times b \times c)$ equals .....

▸ **Answer the following questions:**

(9) Find the solution set of the equation: $x^2 - 6x = -8$ in R

(10) **Factorize each of the following completely:**

① $3x^2 + 2x - 5$ ② $k^2 - 64m^2$ ③ $x^3y^3 - 27$

(11) A parallelogram has an area of 60 cm^2, and its base length is 7 cm more than its corresponding height. What is the height of the parallelogram?

(12) **Factorize each of the following completely:**

① $x^2 - 6x - y^2 + 6y$ ② $8x^5 - 10x^3y^2 + 2xy^4$:

Unit Two Activity

The historical origins of algebraic factorization



▸ **Activity Aim:**

Acknowledge the contributions of Muslim Scientists in the development of algebra and algebraic expressions.

▸ **Implementation Steps:**

In collaboration with one of your friends, try to do the following:

1. Use an artificial intelligence application to find the name of a Muslim Scientist who contributed in the development of algebra.
2. Write about the role of this scientist in the factorization of algebraic expressions.
3. Are there any differences between the old and modern methods of factorization of algebraic expressions?

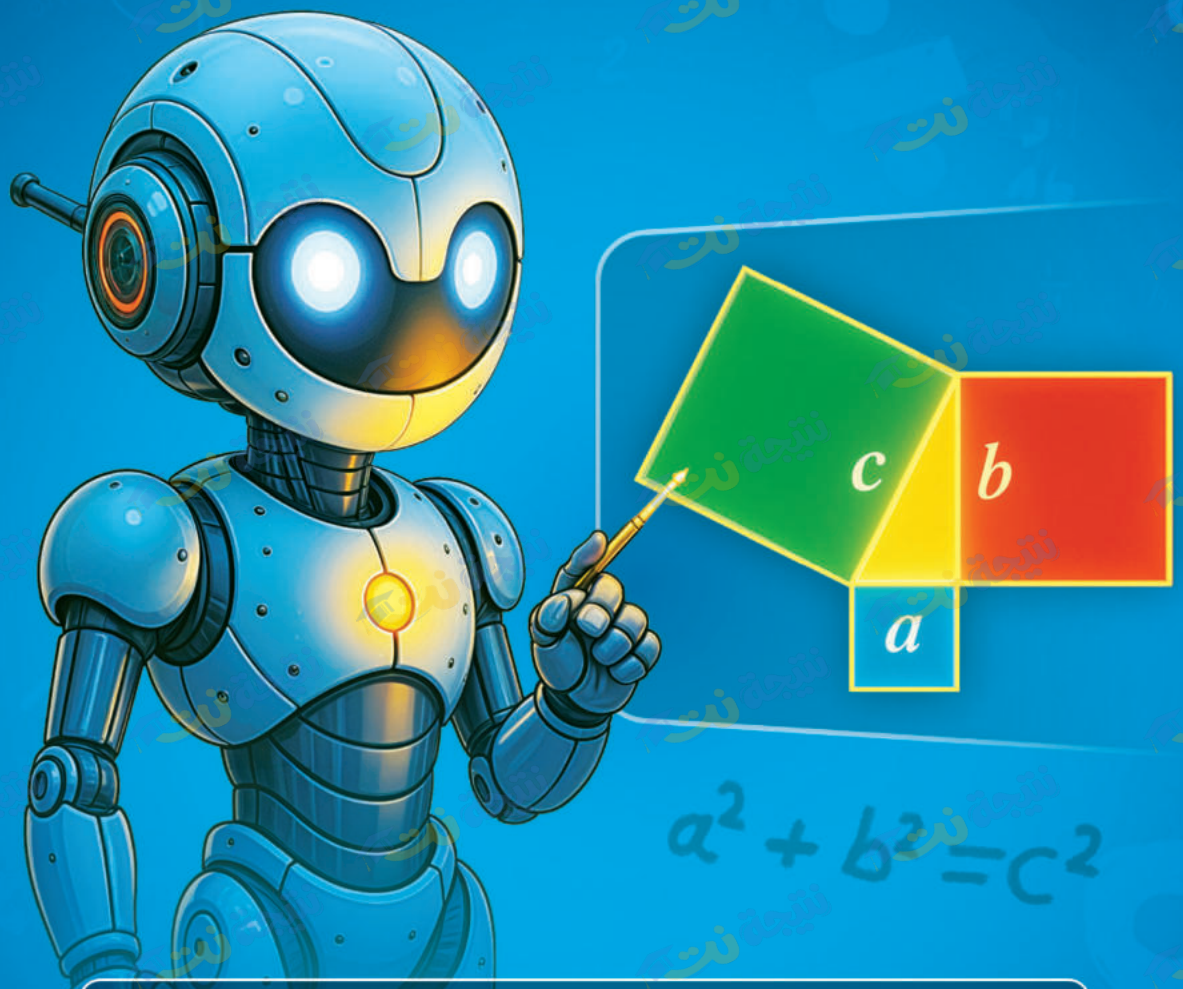
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UNIT 3

Geometry

Unit Lessons

3-1 Congruence
3-2 Pythagoras' Theorem and Its Converse
3-3 Applications of Parallelism
3-4 Medians of the Triangle
3-5 Isosceles Triangle
3-6 Interior and Exterior Angles of Polygons



Geometric theories are used in many artificial intelligence calculations, especially in the analysis of geometric shapes and the design of industrial robots.

▶ **Can machine learning algorithms be developed to solve geometric problems in our daily life?**

Issues and Life Skills
- Mathematical Communication
- Creative Thinking
- Critical Thinking
- Mathematical Language

Values
- Beauty of Nature
- Health Awareness
- Appreciation of Art
- Environmental Awareness

Lesson

3 - 1

Congruence

Learning Outcomes

- Learn the concept of congruence in plane geometric shapes.
- Learn the conditions for two polygons to be congruent.
- Learn the cases of two triangles to be congruent.
- Use congruence to find the measures of angles and lengths of sides.
- Prove the congruence of two triangles.
- Use congruence of geometric shapes to solve real-life problems.

Vocabulary

- Congruence
- Hypotenuse
- Right-angled triangle
- Included angle
- Included side

Notation in Mathematics

The symbol $\cong$ is used to express the congruence of two geometric shapes.

Get Ready!

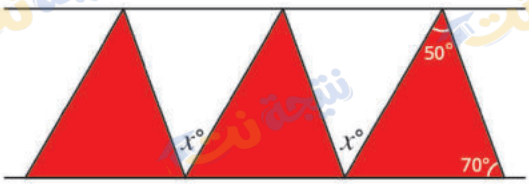
Engineers used congruent triangles to build steel bridges to ensure balance and strength, as congruent triangles contribute distributing of loads equally. In the given picture of a bridge, can you determine the value of x?



In this lesson, you will learn the concept of congruence of plane geometric shapes and the cases of triangle congruence, which will enable you to solve such problems.

Think & Discuss!

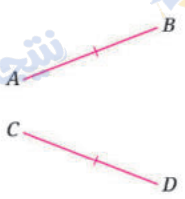
Ibrahim copied the red triangle and pasted it several times to create a decorative strip using a computer program. Can you determine the value of x in this figure?



Learn!

Congruence of two line segments

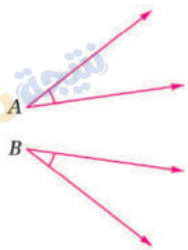
Two line segments are congruent if they are equal in length.



If length of $\overline{AB}$ = length of $\overline{CD}$
then $\overline{AB} \cong \overline{CD}$

Congruence of two angles

Two angles are congruent if they are equal in measure.



If $m(\angle A) = m(\angle B)$
then $\angle A \cong \angle B$

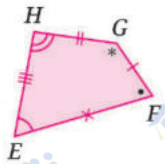
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Congruence of two polygons

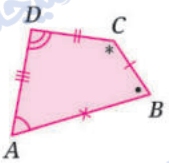
Two polygons are congruent if the following two conditions are satisfied together:

1. Their corresponding sides are equal in length.
2. Their corresponding angles are equal in measure.

If $ABCD$ and $EFGH$ are two polygons in which:



1. $AB = EF$, $BC = FG$,
   $CD = GH$, $AD = EH$

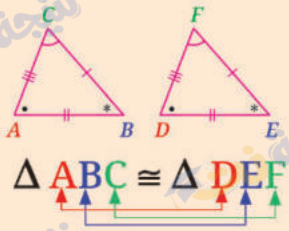


2. $m(\angle A) = m(\angle E)$, $m(\angle B) = m(\angle F)$,
   $m(\angle C) = m(\angle G)$, $m(\angle D) = m(\angle H)$

then The polygon $ABCD \cong$ the polygon $EFGH$

Note that

Congruent polygons must be named in the same order as their corresponding vertices. For example:

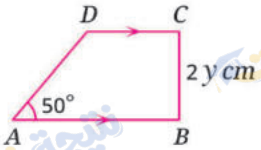


Example 1

In the given figure:

If the polygon $EFGH$ is the image of the polygon $ABCD$ by rotation around a point, what is the value of each of x and y?





$\because \overline{AB} \parallel \overline{DC}$

$\therefore m(\angle A) + m(\angle D) = 180°$

(Two interior angles on the same side of the transversal $\overleftrightarrow{AD}$)

$\therefore m(\angle D) = 180° - 50° = 130°$

$\because$ The polygon $EFGH$ is the image of the polygon $ABCD$ by rotation

$\therefore$ The polygon $ABCD \cong$ the polygon $EFGH$

$\therefore m(\angle H) = m(\angle D) = 130°$ $\quad \therefore x = 130$

$BC = FG = 6$ $\quad \therefore 2y = 6$ $\quad \therefore y = 3$

Remember that

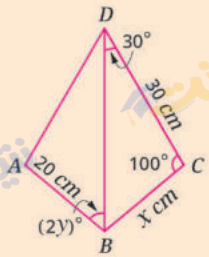
Reflection, translation, and rotation are geometric transformations that give an image congruent to the geometric shape because they preserve the following:

- Angle measures.
- Side lengths.
- Parallelism.

Self-Evaluation 1

In the following figure:

ΔBCD is the image of ΔBAD by reflection in $\overleftrightarrow{DB}$



Find the value of each of x and y.

Congruence of two triangles

The triangle is a polygon with 6 elements (3 sides and 3 angles).
To prove that two triangles are congruent, it is not necessary to prove that all corresponding elements are congruent. It is sufficient to prove that three elements, including at least one side, are congruent.

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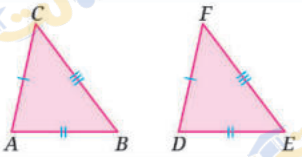
Cases of Congruent Triangles

First Case: Side - Side - Side (SSS)

Two triangles are congruent if each side of one triangle is congruent to its corresponding side of the other triangle.

If $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$

then $\Delta ABC \cong \Delta DEF$

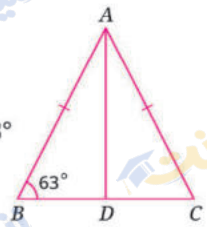


Example (2)

In the given figure:

The point D is the midpoint of $\overline{BC}$, $AB = AC$, $m(\angle B) = 63°$

1. Prove that: $\Delta ADB \cong \Delta ADC$
2. Find by proof: $m(\angle DAC)$



$\because$ The point D is the midpoint of $\overline{BC}$

$\therefore BD = DC$

$\because$ In ΔADB and ΔADC:

$\overline{AB} \cong \overline{AC}$ (given) $\quad$ $\overline{BD} \cong \overline{CD}$ (proved) $\quad$ $\overline{AD} \cong \overline{AD}$ (common side)

$\therefore \Delta ADB \cong \Delta ADC$ (required 1)

From the congruence:

$m(\angle C) = m(\angle B) = 63°$, $m(\angle DAC) = m(\angle DAB)$

In ΔABC:

$\therefore m(\angle CAB) = 180° - (63° + 63°) = 54°$

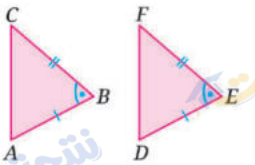
$\therefore m(\angle DAC) = m(\angle DAB) = \frac{54°}{2} = 27°$ (required 2)

Second Case: Side - Angle - Side (SAS)

Two triangles are congruent if two sides and the included angle of one triangle are congruent to their corresponding elements of the other triangle.

If $\overline{AB} \cong \overline{DE}$, $\angle B \cong \angle E$, $\overline{BC} \cong \overline{EF}$

then $\Delta ABC \cong \Delta DEF$



Notation in Mathematics

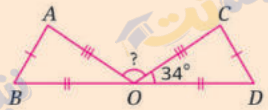
In cases of triangle congruence, the letter S is used to denote a side in a triangle, the letter A is used to denote an angle in a triangle, the letter R is used to denote the right angle, and the letter H is used to denote the hypotenuse.

Critical Thinking

Could two triangles be congruent if the three angles of one triangle are congruent to their corresponding angles of the other triangle?

Self-Evaluation (2)

In the following figure:



1. Prove that:

   $\Delta OAB \cong \Delta OCD$

2. Find: $m(\angle AOC)$

Critical Thinking

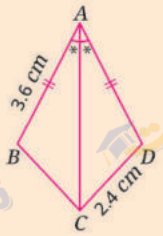
Could two triangles be congruent if two sides and non included angle of one triangle are congruent to their corresponding elements of the other triangle?

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■ Self-Evaluation (3)

In the following figure:

$\overline{AB} \cong \overline{AD}$,
$AB = 3.6\ cm$,
$CD = 2.4\ cm$,
$\overrightarrow{AC}$ bisects $\angle BAD$



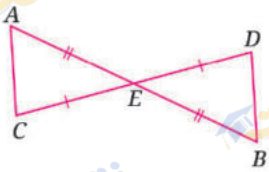
1. Prove that: $\Delta ACB \cong \Delta ACD$
2. Find the perimeter of the polygon $ABCD$

Example 3

In the given figure:

E is the midpoint of each of $\overline{AB}$ and $\overline{CD}$

Prove that: (1) $\Delta AEC \cong \Delta BED$ (2) $\overline{AC} // \overline{DB}$



$\because$ E is the midpoint of each of $\overline{AB}$ and $\overline{CD}$

$\therefore \overline{AE} \cong \overline{BE}$, $\overline{CE} \cong \overline{DE}$

$\because m(\angle AEC) = m(\angle BED)$ (vertically opposite angles)

$\therefore \Delta AEC \cong \Delta BED$ (required 1)

From the congruence, $m(\angle A) = m(\angle B)$

and they are two alternating angles

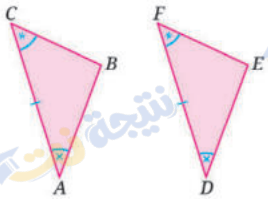
$\therefore \overline{AC} // \overline{DB}$ (required 2)

Third Case: Angle - Side - Angle (*ASA*)

Two triangles are congruent if two angles and the included side of one triangle are congruent to their corresponding elements of the other triangle.

If $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$, $\angle C \cong \angle F$

then $\Delta ABC \cong \Delta DEF$



■ Self-Evaluation (4)

In the following figure:

$\overline{AB} // \overline{DC}$, $\overline{AC} // \overline{DE}$,
$\overline{BC} \cong \overline{EC}$



Prove that: $\Delta ABC \cong \Delta DCE$

Example 4

In the given figure:

$\angle B \cong \angle C$, $\angle ADE \cong \angle AED$, $\overline{BD} \cong \overline{CE}$

Prove that: $\Delta ABD \cong \Delta ACE$



$\because m(\angle ADB) + m(\angle ADE) = 180°$, $m(\angle AEC) + m(\angle AED) = 180°$

$\because m(\angle ADE) = m(\angle AED)$ $\therefore m(\angle ADB) = m(\angle AEC)$

$\therefore \angle ADB \cong \angle AEC$

In ΔABD and ΔACE:

$\angle B \cong \angle C$ (given) , $\overline{BD} \cong \overline{CE}$ (given) , $\angle ADB \cong \angle AEC$ (proved)

$\therefore \Delta ABD \cong \Delta ACE$

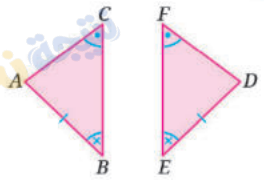
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Fourth Case: Angle - Angle - Side (AAS)

Two triangles are congruent if two angles and a non-included side of one triangle are congruent to their corresponding elements of the other triangle.

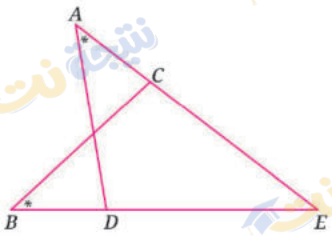
If $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$

then $\Delta ABC \cong \Delta DEF$



Example 5

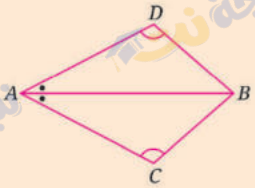
In the given figure:
$\overline{AD} \cong \overline{BC}$, $\angle A \cong \angle B$
Prove that : $AE = BE$



In ΔADE and ΔBCE :
$\because \angle A \cong \angle B$ (given) , $\angle E \cong \angle E$ (common angle) , $\overline{AD} \cong \overline{BC}$ (given)
$\therefore \Delta ADE \cong \Delta BCE$
From the congruence, $AE = BE$

Self-Evaluation (5)

In the following figure:
$\overrightarrow{AB}$ bisects $\angle DAC$,
$\angle ACB \cong \angle ADB$



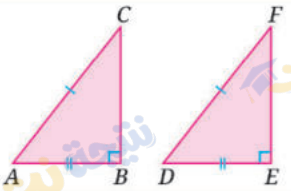
Prove that: $\Delta ACB \cong \Delta ADB$

Fifth Case: Right angle - Hypotenuse - Side (RHS)

Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to their corresponding elements of the other triangle.

If $m(\angle B) = m(\angle E) = 90°$,
$\overline{AC} \cong \overline{DF}$, $\overline{AB} \cong \overline{DE}$

then $\Delta ABC \cong \Delta DEF$

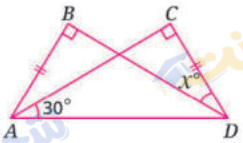


Note that

In a right-angled triangle, the side opposite to the angle of measure 90° is called the hypotenuse and it is the longest side of the triangle.

Example 6

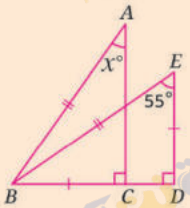
In the given figure:
Find the value of $\mathcal{X}$



$\because$ In ΔABD and ΔDCA :
$m(\angle B) = m(\angle C) = 90°$ (given) , $\overline{AB} \cong \overline{DC}$ (given) , $\overline{AD} \cong \overline{DA}$ (common side)
$\therefore \Delta ABD \cong \Delta DCA$
From the congruence,
$m(\angle BDA) = m(\angle CAD) = 30°$
In ΔDCA :
$\therefore m(\angle CDA) = 180° - (90° + 30°) = 60°$ $\therefore \mathcal{X} = 60 - 30 = 30$

Self-Evaluation (6)

In the following figure:



$BC = DE$, $AB = EB$

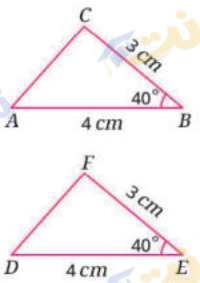
1. Prove that:
   $\Delta ACB \cong \Delta BDE$
2. Find the value of $\mathcal{X}$

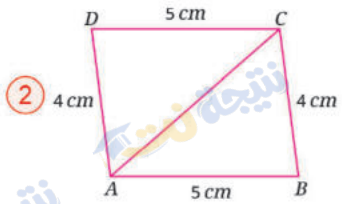
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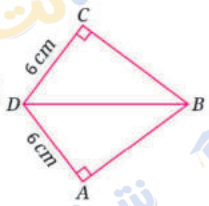
Lesson Assessment

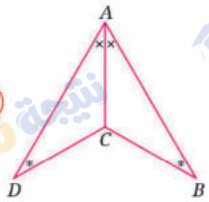
First Measuring Conceptual Understanding

▸ **In each of the following, what is the case used to prove that the two triangles are congruent?**

1 

2 

3 

4 

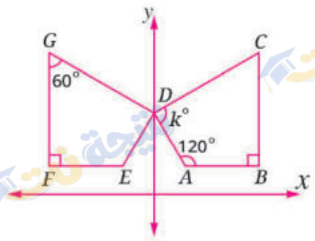
▸ **Choose the correct answer from the given ones:**

5 In the given figure: The polygon $EFGD$ is the image of the polygon $ABCD$ by reflection in the y-axis.

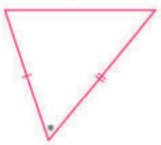
What is the value of k?

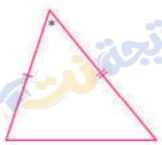
(a) 90 (b) 60

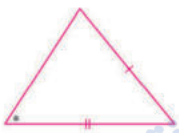
(c) 120 (d) 70

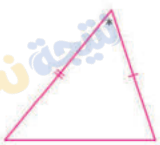


6 Which of the following triangles is not congruent to the other triangles?


(a)


(b)


(c)


(d)

▸ **Spot the mistake:**

7 Ahmed used the triangle congruence to find the value of X.

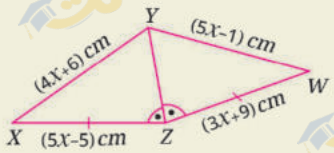
Find the mistake in Ahmed's solution, then correct it.

Ahmed's solution:

$4X + 6 = 3X + 9$

$X + 6 = 9$

$X = 3$



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Second **Applying Concepts**

▸ **Choose the correct answer from the given ones:**

(8) In the given figure:

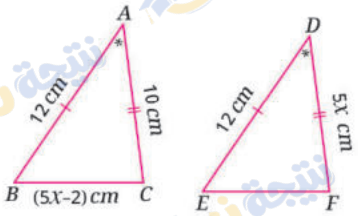
What is the length of $\overline{EF}$?

(a) 6 cm

(b) 8 cm

(c) 10 cm

(d) 12 cm



(9) In the given figure:

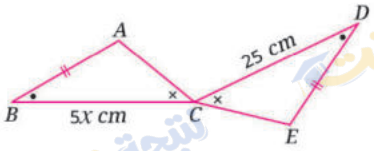
What is the value of X?

(a) 5

(b) 10

(c) 20

(d) 25

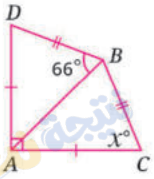


(10) In the given figure:

What is the value of X?

(a) 66 (b) 45

(c) 34 (d) 69

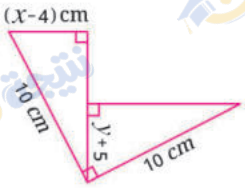


(11) In the given figure:

What is the value of $X - y$?

(a) 9 (b) 6

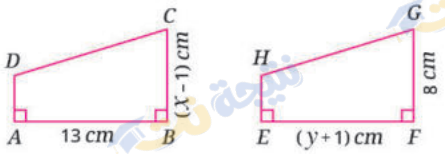
(c) 5 (d) 1



(12) In the given figure: If the polygon $EFGH$ is the image of the polygon $ABCD$ by a translation, what is the value of $X + y$?

(a) 9 (b) 12

(c) 21 (d) 22



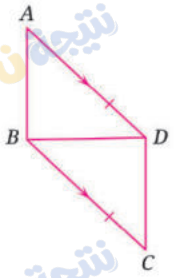
▸ **Answer the following questions:**

(13) In the given figure:

$\overline{AD} \cong \overline{CB}$, $\overline{AD} \parallel \overline{BC}$

Prove that:

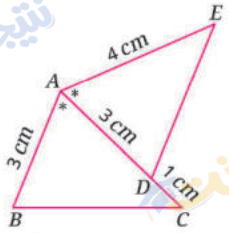
$\Delta ABD \cong \Delta CDB$



(14) In the given figure:

Prove that:

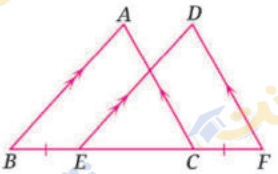
$\Delta ABC \cong \Delta ADE$



(15) In the given figure:

$\overline{BA} \parallel \overline{ED}$, $\overline{CA} \parallel \overline{FD}$, $\overline{BE} \cong \overline{FC}$

Prove that: $\Delta ABC \cong \Delta DEF$



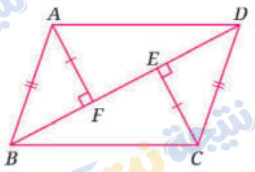
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16 In the given figure:

Prove that:

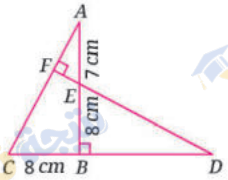
1 $\overline{AB} // \overline{CD}$

2 $DF = BE$



17 In the given figure:

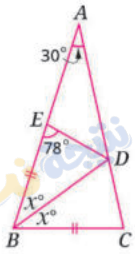
Find the length of $\overline{BD}$



18 In the given figure:

1 Prove that: $\Delta\, BCD \cong \Delta\, BED$

2 Find the value of $\mathcal{X}$



19 In the given figure:

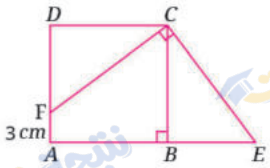
$ABCD$ is a square,

its side length is 12 cm

$\overline{CF} \perp \overline{CE}$, $AF = 3$ cm

1 Prove that: $\Delta\, CBE \cong \Delta\, CDF$

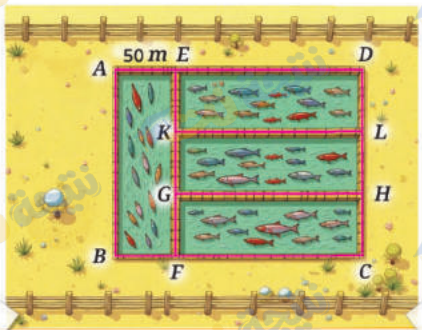
2 Find the length of $\overline{BE}$



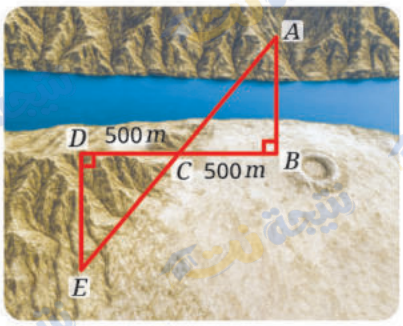
Third Analysis and Subjects Integration

20 **Fish farming:** A rectangular plot of land $ABCD$ has been divided into congruent rectangular ponds for farming 4 types of fish.

What is the perimeter of the plot of land?



Egypt occupies an advanced position in the field of fish farming according to the global classification of the Food and Agriculture Organisation of the United Nations.



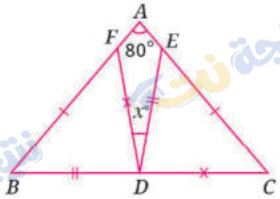
21 **Engineering:** To build a bridge like $\overline{AB}$ over a river, you need to find the distance from the point A to the point B. So you have to identify the points A, C, and E on a straight line as shown in the figure. If $DE = 600$ m, then what is the distance from the point A to the point B?

Creative Thinking

22 In the given figure:

1 Prove that: $\Delta\, BFD \cong \Delta\, ECD$

2 Find the value of $\mathcal{X}$



***Evaluate** your understanding!*

How well do you understand congruence?
Tick the right box



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Lesson

3 - 2

Pythagoras' Theorem and Its Converse

Learning Outcomes

- Learn the relationship among the lengths of the sides of a right-angled triangle.
- Find the length of one of the sides of a right-angled triangle if the length of the hypotenuse and the length of the other side are known.
- Find the length of the hypotenuse of a right-angled triangle if the lengths of the other two sides are known.
- Prove that a triangle is right-angled if the lengths of its three sides are known.
- Determine the type of a triangle according to its angles if the lengths of its sides are known.

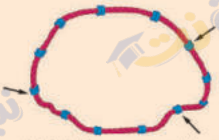
Vocabulary

- Pythagoras' Theorem
- Hypotenuse

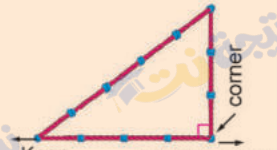
Historical information

The ancient Egyptians used a joined rope with 12 knots at equal distances to form a triangle with side lengths in the ratio 5 : 4 : 3.

This triangle has a right angle between two sides of lengths 3 and 4 units. The Egyptians used this method to construct right angles.



Hold the rope at the indicated knots



Keep the rope taut

Hold the rope aligned with the building

Get Ready!

You have a ladder that is 13 feet long leaning against a vertical wall. If the base of the ladder is 5 feet away from the wall, how high does the ladder reach up the wall?

In this lesson, you will learn about the Pythagoras' theorem and its converse, which will enable you to solve such problems.



Think & Discuss!

If you have four pieces of wood of lengths 9 cm, 10 cm, 12 cm, and 15 cm, which three can you use to make a right-angled triangle?



Learn!

Pythagoras' Theorem

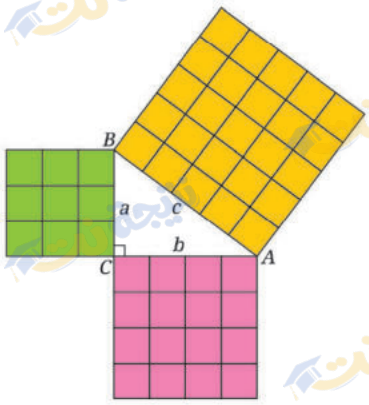
Pythagoras' Theorem is an equation that relates the side lengths of a right-angled triangle. It is named after the Greek mathematician Pythagoras, who lived in the 6th century BC



Pythagoras

Pythagoras' Theorem

The area of the square drawn on the hypotenuse of a right-angled triangle is equal to the sum of the areas of the squares drawn on the other two sides.



Another form of Pythagoras' theorem:

In a right-angled triangle, the square of the hypotenuse length is equal to the sum of the squares of the other two side lengths.

Thus: $c^2 = a^2 + b^2$ where a and b are the lengths of the sides of the right angle of the triangle, and c is the length of the hypotenuse.

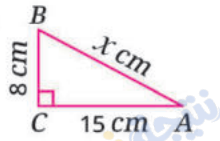
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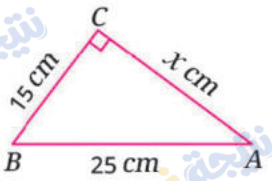
Using Pythagoras' Theorem

You can use Pythagoras' theorem to find the length of a side in a right-angled triangle when the lengths of the other two sides are known.

Example 1

Find the value of X in each of the following, then find the area of the triangle in the second figure.

1 

2 

1 $\because c^2 = a^2 + b^2$

$\therefore X^2 = (8)^2 + (15)^2$

$= 64 + 225 = 289$

$\therefore X = \sqrt{289} = 17$

2 $\because c^2 = a^2 + b^2$

$\therefore (25)^2 = (15)^2 + X^2$

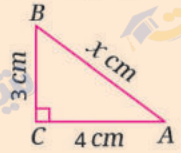
$\therefore X^2 = (25)^2 - (15)^2 = 400$

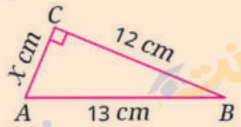
$\therefore X = \sqrt{400} = 20$

$\therefore$ The area $= \frac{1}{2} \times 15 \times 20 = 150\ cm^2$

Self-Evaluation (1)

Find the value of X in each of the following, then find the area of the triangle in the second figure.

1 

2 

Example 2

In the given figure:

$m(\angle B) = 90^\circ$, $AC = 4\sqrt{5}\ cm$

$BD = 6\ cm$, $DC = 2\sqrt{13}\ cm$

Find the length of $\overline{AD}$ with proof.



$\because \Delta DBC$ is right-angled at B :

$\therefore (BC)^2 = (DC)^2 - (DB)^2 = (2\sqrt{13})^2 - (6)^2 = 16$

$\therefore BC = \sqrt{16} = 4$

$\because \Delta ABC$ is right-angled at B :

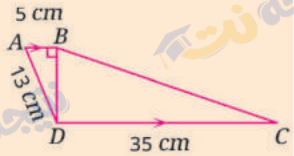
$\therefore (AB)^2 = (AC)^2 - (BC)^2 = (4\sqrt{5})^2 - (4)^2 = 64$

$\therefore AB = \sqrt{64} = 8$

$\therefore AD = AB - BD = 8 - 6 = 2\ cm$

Self-Evaluation (2)

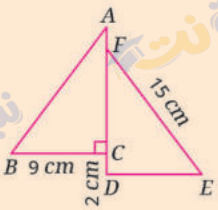
In the following figure:
If $\overline{AB} \parallel \overline{DC}$,
find the perimeter of the polygon $ABCD$.



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■ Self-Evaluation (3)

In the following figure:



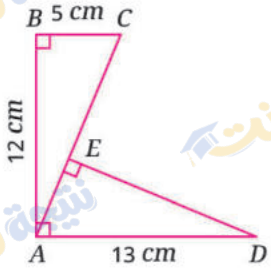
$\Delta ACB \cong \Delta FDE$

Find the length of $\overline{AF}$

Example (3)

In the given figure:

Find the length of $\overline{EC}$



$\because \Delta ABC$ is right-angled at B

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (12)^2 + (5)^2 = 169$

$\therefore AC = \sqrt{169} = 13$

$\because m(\angle C) + m(\angle BAC) = 90^\circ$, $m(\angle DAE) + m(\angle BAC) = 90^\circ$

$\therefore m(\angle C) = m(\angle DAE)$

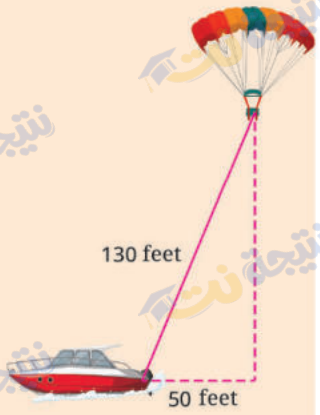
$\because$ In ΔABC and ΔDEA :

$m(\angle ABC) = m(\angle DEA)$, $AC = DA$, $m(\angle C) = m(\angle DAE)$

$\therefore \Delta ABC \cong \Delta DEA$ $\quad\quad$ $\therefore AE = CB = 5$

$\therefore EC = AC - AE = 13 - 5 = 8\ cm$

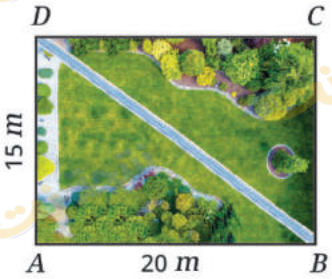
■ Self-Evaluation (4)



The figure shows Omar drifting in a balloon pulled by a boat across the sea. What is the height of Omar above the horizontal level of the boat?

Example (4)

Design: Osama designed a rectangular home garden and plans to build a diagonal path as shown in the figure. Find the length of this path



$\because ABCD$ is a rectangle $\quad\quad$ $\therefore m(\angle A) = 90^\circ$

$\therefore$ In ΔABD :

$(BD)^2 = (AB)^2 + (AD)^2 = (20)^2 + (15)^2 = 625$

$\therefore$ The length of the path $= \sqrt{625} = 25\ m.$

Converse of Pythagoras' Theorem

If the sum of the squares of two side lengths of a triangle is equal to the square of the third side length, then the triangle is right-angled.

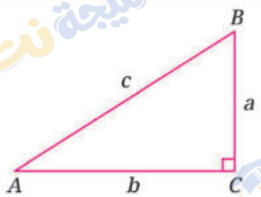
In ΔABC :

If $a^2 + b^2 = c^2$,

then $m(\angle C) = 90^\circ$

Thus:

The triangle ABC is right-angled at C



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Example 5

Which of the following represents the lengths of the sides of a right-angled triangle?

1. $8\ cm, 13\ cm, 15\ cm$
2. $24\ cm, 7\ cm, 25\ cm$

(1) The length of the longest side = 15

$\because (13)^2 + (8)^2 = 233$,

$\because (15)^2 = 225$

$\therefore (13)^2 + (8)^2 \neq (15)^2$

$\therefore$ The triangle is not right-angled.

(2) The length of the longest side = 25

$\because (7)^2 + (24)^2 = 625$

$\because (25)^2 = 625$

$\therefore (7)^2 + (24)^2 = (25)^2$

$\therefore$ The triangle is right-angled.

■ Self-Evaluation (5)

Which of the following represents the lengths of the sides of a right-angled triangle?

1. $41\ cm, 40\ cm, 9\ cm$
2. $5\ cm, 3\sqrt{2}\ cm, \sqrt{7}\ cm$
3. $60\ cm, 40\ cm, 30\ cm$

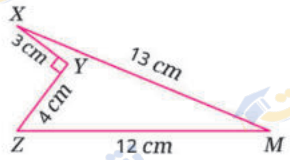
Example 6

In the given figure:

$XYZM$ is a concave quadrilateral,

$m(\angle XYZ) = 90^\circ$.

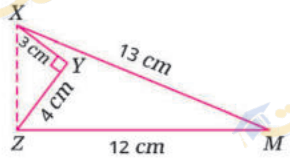
Find the area of the quadrilateral $XYZM$.



Construction: Draw $\overline{XZ}$

$\because \Delta XYZ$ is right-angled at Y

$\therefore (XZ)^2 = (3)^2 + (4)^2 = 9 + 16 = 25$ $\quad \therefore XZ = 5$



In ΔXZM :

$\because (XZ)^2 + (ZM)^2 = (5)^2 + (12)^2 = 25 + 144 = 169$,

$(XM)^2 = (13)^2 = 169$

$\therefore (XM)^2 = (XZ)^2 + (ZM)^2$ $\quad \therefore m(\angle XZM) = 90^\circ$

$\because$ The area of the quadrilateral $XYZM$ = Area of ΔXZM − Area of ΔXYZ

$\therefore A = (\frac{1}{2} \times 5 \times 12) - (\frac{1}{2} \times 3 \times 4) = 30 - 6 = 24\ cm^2$

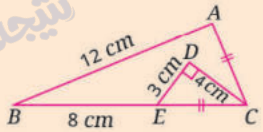
■ Self-Evaluation (6)

In the following figure:

$EC = AC$

Prove that:

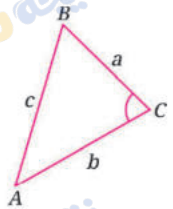
$m(\angle A) = 90^\circ$



Pythagoras' inequality

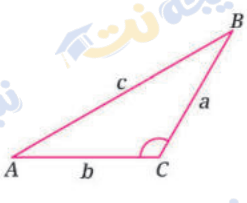
In ΔABC : If c is the length of longest side of the triangle, and:

$c^2 < a^2 + b^2$



ΔABC is acute-angled

$c^2 > a^2 + b^2$



ΔABC is obtuse-angled

■ Note that

- Pythagoras' inequality is used to determine whether a triangle is acute-angled or obtuse-angled.
- The converse of Pythagoras' theorem is used to determine whether a triangle is right-angled or not.

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Example 7

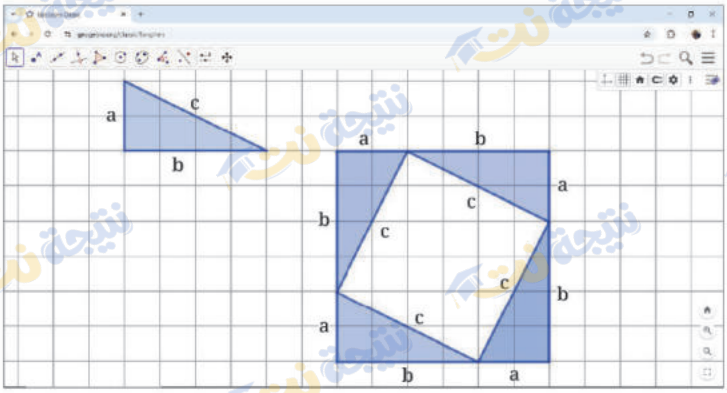
Identify the type of each triangle with respect to its angles if the lengths of its sides are as follows:

1. $9\ cm, 10\ cm, 11\ cm$
2. $3\ cm, 2\sqrt{3}\ cm, 3\sqrt{3}\ cm$
3. $1.5\ cm, 2\ cm, 2.5\ cm$

1. $\because$ The length of the longest side of the triangle $= 11\ cm$
   $\because (11)^2 = 121 \quad , \quad (9)^2 + (10)^2 = 81 + 100 = 181$
   $\therefore (11)^2 < (9)^2 + (10)^2$
   $\therefore$ The triangle is acute-angled.
2. $\because$ The length of the longest side of the triangle $= 3\sqrt{3}\ cm$
   $\because (3\sqrt{3})^2 = 27 \quad , \quad (3)^2 + (2\sqrt{3})^2 = 9 + 12 = 21$
   $\therefore (3\sqrt{3})^2 > (3)^2 + (2\sqrt{3})^2$
   $\therefore$ The triangle is obtuse-angled.
3. $\because$ The length of the longest side of the triangle $= 2.5\ cm$
   $\because (2.5)^2 = 6.25 \ , (1.5)^2 + (2)^2 = 2.25 + 4 = 6.25$
   $\therefore (2.5)^2 = (1.5)^2 + (2)^2$
   $\therefore$ The triangle is right-angled.

Technology activity

Use GeoGebra program and draw a triangle as shown in the figure. The lengths of its sides are a, b, and c. Copy the triangle to get four congruent triangles, and rearrange them to form a square as shown in the following screen.



Find the area of the large square in two ways:

- The area of the large square is equal to the sum of the areas of the four triangles and the small square.

$\therefore A = 4\left(\frac{1}{2}\ a\ b\right) + c^2$ (1)

- The area of the large square is equal to the square of its side length.

$\therefore A = (a + b)^2$ (2)

From (1) and (2) : $4\left(\frac{1}{2}\ a\ b\right) + c^2 = (a + b)^2$

Simplify to verify that: $c^2 = a^2 + b^2$

Self-Evaluation (7)

Determine the type of the triangle with respect to its angles if the lengths of its sides are as follows:

1. $5\ cm, 5\ cm, 5\sqrt{2}\ cm$
2. $11\ cm, 12\ cm, 16\ cm$
3. $4.5\ cm, 20\ cm, 21\ cm$

Enriched Information

Pythagoras' triples are three integers that satisfy the following formula:

$$a^2 + b^2 = c^2$$

where c is the largest number.

▸ The following are common Pythagoras' triples and some of their multiples:

3 , 4 , 5
6 , 8 , 10
9 , 12 , 15
$3x$, $4x$, $5x$

5 , 12 , 13
10 , 24 , 26
15 , 36 , 39
$5x$, $12x$, $13x$

8 , 15 , 17
16 , 30 , 34
24 , 45 , 51
$8x$, $15x$, $17x$

7 , 24 , 25
14 , 48 , 50
21 , 72 , 75
$7x$, $24x$, $25x$

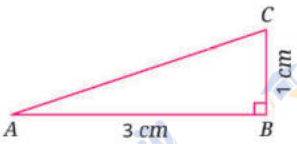
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Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

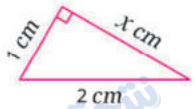
(1) In the given figure: What is the area of the square drawn on $\overline{AC}$?



(a) 4 cm^2 (b) 2 cm^2

(c) 8 cm^2 (d) 10 cm^2

(2) In the given figure : What is the value of x?



(a) 1 (b) 3

(c) $\sqrt{5}$ (d) $\sqrt{3}$

(3) In the given figure : What is the value of x?



(a) 41 (b) 29

(c) $\sqrt{41}$ (d) $\sqrt{29}$

(4) Which of the following numbers can be the lengths of the sides of a right-angled triangle?

(a) 7 , 7 , 4 (b) 12 , 5 , 13 (c) 5 , 8 , 7 (d) 3 , 7 , 5

(5) Which of the following numbers can be the lengths of the sides of an acute-angled triangle?

(a) 4 , 4 , $4\sqrt{2}$ (b) 10 , 8 , 6 (c) 8 , 3.7 , 7.5 (d) 9 , 6 , 6

(6) Which of the following numbers can be the lengths of the sides of an obtuse-angled triangle?

(a) 27 , 10 , 24 (b) 1.5 , 2 , 2.5 (c) 7 , 7 , 7 (d) $10\sqrt{3}$, 10 , 20

Second Applying Concepts

(7) Find the value of x in each of the given figures.

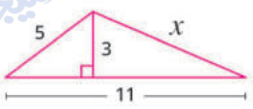


Figure (1)

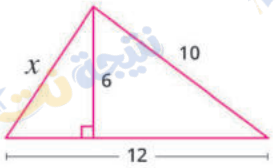


Figure (2)

(8) Find the area of each of the given figures.



Figure (1)

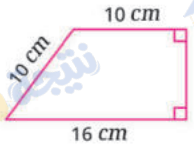


Figure (2)

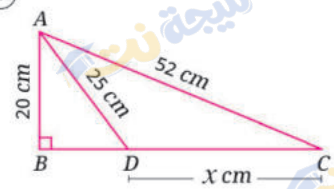
(9) In each of the following, determine the type of the triangle with respect to its angles if the lengths of its sides are as follows:

① 6 cm, 3 cm, 5 cm ② $\sqrt{6}$ cm, $\sqrt{5}$ cm, $\sqrt{7}$ cm ③ 35 cm, 28 cm, 21 cm

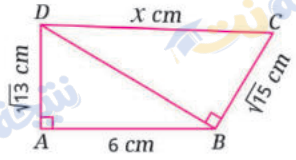
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10 **In each of the following figures, find the value of x:**

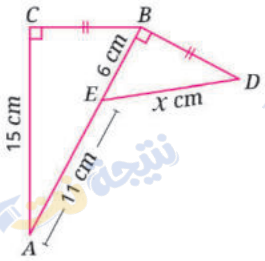
1



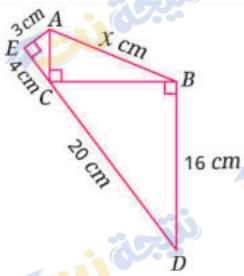
2



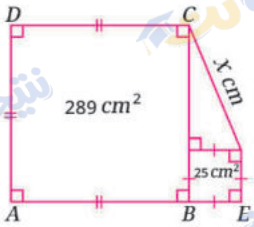
3



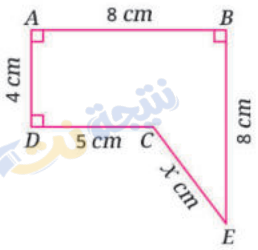
4



5

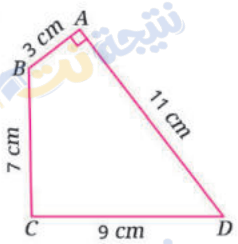


6



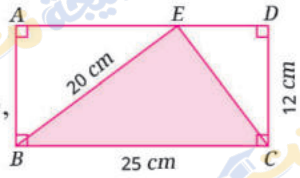
11 **In the given figure:**

Prove that: $m(\angle C) = 90^\circ$



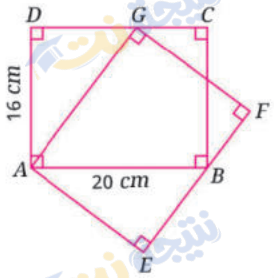
12 **In the given figure:**

$ABCD$ is a rectangle.
Prove that:
$\Delta\, BEC$ is
a right-angled triangle,
then find its area.



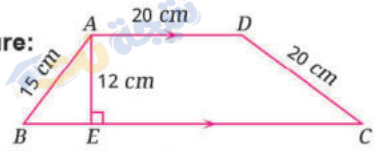
13 **In the given figure:**

The rectangle $ABCD$
is congruent to
the rectangle $AGFE$.
Find by proof the
length of $\overline{BF}$



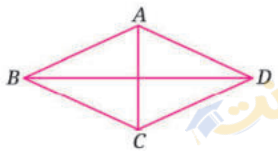
14 **In the given figure:**

$ABCD$ is
a trapezium
in which: $AB = 15\ cm$, $AD = 20\ cm$,
$DC = 20\ cm$. If the height of the trapezium is
$12\ cm$, find its area.



15 **In the given figure:**

$ABCD$ is a rhombus with a perimeter of 52 m
and a diagonal $\overline{BD}$ of length 24 m.
Find the area of the rhombus $ABCD$ in square meters.



16 **Spot the mistake:**

Amal and Maryam are trying to determine whether the numbers 5, 12, and 13 can be the lengths of the sides of a right-angled triangle or not. Which of them followed the correct method? Discuss and correct the mistake if found.

Amal's solution:

$a^2 + b^2 = 5^2 + 12^2$

$= 169$

$c^2 = 13^2 = 169$

$a^2 + b^2 = c^2$

can be

Maryam's solution:

$a^2 + b^2 = 13^2 + 5^2$

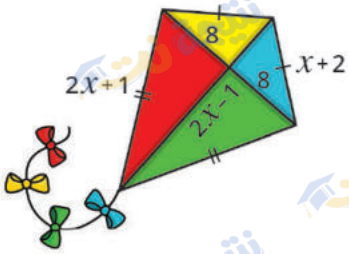
$= 194$

$c^2 = 12^2 = 144$

$a^2 + b^2 \neq c^2$

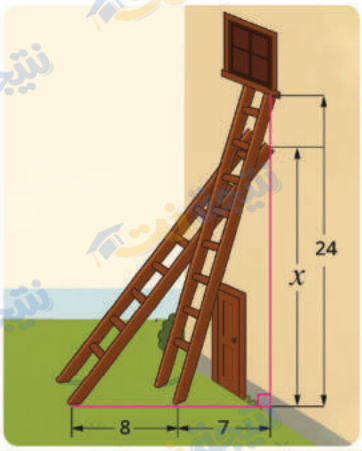
cannot be

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17 Games :
You have a kite with the dimensions shown in the figure. Find the perimeter of the kite.

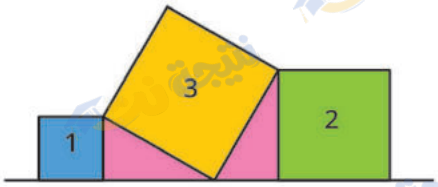
18 Arts : An artist placed a ladder to reach a window 24 feet above the ground. The base of the ladder was 7 feet away from the wall of the house. While mixing paint, the ladder slipped, causing its base to move another 8 feet away from the wall of the house. What is the height of the ladder at that moment?



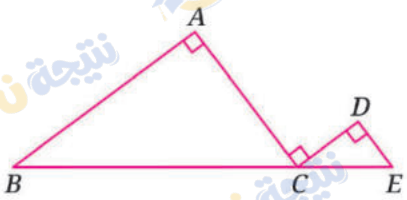
Safety and security rules must be followed when performing home maintenance.

Creative Thinking

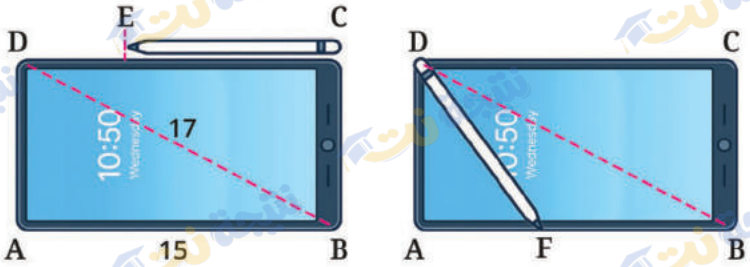
19 In the following figure:
Three squares, the area of square (1) = 16 cm^2 and the area of square (2) = 48 cm^2. Find with proof the length of the side of square (3) .



20 In the following figure:
If $AB + CD = 16\ cm$, $AC + DE = 12\ cm$, find with proof the length of $\overline{BE}$.



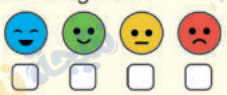
21 Critical Thinking: Khalid uses stylus pen of a tablet device in the shape of a rectangle $ABCD$, the length of one of its sides is 15 units and the length of its diagonal is 17 units.



If the length of $\overline{FB}$ is four units longer than the length of $\overline{DE}$, find the length of the stylus pen.

***Evaluate** your understanding!*

How well do you understand Pythagoras' theorem and its converse?
Tick the right box



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Lesson

3 - 3 Applications of Parallelism

Learning Outcomes

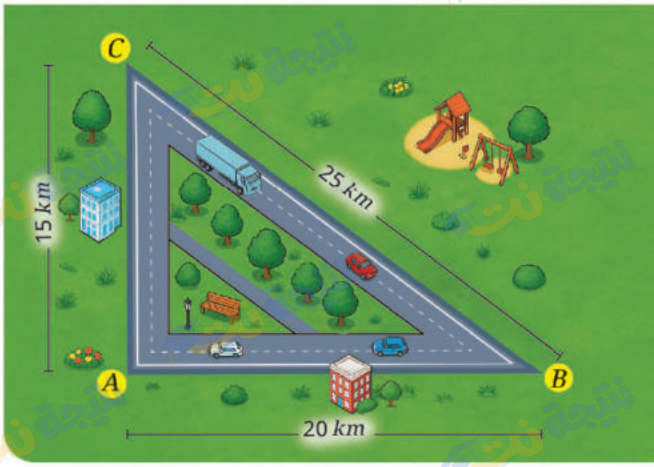
- Learn the relationship between the lengths of line segments enclosed between parallel lines.
- Use the ray drawn from the midpoint of one side of a triangle parallel to another side.
- Learn the midsegment of a triangle.
- Learn the relationship between the midsegment of a triangle and its opposite side.
- Use parallelism to solve problems in geometry.

Vocabulary

- Parallelism
- Midpoint
- Midsegment

Get Ready!

A road engineer is planning to build a new road connecting the school and the hospital, parallel to the main road. If you know that the school is located halfway along the road $\overline{AB}$ and the hospital is located halfway along the road $\overline{AC}$, what is the length of this new road?



In this lesson, you will learn about the relationship between parallel lines and the enclosed line segments between them, as well as the relationship between the midsegment of a triangle and its sides, which will enable you to solve such problems.

Think & Discuss!

In the given figure, if the length of the horizontal bar at the middle of the ladder is 0.75 m, can you determine the distance between the two ends of the ladder on the ground? Discuss.



Learn!

Some applications of parallelism

Theorem 3-1

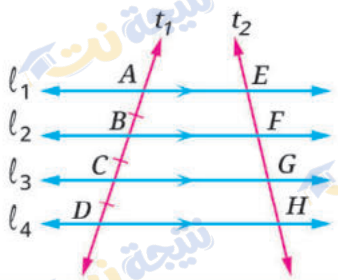
If a straight line intersects several parallel straight lines, and the parts of the transversal between these straight lines are equal in length, then the parts between them for any other transversal are equal in length.

If $\ell_1 // \ell_2 // \ell_3 // \ell_4$,

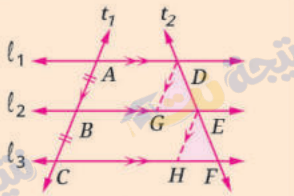
t_1 , t_2 are transversals

such that $AB = BC = CD$,

then $EF = FG = GH$



Discuss the proof of theorem (3-1)

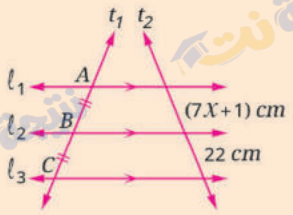


By drawing $\overline{DG}$, $\overline{EH}$ parallel to the straight line t_1, and using the properties of parallelogram and parallelism to prove that the triangles EDG, FEH are congruent, you can prove theorem (3-1). Discuss this.

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■ **Self-Evaluation (1)**

In the following figure :



Find the value of X.

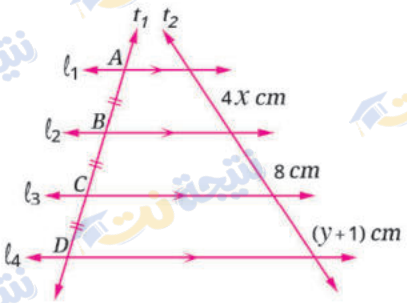
Example (1)

In the given figure:

$l_1 // l_2 // l_3 // l_4$

The straight lines t_1, t_2 are two transversals.

Find the value of each of X and y.

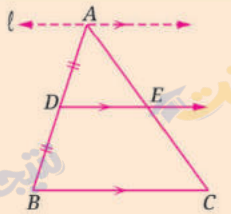


$\because l_1 // l_2 // l_3 // l_4$, $AB = BC = CD$

$\therefore 4X = 8$ $\quad\quad$ $\therefore X = \frac{8}{4} = 2$

$y + 1 = 8$ $\quad\quad$ $\therefore y = 8 - 1 = 7$

■ **Discuss the proof of theorem (3-2)**

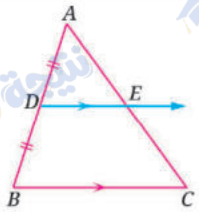


By drawing a straight line from the point A parallel to $\overrightarrow{DE}$ and $\overline{BC}$, and noticing that $AD = DB$ and by using theorem (3-1), you can prove theorem (3-2).
Discuss this.

Theorem 3-2

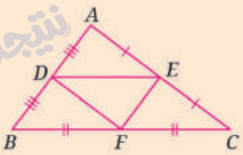
The ray drawn from the midpoint of a side of a triangle parallel to one of the other two sides bisects the third side.

If D is the midpoint of $\overline{AB}$, $\overrightarrow{DE} // \overline{BC}$,
then E is the midpoint of $\overline{AC}$,
i.e. $AE = EC$



■ **Note that**

There are three midsegments in any triangle.

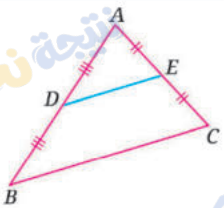


$\overline{DE}$, $\overline{DF}$, $\overline{FE}$ are the midsegments of the triangle ABC

The midsegment of a triangle

The midsegment of a triangle is the line segment drawn between the midpoints of two sides of the triangle.

If D is the midpoint of $\overline{AB}$,
E is the midpoint of $\overline{AC}$,
then $\overline{DE}$ is a midsegment of ΔABC.



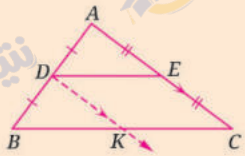
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■ Discuss

How can you prove that the length of the midsegment of a triangle is equal to half the length of its opposite side?

■ Hint

Draw $\overrightarrow{DK}$ parallel to $\overline{AC}$ and intersecting $\overline{BC}$ at K, then prove that $DKCE$ is a parallelogram.

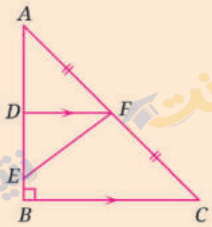


■ Self-Evaluation (2)

In the following figure:

$AB = BC = 16\ cm$

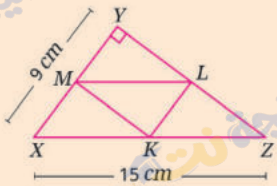
$EB = 2\ cm$



Find the length of $\overline{EF}$.

■ Self-Evaluation (3)

In the following figure:



The sides of the triangle KLM are the midsegments of the triangle XYZ, which is right angled at Y. Calculate the perimeter of the triangle KLM.

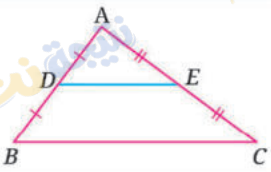
Theorem 3-3 (Theorem of the midsegment of a triangle)

The midsegment of a triangle is parallel to the third side and its length is equal to half the length of that side.

If $\overline{DE}$ is a midsegment of $\Delta\ ABC$,

then : (1) $\overline{DE} \parallel \overline{BC}$

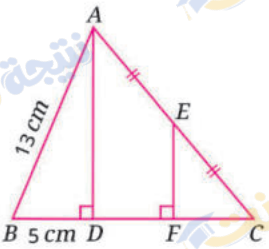
(2) $DE = \frac{1}{2} BC$



Example 2

In the given figure:

Find with proof the length of $\overline{EF}$.



$\because \Delta\ ABD$ is right-angled at D

$\therefore (AD)^2 = (AB)^2 - (BD)^2 = (13)^2 - (5)^2 = 144$

$\therefore AD = \sqrt{144} = 12$

$\because m(\angle ADB) = m(\angle EFB) = 90°$ (and they are corresponding angles)

$\therefore \overline{AD} \parallel \overline{EF}$

In $\Delta\ ADC$: $\because E$ is the midpoint of $\overline{AC}$, $\overline{AD} \parallel \overline{EF}$

$\therefore F$ is the midpoint of $\overline{DC}$

$\therefore EF = \frac{1}{2} AD$ $\quad\quad$ $\therefore EF = \frac{1}{2} \times 12 = 6\ cm.$

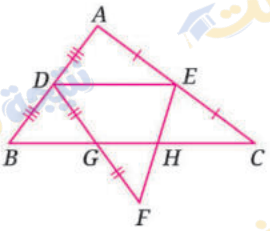
Example 3

In the given figure:

If D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$,

G is the midpoint of $\overline{DF}$,

prove that: $BC = 4\ GH$



In $\Delta\ ABC$:

$\because D$ is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

$\therefore \overline{DE} \parallel \overline{BC}$, $DE = \frac{1}{2} BC$

In $\Delta\ FDE$:

$\because G$ is the midpoint of $\overline{DF}$, $\overline{DE} \parallel \overline{GH}$ $\quad\quad$ $\therefore H$ is the midpoint of $\overline{EF}$

$\therefore GH = \frac{1}{2} DE$ $\quad\quad$ $\therefore GH = \frac{1}{2}(\frac{1}{2} BC) = \frac{1}{4} BC$ $\quad\quad$ $\therefore BC = 4\ GH$

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Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

(1) In the given figure:

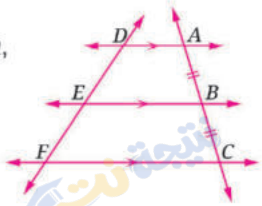
If the length of $\overline{DF} = 15\ cm$,

what is the length of $\overline{EF}$?

(a) 10 cm

(b) 7.5 cm

(c) 5 cm (d) 2.5 cm



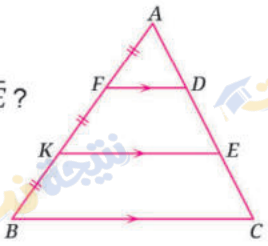
(2) In the given figure:

If $AC = 12\ cm$,

what is the length of $\overline{AE}$?

(a) 3 cm (b) 4 cm

(c) 6 cm (d) 8 cm



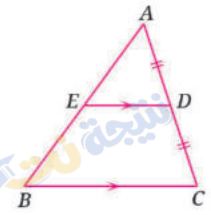
(3) In the given figure:

If the length of $\overline{EB} = 10\ cm$,

what is the length of $\overline{AB}$?

(a) 20 cm (b) 10 cm

(c) 7.5 cm (d) 5 cm

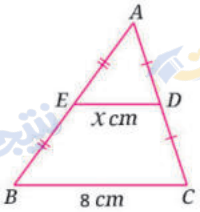


(4) In the given figure:

What is the value of $\mathcal{X}$?

(a) 16 (b) 8

(c) 4 (d) 2

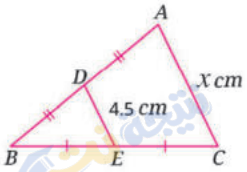


(5) In the given figure:

What is the value of $\mathcal{X}$?

(a) 9 (b) 4.5

(c) 13.5 (d) 2.25



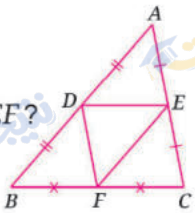
(6) In the given figure:

If the perimeter of $\Delta\ ABC$ is equal to 18 cm,

what is the perimeter of $\Delta\ DEF$?

(a) 6 cm (b) 12 cm

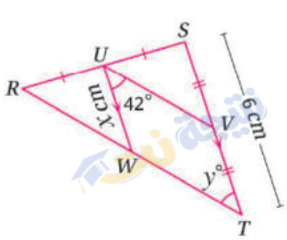
(c) 36 cm (d) 9 cm



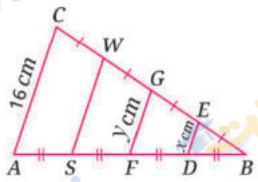
Second Applying Concepts

(7) In each of the following figures, find the value of each of $\mathcal{X}$ and $\mathcal{Y}$.

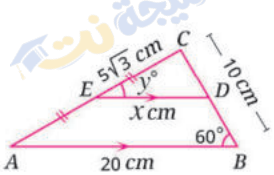
(1)



(2)



(3)

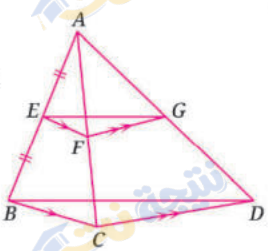


(8) In the given figure:

If E is the midpoint of $\overline{AB}$,

$\overline{EF} \parallel \overline{BC}$, $\overline{FG} \parallel \overline{CD}$,

prove that $BD = 2\ EG$

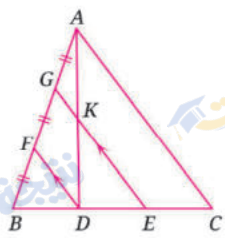


(9) In the given figure:

$GK = 8\ cm$, $\overline{FD} \parallel \overline{GE}$,

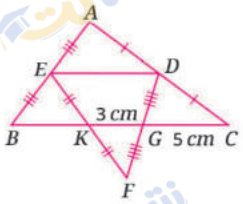
$AG = GF = FB$

Find the length of $\overline{KE}$.

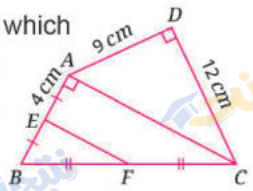


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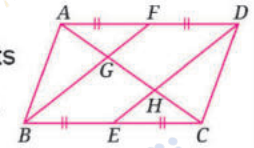
10 **In the given figure:**
Find with proof the length of $\overline{BK}$



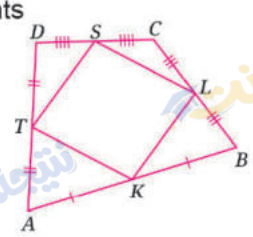
11 **In the given figure:**
$ABCD$ is a quadrilateral in which E and F are the midpoints of $\overline{AB}$ and $\overline{BC}$ respectively.
Find the area of $\Delta\, BEF$.



12 **In the given figure:**
$ABCD$ is a parallelogram.
If E and F are the midpoints of $\overline{BC}$ and $\overline{AD}$ respectively, prove that: $AC = 3\ AG$



13 **In the given figure:**
K, L, S, T are the midpoints of the sides of the quadrilateral $ABCD$
Prove that:
the figure $KLST$ is a parallelogram.



Third Analysis and Subjects Integration

14 **Decoration:** The given figure represents a wooden bookcase in the shape of an equilateral triangle with side length 3 m and three shelves made of glass.
What is the length of each of the three shelves?



The optimal distribution of decorative elements according to the area reflects aesthetic awareness and creates a feeling of comfort.



Choosing safe places to play reduces the risk of injury.

15 **Engineering design:** When designing a swing as shown in the given picture, the length of the horizontal beam connecting the midpoints of the legs is equal to x feet, where $3 \le x \le 4$. Find the shortest distance between the ends of the legs on the ground.

Creative Thinking

16

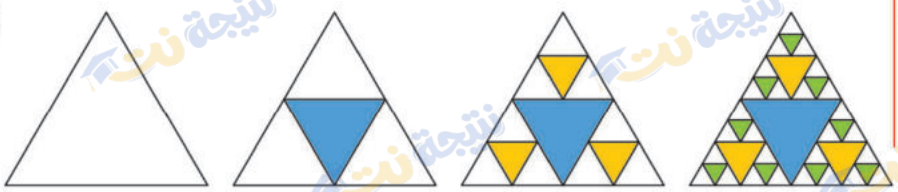


Figure (1) Figure (2) Figure (3) Figure (4)

Figure (1) represents a triangle with a perimeter of 8 cm. The triangle formed by the three midsegments was coloured as shown in Figure (2), then this was repeated on the uncoloured triangles to obtain Figure (3) and in the same pattern, Figure (4) is formed.
Calculate the sum of the perimeters of the coloured triangles in Figure (3). Can you calculate the sum of the perimeters of the coloured triangles in Figure (4)?

***Evaluate** your understanding!*
How well do you understand applications of parallelism?
Tick the right box

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Lesson

3 - 4

Medians of the Triangle

■ Learning Outcomes

- Learn the concept of the median of a triangle.
- Learn the point of intersection of the medians of a triangle.
- Learn the relationship between the length of the median drawn from the right vertex and the length of the hypotenuse in a right-angled triangle.
- Use the medians of a triangle to solve real-life problems.

■ Vocabulary

- Medians
- Centroid
- Equilibrium point.

Get Ready!



Drones are used in various fields, such as aerial photography, rescue operations, surveillance, etc., and their importance is increasing with the development of technology.

You have a drone with three arms, each ending with a motor.

If these motors are identical, where is the geometric center of the drone?

In this lesson, you will learn how to find the intersection point of the medians of a triangle, which will enable you to solve such real-life problems.

Think & Discuss!

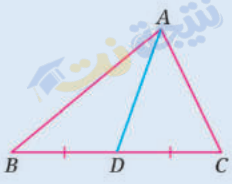


An interior designer is creating a custom table for a client. The tabletop is made of glass and is shaped like a triangle that needs to be balanced on a single leg. At which point should the leg be fixed? Discuss.

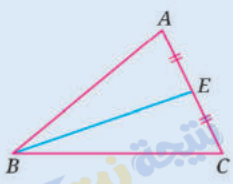
Learn!

Median of a triangle

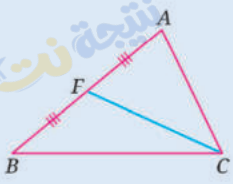
The median of a triangle is a line segment drawn from one vertex of a triangle to the midpoint of the opposite side. Every triangle has three medians, as follows:



$\overline{AD}$ is the median drawn from vertex A



$\overline{BE}$ is the median drawn from vertex B



$\overline{CF}$ is the median drawn from vertex C

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■ Think with your classmates

If two medians of a triangle intersect at a point, is this point the concurrency point of all medians of the triangle?

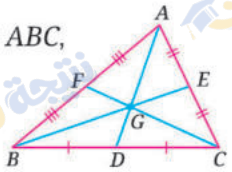
Theorem 4-1

The medians of a triangle are concurrent (intersect at one point).

In the given figure:

If $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are the medians of the triangle ABC, then $\overline{AD} \cap \overline{BE} \cap \overline{CF} = \{G\}$

Thus the point G is the concurrency point of the medians of the triangle ABC.



■ Add to your knowledge

The point of concurrency of the medians of a triangle is called the centroid of the triangle. Also known as the geometric center or the equilibrium point.

■ Think with your classmates

What is the ratio by which the centroid of the triangle divides any of its medians from the base?

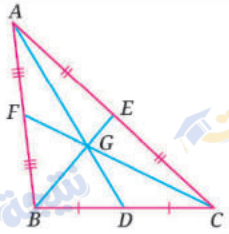
Theorem 4-2

The centroid of a triangle divides each median in the ratio 2 : 1 from the vertex.

In the given figure:

If G is the intersection point of the medians of the triangle ABC, then:

$AG = 2\,GD$, $AG = \frac{2}{3}\,AD$, $GD = \frac{1}{3}\,AD$

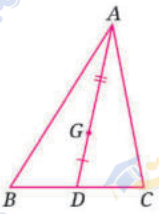


Corollary:

The point which divides a median of a triangle in the ratio 2 : 1 from the vertex is the centroid of the triangle.

In the given figure:

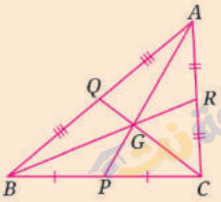
If $\overline{AD}$ is a median of the triangle ABC and $G \in \overline{AD}$ such that $AG = 2\,GD$ then G is the centroid of the triangle ABC.



■ Self-Evaluation (1)

In the following figure:

If G is the centroid of the triangle ABC and $AP = 21\ cm$, $GQ = 5\ cm$, and $BG = 16\ cm$.



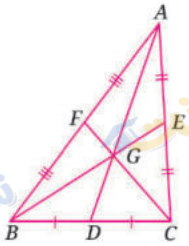
Find the length of each of: $\overline{PG}$, $\overline{CG}$, and $\overline{BR}$.

Example 1

In the given figure:

If G is the centroid of the triangle ABC and $AG = 8\ cm$, $GE = 3.5\ cm$, $CF = 7.5\ cm$

Find the length of each of $\overline{AD}$, $\overline{BE}$, and $\overline{CG}$

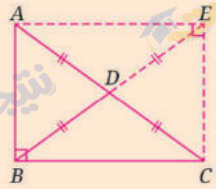


- $AD = \frac{3}{2}\,AG = \frac{3}{2} \times 8 = 12\ cm$
- $BE = 3\,GE = 3 \times 3.5 = 10.5\ cm$
- $CG = \frac{2}{3}\,CF = \frac{2}{3} \times 7.5 = 5\ cm$

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■ Discuss the proof of the theorem

Complete the rectangle.



The diagonals of a rectangle are equal in length and bisect each other.

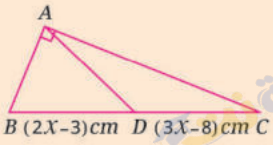
Thus:

$BD = AD = DC = DE = \frac{1}{2} AC$

■ Self-Evaluation (2)

In the following figure:

If $\overline{AD}$ is a median in the triangle ABC which is right-angled at A

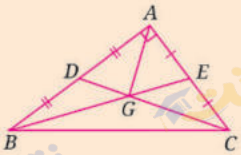


Find the length of $\overline{AD}$.

■ Self-Evaluation (3)

In the following figure:

If D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$, respectively, $\overline{BE} \cap \overline{CD} = \{G\}$, $AC = 9\ cm$, $AG = 5\ cm$



Find the length of $\overline{AB}$.

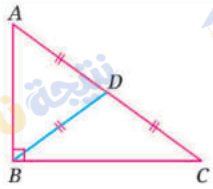
Theorem 4-3

In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.

In the given figure:

If $\overline{BD}$ is the median drawn from the vertex of the right angle B

then $BD = AD = DC = \frac{1}{2} AC$

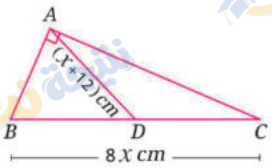


Example 2

In the given figure:

The triangle ABC is right-angled at A, and $\overline{AD}$ is a median.

Find the length of $\overline{AD}$.



$\because \overline{AD}$ is the median drawn from the right angle at vertex A in the triangle ABC

$\therefore AD = \frac{1}{2} BC$ $\quad \therefore X + 12 = \frac{1}{2} \times 8X$

$\therefore X + 12 = 4X$ $\quad \therefore 3X = 12$ $\quad \therefore X = 4$

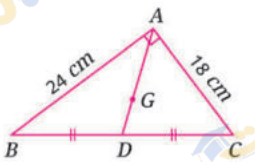
$\therefore AD = X + 12 = 4 + 12 = 16\ cm$

Example 3

In the given figure:

If G is the centroid of ΔABC, which is right-angled at A

Find the length of $\overline{AG}$



$\because \Delta ABC$ is right-angled at A

$\therefore (BC)^2 = (AB)^2 + (AC)^2$ (Pythagoras' theorem)

$= (24)^2 + (18)^2 = 900$

$\therefore BC = \sqrt{900} = 30\ cm$

$\because \overline{AD}$ is the median drawn from the vertex of the right angle

$\therefore AD = \frac{1}{2} BC = \frac{1}{2} \times 30 = 15\ cm$

$\because G$ is the centroid of the triangle ABC

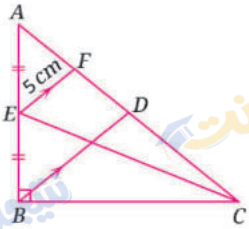
$\therefore AG = \frac{2}{3} AD = \frac{2}{3} \times 15 = 10\ cm$

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Example 4

In the given figure:

If ABC is a right-angled triangle at B, $\overline{BD}$ and $\overline{CE}$ are two medians in it, $\overline{EF} // \overline{BD}$, $EF = 5\ cm$, find the length of $\overline{AC}$



In $\Delta\ ABD$:

$\because$ E is the midpoint of $\overline{AB}$, $\overline{EF} // \overline{BD}$ $\therefore$ F is the midpoint of $\overline{AD}$

$\therefore EF = \frac{1}{2} BD$ $\therefore BD = 2 \times 5 = 10\ cm$

$\because$ $\overline{BD}$ is the median drawn from the right angle at vertex B

$\therefore BD = \frac{1}{2} AC$ $\therefore AC = 2 \times 10 = 20\ cm$

Converse of theorem 4-3

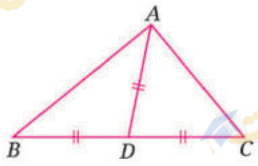
If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is right.

In the given figure:

If $\overline{AD}$ is a median drawn from the vertex A,

and $AD = BD = CD = \frac{1}{2} BC$,

then $m(\angle BAC) = 90°$



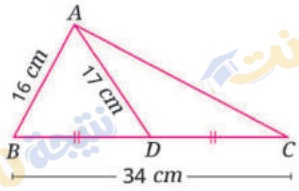
Example 5

In the given figure:

If $\overline{AD}$ is a median of the triangle ABC,

prove that: $m(\angle BAC) = 90°$,

then find the area of the triangle ABC



$\because$ $AD = 17\ cm$, $BC = 34\ cm$.

$\therefore AD = \frac{1}{2} BC$ $\therefore m(\angle BAC) = 90°$

From Pythagoras' Theorem,

$(AC)^2 = (BC)^2 - (AB)^2$

$= (34)^2 - (16)^2 = 900$

$\therefore AC = \sqrt{900} = 30\ cm$

$\therefore$ The area of $\Delta\ ABC = \frac{1}{2} \times AB \times AC = \frac{1}{2} \times 16 \times 30 = 240\ cm^2$.

■ Self-Evaluation (4)

In the following figure:

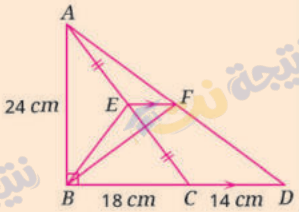
If $C \in \overline{BD}$, $\overline{AB} \perp \overline{BD}$,

E is the midpoint of $\overline{AC}$,

$\overline{EF} // \overline{CD}$

$AB = 24\ cm$, $BC = 18\ cm$,

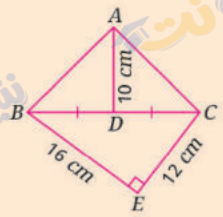
$CD = 14\ cm$



Find the perimeter of the triangle BEF.

■ Self-Evaluation (5)

In the following figure:



If D is the midpoint of $\overline{BC}$,

prove that: $m(\angle BAC) = 90°$

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Lesson Assessment



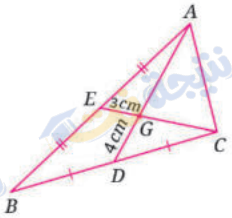
First **Measuring Conceptual Understanding**



▸ **Choose the correct answer from the given ones:**

(1) In the given figure:

If G is the centroid of the triangle ABC, $GE = 3\ cm$, $GD = 4\ cm$, which of the following is correct?



(a) $AG = 6\ cm$

(b) $CG = 8\ cm$

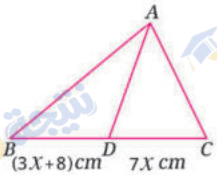
(c) $CE = 9\ cm$

(d) $AG = 12\ cm$

(2) In the given figure:

$\overline{AD}$ is a median of the triangle ABC

What is the length of $\overline{BC}$?



(a) 3 cm

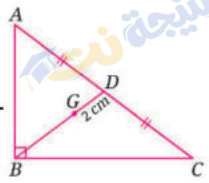
(b) 6 cm

(c) 14 cm

(d) 28 cm

(3) In the given figure:

If G is the centroid of the triangle ABC which is right-angled at B, $GD = 2\ cm$, what is the length of $\overline{AC}$?

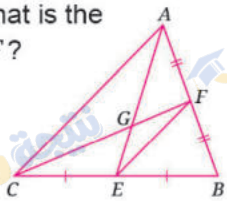


(a) 8 cm (b) 12 cm

(c) 18 cm (d) 24 cm

(4) In the given figure:

$\overline{AE}$ and $\overline{CF}$ are two medians of the triangle ABC, $\overline{AE} \cap \overline{CF} = \{G\}$, $CG = 12\ cm$, $AE = 15\ cm$, $AC = 20$ cm. What is the perimeter of the triangle GEF?



(a) 18 cm

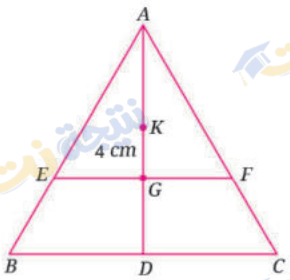
(b) 21 cm

(c) 24 cm

(d) 27 cm

(5) In the given figure:

If G is the centroid of the triangle ABC, and K is the centroid of the triangle AEF, what is the length of $\overline{GD}$?



(a) 2 cm

(b) 4 cm

(c) 6 cm

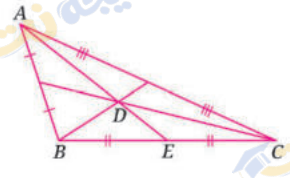
(d) 8 cm

▸ **Spot the mistake:**

(6) Bassem and Sameh solved the following problem:

In the given figure: If the length of $\overline{AE} = 48\ cm$, what is the length of $\overline{DE}$?

Bassem's solution:	**Sameh's solution:**
$DE = \frac{2}{3} AE$	$DE = \frac{1}{3} AE$
$DE = \frac{2}{3} (48)$	$DE = \frac{1}{3} (48)$
$DE = 32$	$DE = 16$



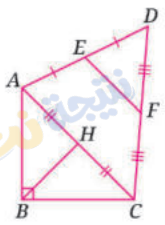
Which of them followed the right method? Discuss.

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Second Applying Concepts

(7) In the given figure:

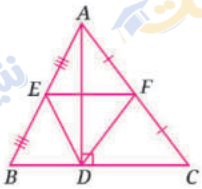
If $\overline{BH}$ is a median of the triangle ABC, which is right-angled at B, E and F are the midpoints of $\overline{AD}$ and $\overline{CD}$, respectively, prove that: $BH = EF$



(8) In the given figure:

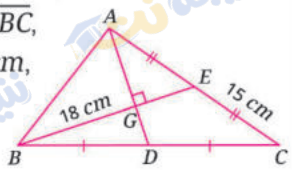
ABC is a triangle in which $\overline{AD} \perp \overline{BC}$, E and F are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.

Prove that : the perimeter of $\Delta EFD = \frac{1}{2}$ the perimeter of ΔABC



(9) In the given figure:

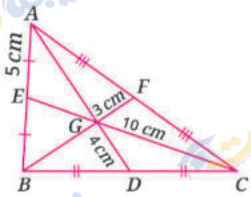
If E is the midpoint of $\overline{AC}$, D is the midpoint of $\overline{BC}$, $\overline{AD} \perp \overline{BE}$, $CE = 15\ cm$, $BG = 18\ cm$, find with proof the length of $\overline{AD}$.



(10) In the given figure:

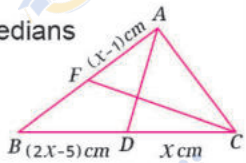
G is the centroid of the triangle ABC, $AE = 5\ cm$, $CG = 10\ cm$, $GD = 4\ cm$, $GF = 3\ cm$

Find with proof the area of the triangle ABD.



(11) In the given figure:

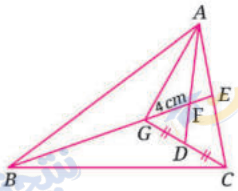
If $\overline{AD}$ and $\overline{CF}$ are two medians of the triangle ABC, find the length of $\overline{FB}$



(12) In the given figure:

G is the centroid of the triangle ABC, $DC = DG$, $GF = 4\ cm$

Find with proof the length of $\overline{BE}$.

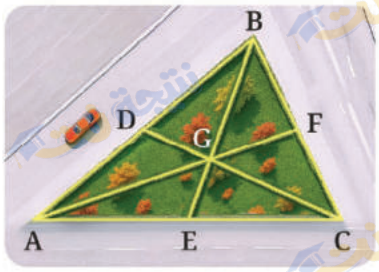


Third Analysis and Subjects Integration

(13) Engineering design : There are three paths inside a triangular garden, each path is a median of the triangle that represents the garden, and these paths intersect at the point of concurrency of the medians.

① If $DC = 81\ m$, find DG, CG

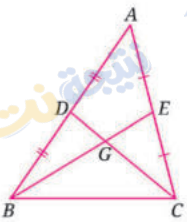
② If $GF = 25\ m$, find GA, AF



Creative Thinking

(14) In the given figure:

If $BE = 15\ cm$, $CD = 9\ cm$, find the interval to which the length of $\overline{BC}$ belongs.



***Evaluate** your understanding!*

How well do you understand medians of the triangle? Tick the right box



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Lesson

3 - 5

Isosceles Triangle

■ Learning Outcomes

- Learn the properties of an isosceles triangle.
- Learn the properties of an equilateral triangle.
- Learn the symmetry in a triangle.
- Learn the relationship among the lengths of the sides of a 30°-60°-90° triangle.
- Use the properties of isosceles triangles and equilateral triangles to solve real-life problems.

■ Vocabulary

- Isosceles triangle
- Equilateral triangle
- Leg
- Base
- Axis of symmetry

Get Ready!

The Egyptian city of Dahab, located in South Sinai Governorate, is not just a coastal city, but one of the most beautiful tourist cities in the world. If Adel and his friend Yasser set up a triangular tent for camping on one of Dahab's beaches, what is the approximate height of the tent?

In this lesson, you will learn about isosceles triangles, equilateral triangles, and their axes of symmetry, which will enable you to solve such real-life problems.



Think & Discuss!

Artists use the colour spinner to show the relationships between colours.The twelve triangles in the figure are isosceles triangles with congruent vertices angles.

Can you find the measures of the interior angles of any of these triangles? Discuss.

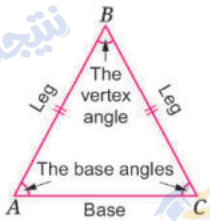


Learn!

Isosceles triangle

An isosceles triangle is a triangle with two congruent sides.

- The two congruent sides are called the legs of the triangle.
- The angle between them is called the vertex angle.
- The other two angles are called the base angles.



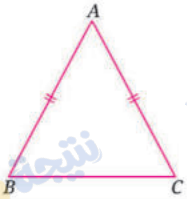
Theorem 5-1 **The isosceles triangle theorem.**

The base angles of an isosceles triangle are congruent.

In ΔABC:

If $\overline{AB} \cong \overline{AC}$,

then $\angle B \cong \angle C$

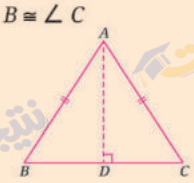


■ Discuss the proof of theorem (5-1)

Draw $\overline{AD} \perp \overline{BC}$, and from the congruence of the two triangles ABD, ACD,

You can prove that:

$\angle B \cong \angle C$



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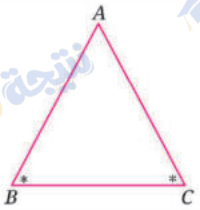
Converse of theorem 5-1 **Converse of the isosceles triangle theorem**

If two angles of a triangle are congruent, then the two sides opposite to these two angles are congruent.

In ΔABC:

If $\angle B \cong \angle C$,

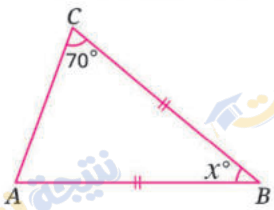
then $\overline{AC} \cong \overline{AB}$



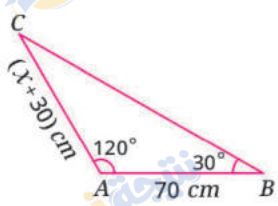
Example 1

Find the value of $\mathcal{X}$ in each of the following figures:

1



2



1

$\because \overline{AB} \cong \overline{CB} \quad \therefore \angle A \cong \angle C$

$\therefore m(\angle A) = m(\angle C) = 70°$

$\therefore m(\angle B) = 180° - (70° + 70°)$

$= 40°$

$\therefore \mathcal{X} = 40$

2

$\because m(\angle C) = 180° - (120° + 30°)$

$= 30°$

$\therefore \angle C \cong \angle B \quad \therefore \overline{AC} \cong \overline{AB}$

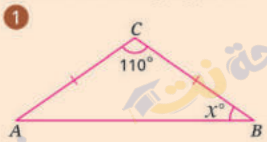
$\therefore \mathcal{X} + 30 = 70$

$\therefore \mathcal{X} = 70 - 30 = 40$

■ Self-Evaluation (1)

Find the value of $\mathcal{X}$ in each of the following figures:

1



2

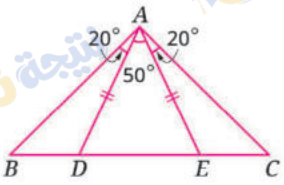


Example 2

In the given figure:

$D \in \overline{BC}$, $E \in \overline{BC}$

Prove that: ΔABC is an isosceles triangle.



In ΔADE:

$\because \overline{AD} \cong \overline{AE}$

$\therefore m(\angle ADE) = m(\angle AED) = \frac{180° - 50°}{2} = 65°$

$\therefore m(\angle ADB) = m(\angle AEC) = 180° - 65° = 115°$

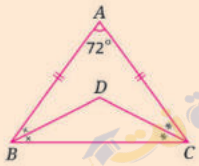
$\therefore m(\angle B) = m(\angle C) = 180° - (20° + 115°) = 45°$

$\therefore \angle B \cong \angle C \quad \therefore \overline{AB} \cong \overline{AC}$

$\therefore \Delta ABC$ is an isosceles triangle.

■ Self-Evaluation (2)

In the following figure:



$AB = AC$, $m(\angle A) = 72°$,

$\overrightarrow{BD}$ bisects $\angle ABC$,

$\overrightarrow{CD}$ bisects $\angle ACB$

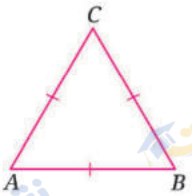
Prove that:

ΔDBC is an isosceles triangle.

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Equilateral triangle

An equilateral triangle is a triangle whose three sides are congruent.



Corollary:

If the triangle is equilateral, then it is equiangular (has three congruent angles).

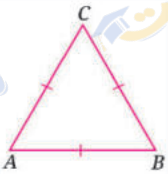
In $\Delta\ ABC$:

If

$AB = AC = BC$,

then:

$\angle A \cong \angle B \cong \angle C$



Corollary:

If a triangle is equiangular, then it is equilateral.

In $\Delta\ ABC$:

If

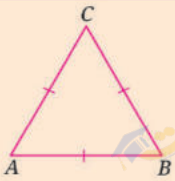
$\angle A \cong \angle B \cong \angle C$,

then:

$AB = AC = BC$



■ **Discuss the proof of the corollaries**



Use the theorem of the isosceles triangle and its converse to prove the adjacent corollaries.

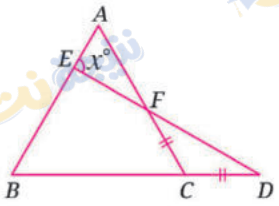
Example 3

In the given figure:

ABC is an equilateral triangle,

$\overline{AC} \cap \overline{ED} = \{F\}$, $CF = CD$

Find the value of x



✓ $\because \Delta\ ABC$ is an equilateral triangle.

$\therefore m(\angle ACB) = m(\angle A) = m(\angle B) = 60°$

$\therefore m(\angle FCD) = 180° - 60° = 120°$

$\because \Delta\ FCD$ is an isosceles triangle in which $CF = CD$

$\therefore m(\angle D) = m(\angle CFD) = \frac{180° - 120°}{2} = 30°$

$\because \angle AED$ is an exterior angle of $\Delta\ EBD$

$\therefore m(\angle AED) = m(\angle B) + m(\angle D) = 60° + 30° = 90°$

$\therefore x = 90$

■ **Self-Evaluation (3)**

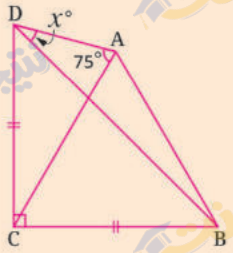
In the following figure:

ABC is an equilateral triangle,

$BC = CD$,

$m(\angle DAC) = 75°$,

$m(\angle DCB) = 90°$



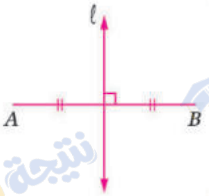
Find with proof the value of x

Axis of symmetry of a line segment

The axis of symmetry of a line segment is the perpendicular straight line passing through its midpoint.

If the straight line $\ell \perp \overline{AB}$, the straight line ℓ bisects $\overline{AB}$,

then the straight line ℓ is the axis of symmetry of $\overline{AB}$



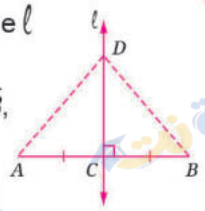
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Corollaries of the axis of symmetry of a line segment

Corollary:

Any point belongs to the axis of symmetry of a line segment is equidistant from its terminals.

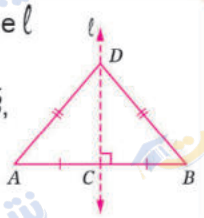
If the straight line ℓ is the axis of symmetry of $\overline{AB}$, $D \in \ell$, then $DA = DB$



Corollary:

Any point that is equidistant from the terminals of a line segment lies on the axis of symmetry of that line segment.

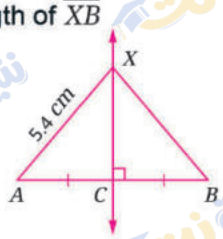
If the straight line ℓ is the axis of symmetry of $\overline{AB}$, $DA = DB$, then $D \in \ell$



Example 4

1 In the given figure:

Find the length of $\overline{XB}$



① $\because CA = CB$, $\overleftrightarrow{XC} \perp \overline{AB}$

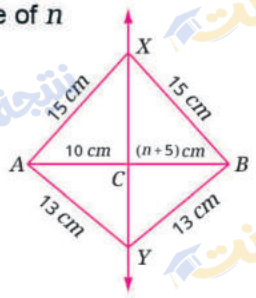
$\therefore \overleftrightarrow{XC}$ is the axis of symmetry of $\overline{AB}$

$\because X \in \overleftrightarrow{XC}$

$\therefore XB = XA = 5.4\ cm$

2 In the given figure:

Find the value of n



② $\because XA = XB = 15\ cm$

$\therefore X$ lies on the axis of symmetry of $\overline{AB}$

$\because YA = YB = 13\ cm$

$\therefore Y$ lies on the axis of symmetry of $\overline{AB}$

$\therefore \overleftrightarrow{XY}$ is the axis of symmetry of $\overline{AB}$

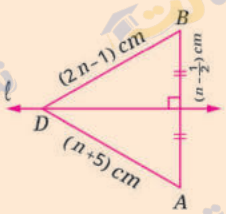
$\therefore \overleftrightarrow{XY} \perp \overline{AB}$, $CB = AC$

$\therefore n + 5 = 10$

$\therefore n = 10 - 5 = 5$

Self-Evaluation (4)

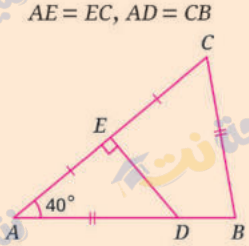
1 In the following figure:



Find: $m(\angle A)$

2 In the following figure:

$AE = EC$, $AD = CB$



Find: $m(\angle C)$

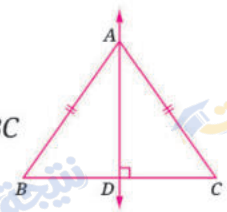
Note that

- The isosceles triangle has only one axis of symmetry.
- The equilateral triangle has three axes of symmetry.
- The scalene triangle has no axis of symmetry.

Axis of symmetry of an isosceles triangle

The axis of symmetry of an isosceles triangle is a straight line passing through the vertex of the triangle perpendicular to its base.

If $AB = AC$, $\overleftrightarrow{AD} \perp \overline{BC}$, then $\overleftrightarrow{AD}$ is the axis of symmetry of the triangle ABC



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Corollaries of the axis of symmetry of an isosceles triangle

Corollary:

The median of an isosceles triangle drawn from the vertex bisects the vertex angle and is perpendicular to the base.

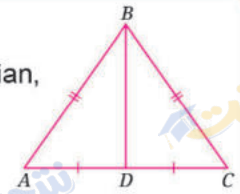
If $AB = CB$,

$\overline{BD}$ is a median,

then:

① $\overline{BD} \perp \overline{AC}$

② $m(\angle ABD) = m(\angle CBD)$



Corollary:

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

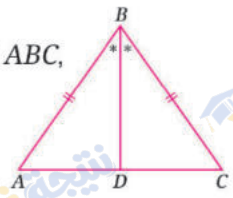
If $AB = CB$,

$\overrightarrow{BD}$ bisects $\angle ABC$,

then:

① $AD = CD$

② $\overline{BD} \perp \overline{AC}$



Corollary:

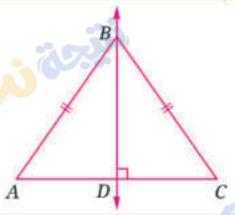
The straight line drawn from the vertex of an isosceles triangle perpendicular to the base bisects both the base and the vertex angle.

If $AB = CB$, $\overleftrightarrow{BD} \perp \overline{AC}$,

then:

① $AD = DC$

② $m(\angle ABD) = m(\angle CBD)$



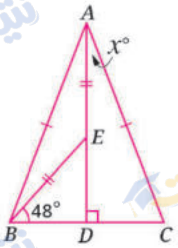
Example 5

In the given figure:

$AB = AC$, $EA = EB$, $\overline{AD} \perp \overline{BC}$,

$m(\angle EBD) = 48°$

Find the value of $\mathcal{X}$



In ΔEDB:

$m(\angle DEB) = 180° - (90° + 48°) = 42°$

$\because \angle DEB$ is an exterior angle of ΔAEB, $EA = EB$

$\therefore m(\angle EAB) = m(\angle EBA) = \frac{42°}{2} = 21°$

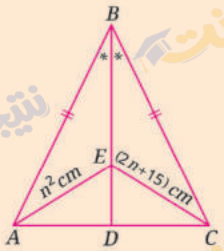
$\because \Delta ABC$ is an isosceles triangle, $\overline{AD} \perp \overline{BC}$

$\therefore m(\angle CAD) = m(\angle BAD) = 21°$ $\quad \therefore \mathcal{X} = 21$

■ Self-Evaluation (5)

In the following figure:

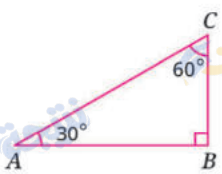
$E \in \overline{BD}$



Find the value of n

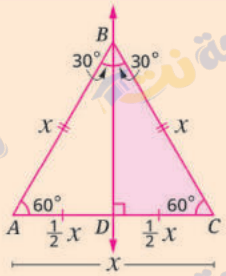
30°–60°–90° triangle

30°–60°–90° triangle is a right-angled triangle with two acute angles of measures 30° and 60°.



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■ Note that

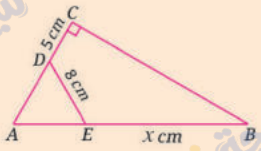


The axis of symmetry of an equilateral triangle divides it into two congruent triangles, each of which is a 30°-60°-90° triangle.

■ Self-Evaluation (6)

In the following figure:

ΔADE is equilateral.



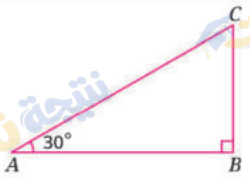
Find the value of X

Corollary:

In a 30°-60°-90° triangle, the length of the side opposite to the angle of measure 30° is equal to half the length of the hypotenuse.

If ΔABC is a right-angled triangle at B,
$m(\angle A) = 30°$,

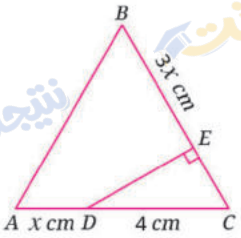
then $BC = \frac{1}{2} AC$



Example (6)

In the given figure:

If ΔABC is an equilateral triangle,
find the value of X



$\because \Delta ABC$ is an equilateral triangle

$\therefore m(\angle C) = 60°$

$\because m(\angle DEC) = 90°$ $\quad \therefore m(\angle EDC) = 30°$

$\therefore \Delta DEC$ is a 30°-60°-90° triangle

$\therefore EC = \frac{1}{2} DC = \frac{1}{2} \times 4 = 2\ cm$

$\because BC = AC$ $\quad \therefore 3X + 2 = X + 4$ $\quad \therefore 3X - X = 4 - 2$

$\therefore 2X = 2$ $\quad \therefore X = 1$

Lesson Assessment



First **Measuring Conceptual Understanding**

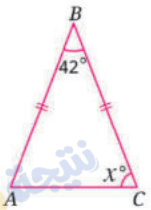
▸ **Choose the correct answer from the given ones:**

(1) **In the given figure:**

What is the value of X?

(a) 42 (b) 21

(c) 69 (d) 84

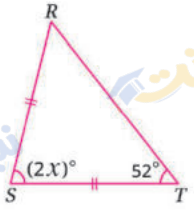


(2) **In the given figure:**

What is the value of X?

(a) 26 (b) 52

(c) 76 (d) 38

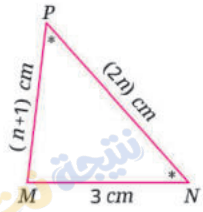


(3) **In the given figure:**

What is the length of $\overline{PN}$?

(a) 8 cm (b) 3 cm

(c) 6 cm (d) 4 cm

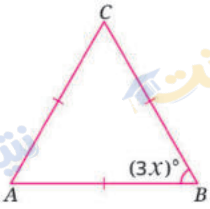


(4) **In the given figure:**

What is the value of X?

(a) 10 (b) 20

(c) 30 (d) 60



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(5) If the lengths of two sides of an isosceles triangle are 3 cm and 7 cm, what is the length of the third side?

(a) 3 cm (b) 4 cm

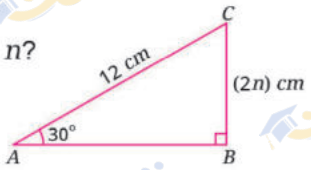
(c) 7 cm (d) 10 cm

(6) **In the given figure:**

What is the value of n?

(a) 3 (b) 6

(c) 9 (d) 12

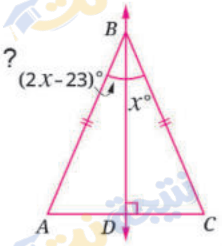


(7) **In the given figure:**

What is the measure of $\angle C$?

(a) 23° (b) 46°

(c) 67° (d) 60°

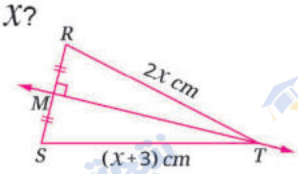


(8) **In the given figure:**

What is the value of X?

(a) 10 (b) 7

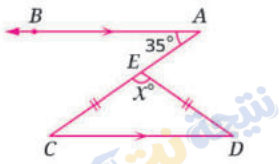
(c) 5 (d) 3



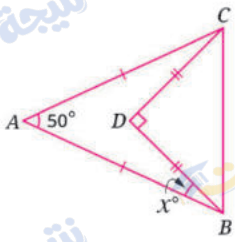
Second Applying Concepts

▸ **In each of the following figures, find the value of X :**

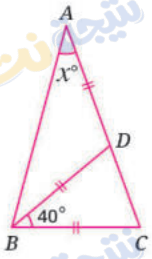
(9)



(10)



(11)

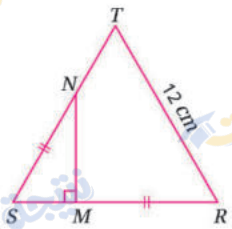


(12) **In the given figure:**

ΔSRT is an equilateral triangle, and its side length is 12 cm,

$MR = SN$, $\overline{NM} \perp \overline{SR}$

Find with proof the length of $\overline{NM}$

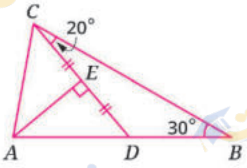


(13) **In the given figure:**

$m(\angle B) = 30°$, $m(\angle DCB) = 20°$,

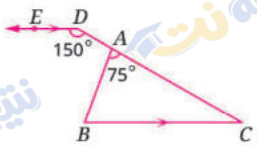
$CE = DE$, $\overline{AE} \perp \overline{CD}$

Find with proof $m(\angle CAE)$

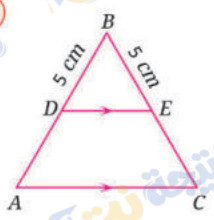


▸ **In each of the following figures, prove that the triangle ABC is an isosceles triangle:**

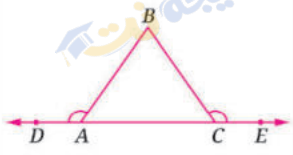
(14)



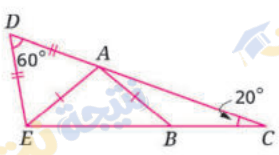
(15)



(16)



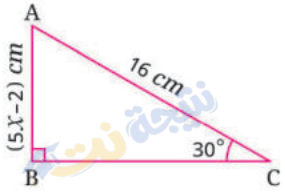
(17)



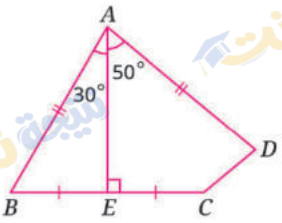
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▸ **Answer the following questions:**

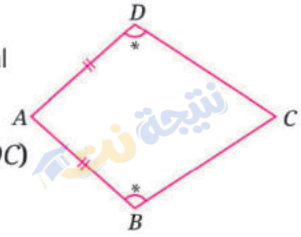
(18) **In the given figure:**
Find the value of x, then find the area of ΔABC



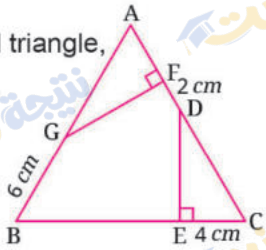
(19) **In the given figure:**
Find $m(\angle D)$



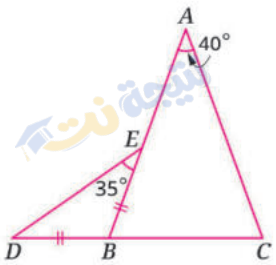
(20) **In the given figure:**
$ABCD$ is a quadrilateral in which $AB = AD$,
$m(\angle ABC) = m(\angle ADC)$
Prove that: $BC = DC$



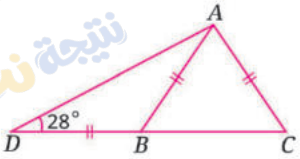
(21) **In the given figure:**
ΔABC is an equilateral triangle,
$EC = 4\ cm$, $FD = 2\ cm$,
$GB = 6\ cm$
Find with proof the perimeter of ΔABC



(22) **In the given figure:**
$D \in \overrightarrow{CB}$,
$m(\angle DEB) = 35°$,
$m(\angle A) = 40°$,
$BE = BD$
Prove that: $AB = AC$

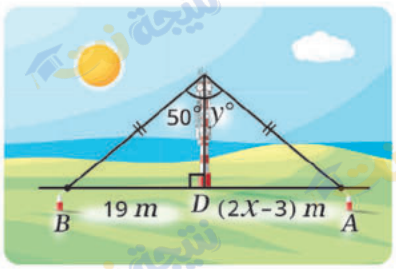


(23) **In the given figure:**
$B \in \overline{DC}$,
$m(\angle D) = 28°$
Find with proof $m(\angle CAD)$

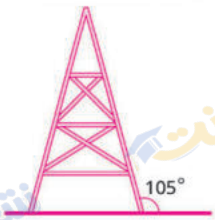


Third Analysis and Subjects Integration

(24) **Communications:** A communications tower is fixed to the ground by two wires of equal length at the points A, B such that the base of the tower D lies on $\overleftrightarrow{AB}$ as shown in the figure. What are the values of x and y that make the tower perpendicular to the ground?

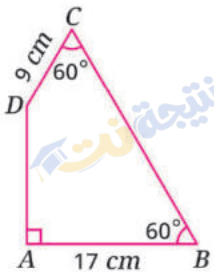


(25) **Observation towers:** An observation tower was built in the shape of an isosceles triangle, as shown in the figure. If a side of the tower meets the ground at an angle of measure 105°, what is the measure of the angle at its top?



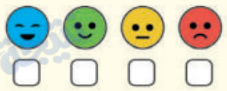
Creative Thinking

(26) **In the given figure:**
Find with proof the length of $\overline{BC}$



***Evaluate** your understanding!*

How well do you understand isosceles triangle?
Tick the right box



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Lesson

3 - 6

Interior and Exterior Angles of Polygons

Learning Outcomes

- Learn the mathematical formula for calculating the sum of the measures of the interior angles of a convex polygon.
- Learn the sum of the measures of the exterior angles of a convex polygon.
- Learn the mathematical formula for calculating the measure of an interior angle of a regular polygon.
- Learn the mathematical formula for calculating the number of diagonals of a convex polygon.

Vocabulary

- Polygon
- Regular polygon
- Convex polygon
- Interior angle
- Exterior angle

Get Ready!

The Octagon building is the new headquarters of the Egyptian Ministry of Defence in the new administrative capital. It is one of the largest and most modern defence ministry headquarters in the world.

Can you find the measure of the interior angle of the polygon represented by this building?



The Octagon Building in the New Administrative Capital

In this lesson, you will learn how to calculate the sum of the measures of the interior and exterior angles of a convex polygon and how to calculate the measure of each angle of a regular polygon, which will enable you to solve such real-life problems.

Think & Discuss!

While organizing a conference hall, the organizer used identical tables, each in the shape of an isosceles trapezoid. How many tables does the organizer need to form a regular polygon? Discuss.



Learn!

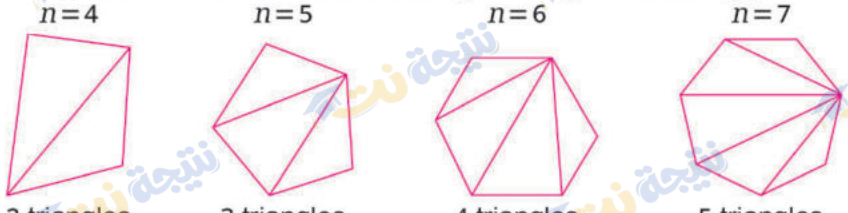
The mathematical formula for the sum of the measures of the interior angles of a convex polygon

The sum of the measures of the interior angles of a convex polygon $= (n - 2) \times 180°$, where n is the number of sides of the polygon.

For example:

- The sum of the measures of the interior angles of a convex pentagon $= (5 - 2) \times 180° = 540°$
- The sum of the measures of the interior angles of a convex octagon $= (8 - 2) \times 180° = 1080°$

▸ **The previous formula can be deduced by noticing the following pattern:**



2 triangles | 3 triangles | 4 triangles | 5 triangles

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- The diagonals drawn from one of the vertices of a polygon divide the polygon into a number of triangles = $(n-2)$ triangles.
- The sum of the measures of the interior angles of each triangle = 180°.
- The sum of the measures of the interior angles of the polygon
  = 180° × number of triangles
  = $(n-2) \times 180°$

Example 1

In each of the following, find the number of sides of the convex polygon whose sum of measures of its interior angles is:

1 720°

$(n-2) \times 180° = 720°$

$\therefore n-2 = \frac{720°}{180°} = 4$

$\therefore n = 4 + 2 = 6$ sides

2 1440°

$(n-2) \times 180° = 1440°$

$\therefore n-2 = \frac{1440°}{180°} = 8$

$\therefore n = 8 + 2 = 10$ sides

Self-Evaluation (1)

What is the number of sides of the convex polygon whose sum of measures of its interior angles is 1260°?

Example 2

Carpentry: A carpenter wants to make a wooden locker to be fit properly in one corner of a rectangular room. If the carpenter makes the locker with the angles shown in the figure, does it fit properly or not?



For the locker to be fit properly,
$m(\angle A)$ must be 90°

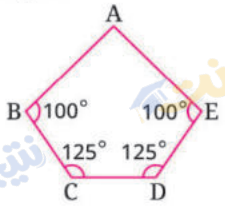
- Calculate the sum of the measures of the interior angles of the pentagon as follows:

$(n-2) \times 180° = (5-2) \times 180° = 540°$

$\therefore m(\angle A) + 100° + 125° + 125° + 100° = 540°$

$\therefore m(\angle A) = 540° - 450° = 90°$

$\therefore$ The locker fits properly the corners of the room.



Self-Evaluation (2)

In the following figure:
A dining table has a surface in the shape of a parallelogram.



Find: $m(\angle D)$

Note that

For any convex polygon, there are two exterior angles at each vertex, as shown in the figure.



These two angles are vertically opposite angles, so they are equal in measure.

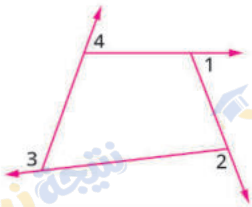
The sum of the measures of the exterior angles of a convex polygon

Rule

The sum of the measures of the exterior angles of a convex polygon (one angle at each vertex) is equal to 360°.

In the given figure:

$m(\angle 1) + m(\angle 2) + m(\angle 3) + m(\angle 4) = 360°$

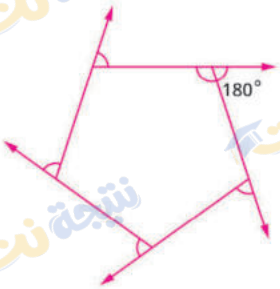


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■ Think & Discuss

Discuss the validity of the following statement:
The sum of the measures of the exterior angles (one at each vertex) of the convex hexagon and the convex 12-side polygon are equal.

The previous rule can be deduced by noticing that for any convex polygon with n sides:

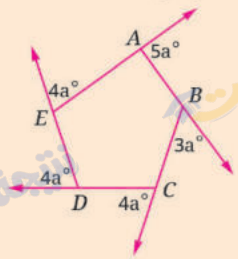


- At each vertex there is a pair of angles, one of them is exterior and the other is interior the sum of their measures is 180°.
- The sum of the measures of the pairs of angles at all vestices of the polygon = $180° n$
- The sum of the measures of the exterior angles (one at each vertex) is :

$180° n - (n-2) \times 180° = 180° n - (180° n - 360°) = 360°$

■ Self-Evaluation (3)

In the following figure:

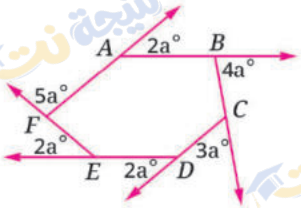


Find: $m(\angle BCD)$

Example 3

In the given figure:

Find: (1) The value of a (2) $m(\angle BCD)$



$\because$ The sum of the measures of the exterior angles of a convex polygon = 360°

$\therefore 2a° + 4a° + 3a° + 2a° + 2a° + 5a° = 360°$

$\therefore 18a = 360$ $\quad \therefore a = \frac{360}{18} = 20$ (Required 1)

$\therefore$ The measure of the exterior angle at vertex $C = 20° \times 3 = 60°$

$\therefore m(\angle BCD) = 180° - 60° = 120°$ (Required 2)

■ Remember that

The regular polygon is a polygon that has the following two properties:
1. All sides are equal in length.
2. All interior angles are equal in measure.

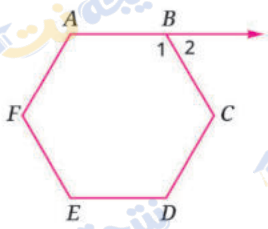
Mathematical formula for the measure of the angle of a regular polygon

In a regular polygon with n sides:

- The measure of the interior angle $= \frac{(n-2) \times 180°}{n}$
- The measure of the exterior angle at one of its vertices $= \frac{360°}{n}$

Example:

The given figure is a regular hexagon.



- $m(\angle 1) = \frac{(n-2) \times 180°}{n}$

$= \frac{(6-2) \times 180°}{6} = 120°$

- $m(\angle 2) = \frac{360°}{n} = \frac{360°}{6} = 60°$

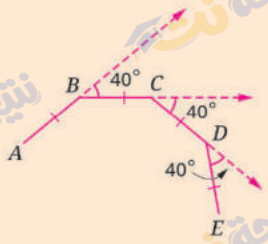
■ Think & Discuss

Discuss the validity of the following statements :
1. The measure of the interior angle of a regular nonagon is 140°.
2. The measure of the exterior angle of a regular pentagon at one of its vertices is 108°

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■ Self-Evaluation (4)

In the following figure:



How many sides does the regular polygon have when it is completed in the same way?

Example 4

Find the number of sides of the regular polygon in each of the following :

1. If the measure of one of its interior angles = 150°
2. If the measure of one of its exterior angles = 24°

(1) ∵ The polygon is regular and the measure of one of its interior angles = 150°

$\therefore \frac{(n-2)\times 180^\circ}{n} = 150^\circ$

$\therefore 180^\circ n - 360^\circ = 150^\circ n$ $\quad\quad$ $\therefore 180^\circ n - 150^\circ n = 360^\circ$

$\therefore 30^\circ n = 360^\circ$ $\quad\quad$ $\therefore n = \frac{360^\circ}{30^\circ} = 12$

∴ The number of sides of the polygon = 12 sides.

(2) ∵ The polygon is regular and the measure of one of its exterior angles = 24°

$\therefore \frac{360^\circ}{n} = 24^\circ$ $\quad\quad$ $\therefore 24^\circ n = 360^\circ$ $\quad\quad$ $\therefore n = \frac{360^\circ}{24^\circ} = 15$

∴ The number of sides of the polygon = 15 sides.

The mathematical formula for the number of diagonals of a convex polygon

The number of diagonals of a convex polygon with n sides is equal to $\frac{n(n-3)}{2}$

■ Self-Evaluation (5)

Find each of the following:

1. The number of diagonals of a convex pentagon.
2. The number of sides of a convex polygon with 27 diagonals.

Example 5

1. Find the number of diagonals of a convex octagon.
2. Find the number of sides of a convex polygon with 35 diagonals.

(1) The number of diagonals of a convex octagon $= \frac{n(n-3)}{2} = \frac{8(8-3)}{2}$ = 20 diagonals.

(2) ∵ The number of diagonals = 35

$\therefore \frac{n(n-3)}{2} = 35$ $\quad\quad$ $\therefore n(n-3) = 70$

$\therefore n^2 - 3n - 70 = 0$ $\quad\quad$ $\therefore (n+7)(n-10) = 0$

∴ Either $n + 7 = 0$ $\quad\quad$ hence $n = -7$ (refused)

or $n - 10 = 0$ $\quad\quad$ hence $\boxed{n = 10}$

∴ The number of sides = 10 sides.

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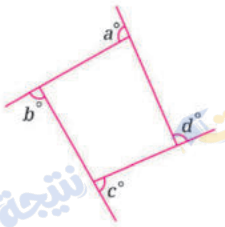
Lesson Assessment

First Measuring Conceptual Understanding

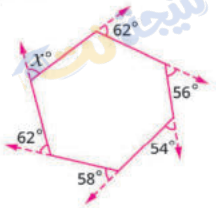
▸ **Choose the correct answer from the given ones:**

1. What is the sum of the measures of the interior angles of the convex hexagon?
   (a) 900° (b) 720°
   (c) 540° (d) 360°

2. **In the given figure:**
   What is the value of $a + b + c + d$?
   (a) 180 (b) 360
   (c) 540 (d) 720



3. **In the given figure:**
   What is the value of X ?
   (a) 58 (b) 62
   (c) 68 (d) 72



4. What is the measure of the interior angle of a regular polygon with 18 sides?
   (a) 135° (b) 144°
   (c) 150° (d) 160°

5. How many sides does a convex polygon have if the sum of the measures of its interior angles is 2340°?
   (a) 12 (b) 13 (c) 14 (d) 15

6. How many sides does a regular polygon have if the measure of one of its interior angles is 144°?
   (a) 10 (b) 11 (c) 12 (d) 14

7. How many sides does a regular polygon have if the measure of one of its exterior angles is 20°?
   (a) 12 (b) 14 (c) 16 (d) 18

8. How many diagonals does a convex heptagon have?
   (a) 7 (b) 11 (c) 13 (d) 14

▸ **Spot the mistake:**

9. Amal and Samar tried to find the measure of the exterior angle of a regular pentagon. Which of them got the right answer? Why?

Amal's solution :

$(n - 2)\ 180° = (5 - 2)\ 180°$

$= 3 \times 180° = 540°$

The measure of the exterior angle E

$E = \frac{540°}{5} = 108°$

Samar's solution :

There are 10 exterior angles of the pentagon.

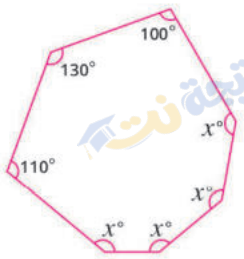
The measure of the exterior angle E

$E = \frac{360°}{10} = 36°$

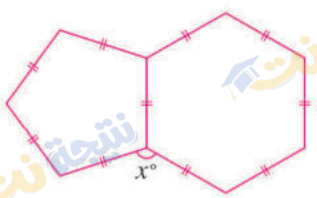
Second Applying Concepts

▸ **In each of the following figures, find the value of X:**

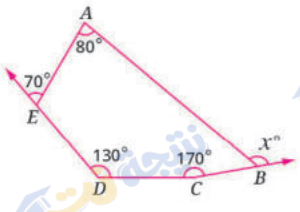
10.



11.

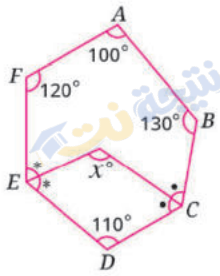


12.

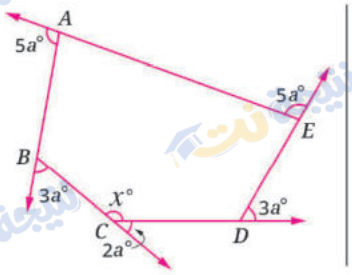


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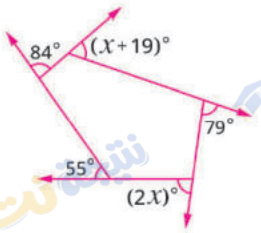
(13)



(14)



(15)



(16) Find each of the following:

(1) The number of sides of a convex polygon whose sum of measures of its interior angles is 2700°, then find the number of its diagonals.

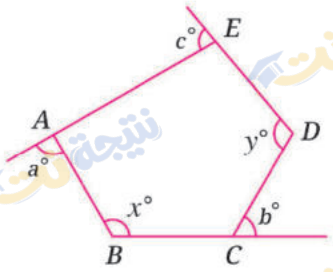
(2) The number of sides of a convex polygon that has 9 diagonals.

(3) The number of sides of a regular polygon whose interior angle measure is 156°, then find the number of its diagonals.

(17) In the given figure:

$x + y = 230$

Find with proof the value of: $a + b + c$



Third **Analysis and Subjects Integration**

(18) Old Egyptian coins:

Find the measure of the interior angle of each of the following regular polygons:





Creative Thinking

(19) If the number of sides of a regular polygon increases by two, the measure of the exterior angle decreases by 9°.
How many sides does the polygon have?

***Evaluate** your understanding!*

How well do you understand interior and exterior angles of polygons?
Tick the right box



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Unit Three Assessment

Evaluate your understanding!
How well do you understand Unit Three? Tick the right box

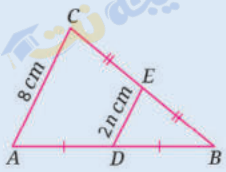
▸ **Choose the correct answer from the given ones:**

(1) In the given figure:

What is the value of n ?

(a) 2 (b) 4

(c) 6 (d) 8

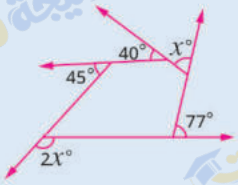


(2) In the given figure:

What is the value of $\mathcal{X}$?

(a) 77 (b) 45

(c) 55 (d) 66



(3) In the given figure:

Q and N are the midpoints of $\overline{SR}$ and $\overline{TR}$ respectively,

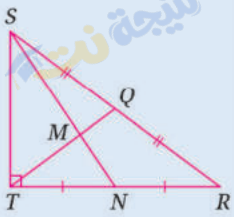
$TR = 12$ cm , $ST = 9$ cm,

$m(\angle STR) = 90°$.

What is the length of $\overline{MT}$?

(a) 2.5 cm (b) 5 cm

(c) 7.5 cm (d) 15 cm



(4) In the given figure:

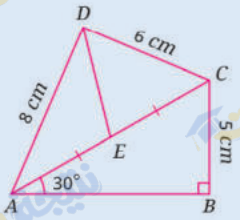
What is the length of $\overline{DE}$?

(a) 6 cm

(b) 8 cm

(c) 10 cm

(d) 5 cm



▸ **Complete each of the following with the correct answer:**

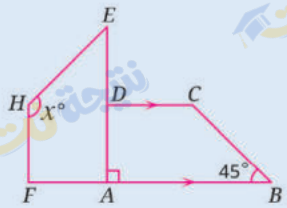
(5) The number of sides of a convex polygon with 20 diagonals is .....

(6) The type of the triangle whose side lengths are 6 cm, 7 cm, and 8 cm according to its angles is .....

(7) In the given figure:

If the polygon $AEHF$ is the image of the polygon $ABCD$ by a rotation,

then $\mathcal{X}$ = ...



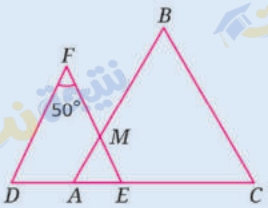
(8) In the given figure:

ΔABC is an equilateral triangle,

ΔDEF is an isosceles triangle in which

$FD = FE$, $m(\angle F) = 50°$,

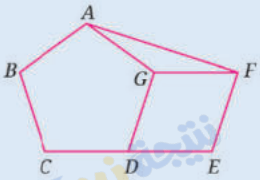
then $m(\angle AME)$ = ...°



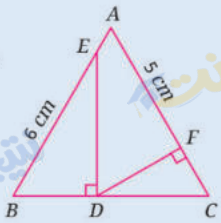
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▸ **Answer the following questions:**

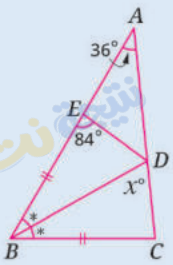
(9) **In the given figure :**
$ABCDG$ is a regular pentagon and $GDEF$ is a rhombus.
Find $m(\angle FAG)$



(10) **In the given figure:**
ABC is an equilateral triangle,
$\overline{DF} \perp \overline{AC}$, $\overline{ED} \perp \overline{BC}$,
$AF = 5\ cm$, $BE = 6\ cm$.
Find with proof the length of $\overline{FC}$



(11) **In the given figure:**
$BC = BE$,
$\overrightarrow{BD}$ bisects $\angle ABC$
① Prove that:
$\Delta BCD \cong \Delta BED$
② Find the value of x



(12) **In the given figure:**
$BD = CD$, $\overline{AD} \perp \overline{BC}$,
$m(\angle EBD) = 48°$, $AE = BE$
Find with proof the value of x



Unit Three Activity — Pythagoras' Theorem



▸ **Activity Aim:**
To prove Pythagoras' theorem practically.

▸ **Implementation Steps:**
There are many videos available on YouTube that prove Pythagoras' theorem practically. collaborate with two of your friends and try to do the following:

1. Search for different practical methods to prove Pythagoras' theorem.
2. Choose only two methods and compare them in terms of (method used, materials used, level of accuracy) or any other elements for comparison selected by the team.
3. Choose one of these two methods (the simplest one) and try to apply it in collaboration with your peers.

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UNIT 4

Statistics

Unit Lessons

4-1 Ascending Cumulative Frequency Table and Its Graphical Representation

4-2 Measurements of Central Tendency of the Frequency Table with Intervals



Artificial intelligence learning algorithms are used in statistical methods such as regression analysis and normal distribution to analyse student performance and suggest appropriate educational paths.

▶ Can artificial intelligence algorithms be developed to solve complex statistical problems?

Issues and Life Skills		Values	
- Information Technology	- Decision-making	- Responsibility	- Rational Consumption
- Creative Thinking	- Prediction	- Fair Distribution	- Respecting Others

Lesson

4 - 1

Ascending Cumulative Frequency Table and Its Graphical Representation

■ Learning Outcomes

- Form an ascending cumulative frequency table for a frequency table with intervals.
- Draw the ascending cumulative frequency curve.

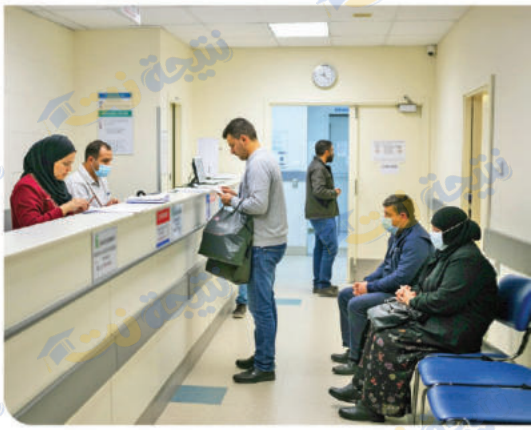
■ Vocabulary

- Frequency table with intervals
- Ascending cumulative frequency table
- Ascending cumulative frequency curve

■ Previous Knowledge

If the amount of data is large, it should be organized into equal sets or intervals, from which we can form a frequency table with intervals, which can be represented graphically by a histogram.

Get Ready!



The administration of a hospital recorded the waiting times of 200 patients, in minutes, in order to improve the appointment schedule based on the busiest periods.

Waiting times (in minutes)	0 –	10 –	20 –	30 –	40 –
Number of patients	30	50	70	40	10

Can you calculate the percentage of patients who waited less than half an hour?

In this lesson, you will learn how to form an ascending cumulative frequency table, which will enable you to solve such real-life problems.

Think & Discuss!

If the number of pages in 50 books in the school library is as follows:

57	10	50	44	21	49	61	31	21	48
46	30	11	48	51	48	57	49	31	72
61	28	39	52	24	47	38	20	23	47
69	49	41	31	67	41	17	52	63	19
58	49	37	43	52	22	59	45	31	35

Can you form a frequency table with intervals using the intervals (–10 , – 20 , – 30 , ...)?

Learn!

You have previously studied how to organize data into equal sets or intervals and write them in a frequency table with intervals.

For example, if the given stem-and-leaf plot shows the science marks of 28 students in the second year of a preparatory school, you can form a frequency table with intervals using equal intervals such as 10 –, 20 –, ...
The table will be as follows:

Marks	10 –	20 –	30 –	40 –	50 –
Frequency	2	10	9	5	2

Stems	Leaves
1	3 8
2	4 4 6 6 7 7 8 8 8 9
3	0 2 4 5 5 5 7 9 9
4	1 4 4 7 8
5	2 5

Key: 3 | 0 means 30 marks

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▸ **Some information can be obtained directly from the table, such as:**

- The number of students who got marks belong to the interval [20, 30[is equal to 10 students.
- The number of students who got marks belong to the interval [40, 50[is equal to 5 students.

▸ **However, you cannot obtain some other information directly from the table, such as:**

- The number of students who got less than 30 marks.
- The number of students who got 20 marks or more.

To obtain this information directly, you must learn how to form an ascending cumulative frequency table as follows:

Marks	Frequency	
10 –	2	2 (less than 20)
20 –	10	2 + 10 = 12 (less than 30)
30 –	9	12 + 9 = 21 (less than 40)
40 –	5	21 + 5 = 26 (less than 50)
50 –	2	26 + 2 = 28 (less than 60)

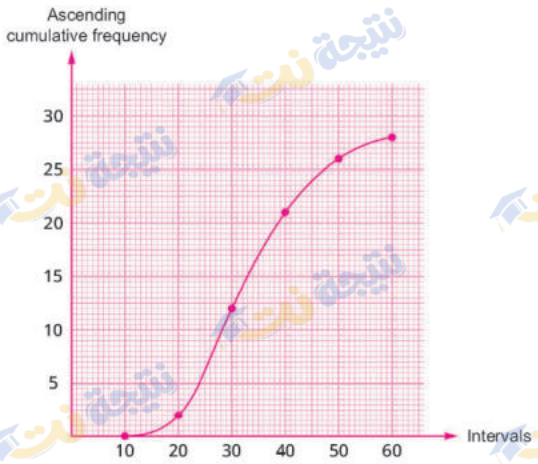
Write this data in a table as follows, which is represented graphically as follows:

■ Note that

There is no student with a mark less than 10, which means:
The ascending cumulative frequency of students who got less than 10 marks is 0.

Upper limits of intervals	Ascending cumulative frequency
less than 10	0
less than 20	2
less than 30	12
less than 40	21
less than 50	26
less than 60	28

«Ascending cumulative frequency table»



«Ascending cumulative frequency curve»

▸ **From the table, note that:**

- The number of students who got less than 30 marks = 12 students.
- The number of students who got 20 marks or more is equal to:
  Number (2) subtracted from the total number (28) = 26 students.

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■ Self-Evaluation (1)

The following table shows the masses of 20 boxes in kilograms.

Intervals (kg)	Frequency
45 –	2
50 –	3
55 –	3
60 –	6
65 –	4
70 –	2

Form an ascending cumulative frequency table and represent it graphically.

Example 1

The given frequency table shows the best record times achieved by 100 swimmers in a 50 meters race.

Intervals in seconds	Frequency
25 –	5
30 –	17
35 –	34
40 –	29
45 –	15

Form an ascending cumulative frequency table and represent it graphically, then find:

1. The number of swimmers who achieved record less than 40 seconds.
2. The number of swimmers who achieved record 35 seconds or more.
3. The percentage of swimmers who achieved record of less than 45 seconds.

Upper limits of intervals	Ascending cumulative frequency
less than 25	0
less than 30	5
less than 35	22
less than 40	56
less than 45	85
less than 50	100



(1) The number of swimmers who achieved record less than 40 seconds is: 56 swimmers.

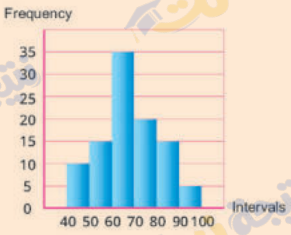
(2) The number of swimmers who achieved record 35 seconds or more is: $100 - 22 = 78$ swimmers

(3) The percentage of swimmers who achieved record less than 45 seconds is:

$$\frac{85}{100} \times 100\% = 85\%$$

■ Self-Evaluation (2)

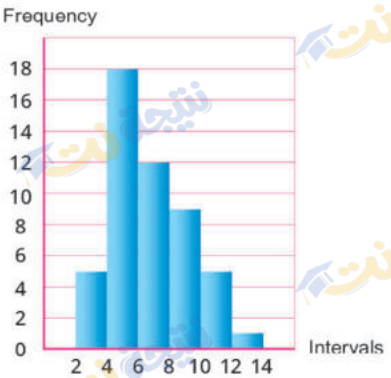
If the following histogram shows the lengths of 100 fish caught in a fishing competition, measured to the nearest centimeter.



Form the frequency table with intervals, the ascending cumulative frequency table, and represent it graphically.

Example 2

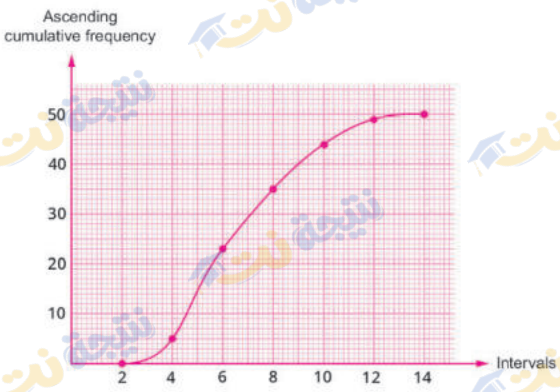
If the given histogram shows the masses of 50 pieces of a product for sale online, form a frequency table with intervals and an ascending cumulative frequency table, and represent it graphically.



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Intervals	2 –	4 –	6 –	8 –	10 –	12 –
Frequency	5	18	12	9	5	1

Upper limits of intervals	Ascending cumulative frequency
less than 2	0
less than 4	5
less than 6	23
less than 8	35
less than 10	44
less than 12	49
less than 14	50

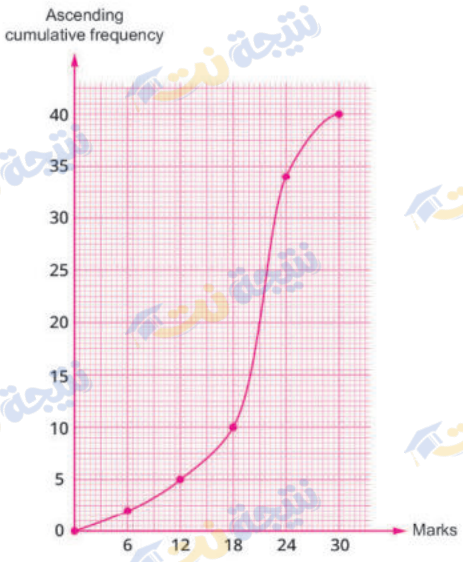


Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

The given figure represents the ascending cumulative curve of the marks of a class of 40 students in an exam.



1. How many students got less than 12 marks?
   (a) 5 (b) 10
   (c) 20 (d) 40
2. How many students got 18 marks or more?
   (a) 20 (b) 30
   (c) 35 (d) 40
3. What is the percentage of students who got less than 6 marks?
   (a) 5 % (b) 10 %
   (c) 15 % (d) 20 %

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Second Applying Concepts

(4) The management of a factory studied the time taken for employees to arrive at the factory in order to determine the most appropriate means of transportation to save employees' time and maintain productivity.

Intervals (time in minutes)	Frequency (number of employees)
0 –	6
20 –	39
40 –	31
60 –	14
80 –	6
100 –	4

Form an ascending cumulative frequency table and represent it graphically.

(5) The following table shows the delivery times in seconds for a fast food restaurant.

Intervals (time in seconds)	Frequency (number of meals)
75 –	11
120 –	24
165 –	10
210 –	3
255 –	2

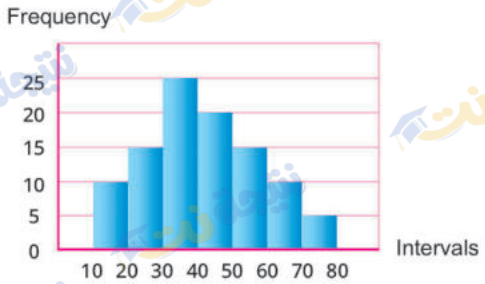
Form an ascending cumulative frequency table and represent it graphically.

(6) The following table shows the number of houses with different areas (measured in square meters).

Intervals (m^2)	100 –	150 –	200 –	250 –	300 –	350 –
Frequency	5	23	27	21	9	5

(1) Form an ascending cumulative frequency table and represent it graphically.
(2) Find the number of houses with an area less than 300 square meters.
(3) Find the number of houses with an area of 150 square meters or more.
(4) Find the percentage of houses with an area of 250 square meters or more.

(7) From the given histogram, form an ascending cumulative frequency table and represent it graphically.



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(8) The given table shows the ascending cumulative frequency distribution of the marks of 100 students in the second preparatory class in mathematics

(1) How many students got less than 25 marks?

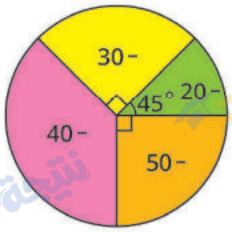
(2) How many students got 30 marks or more?

(3) Form a frequency table with intervals.

Upper limits of intervals	Ascending cumulative frequency
less than 5	0
less than 10	10
less than 15	36
less than 20	61
less than 25	79
less than 30	89
less than 35	95
less than 40	100

Third Analysis and Subjects Integration

(9) **Tourism:** The given pie chart shows the number of tourists visited the Khan Al-Khalili area on one day according to their ages in years (20 – , 30 – , ...).
If the total number of tourists is 600, form an ascending cumulative frequency table and represent it graphically.



(10) **Shopping:** A survey was conducted on 40 people who purchased tablet devices to find out the screen size of their devices in inches. The following table summarises the results of this survey.



Size (inches)	Frequency
7 –	5
8 –	6
9 –	9
10 –	16
11 –	3
12 –	1

Form an ascending cumulative frequency table and represent it graphically, then answer the following questions:

(1) How many tablets have the following sizes:
From 7 to less than 8 inches?
Less than 11 inches?
9 inches or more?

(2) What is the percentage of tablets that have the following sizes:
From 10 to less than 11 inches?
Less than 9 inches?

***Evaluate** your understanding!*

How well do you understand ascending cumulative frequency table and its graphical representation?
Tick the right box



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Lesson

4 - 2

Measurements of Central Tendency of the Frequency Table with Intervals

Learning Outcomes

- Estimate the arithmetic mean of the frequency table with intervals.
- Estimate the median of the frequency table with intervals.
- Estimate the mode of the frequency table with intervals.
- Read and interpret frequency tables and graphs for a given society.

Vocabulary

- Mean $(\bar{x})$
- Mode
- Median (Q_2) (Second quartile)

Previous Knowledge

Measurements of central tendency are values that describe the center of a set of data, such as the arithmetic mean, median, and mode, where:

- Arithmetic mean = $\frac{\text{Sum of values}}{\text{Their number}}$
- Median is the value that lies in the middle of the values after they have been arranged in order.
- Mode is the most common value (more repeated)

Get Ready!



Looking at the heights of the students in the following table:

Height (cm)	Number of students
155 –	5
160 –	10
165 –	25
170 –	6
175 –	4

Can you estimate the mean height of the students?

In this lesson, you will learn how to find the measurements of central tendency (arithmetic mean, median, and mode) for a frequency table with intervals, which will enable you to solve such problems.

Think & Discuss!

The given frequency table shows the number of hours that 50 boys and girls spend on the internet and social media on weekends. Can you calculate the arithmetic mean, median, and mode for the number of hours spent on the internet and social media? Discuss.

Hours spent	10	12	16
Number of followers	15	17	18

Learn!

1 The arithmetic mean of a frequency distribution with intervals

You can estimate the arithmetic mean of a frequency distribution with intervals using the following mathematical formula:

$$\text{The arithmetic mean } (\bar{x}) = \frac{\Sigma (f.x_m)}{\Sigma f}$$

where Σf is the sum of all frequencies

x_m is the center of the interval and is equal to $\frac{\text{Lower limit} + \text{Upper limit}}{2}$

$\Sigma (f.x_m)$ is the sum of the products of center of each interval x_m by its corresponding frequency f

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■ Self-Evaluation (1)

The following frequency table shows the heights of a sample of plants in a nursery.

Intervals (mm)	Frequency
300 –	3
320 –	18
340 –	47
360 –	32
380 –	14
400 –	6

Estimate the arithmetic mean of the plant heights.

■ Note that

The lengths of the intervals are equal. In example (1), the length of each interval = 10

∵ The first interval is (80 – 90)

∴ The center of the first interval $= \frac{80+90}{2} = 85$

∵ The second interval is (90 – 100)

∴ The center of the second interval $= \frac{90+100}{2} = 95$

And so on ...

∵ The last interval is (120 – 130)

∴ The center of the last interval $= \frac{120+130}{2} = 125$

Then form the table as shown.

■ Self-Evaluation (2)

The number of patients visiting a clinic in a hospital over a number of days was recorded as follows:

Number of patients	Frequency
10 –	16
20 –	15
30 –	m
40 –	10
50 –	3
60 –	2

If the mean of patients per day is 30 patients, find the value of m.

Example (1)

When measuring the speeds of some cars passing on a motorway, the traffic department found the following data when observing the speeds of 100 cars on one day.

Speed (km/h)	Frequency
80 –	13
90 –	17
100 –	24
110 –	32
120 –	14



Estimate the arithmetic mean of the speeds of cars passing on that day.

Intervals	Frequency (f)	Centers of intervals (x_m)	$f.x_m$
80 –	13	85	1105
90 –	17	95	1615
100 –	24	105	2520
110 –	32	115	3680
120 –	14	125	1750
Total	100		10670

$$\overline{x} = \frac{\Sigma (f.x_m)}{\Sigma f} = \frac{10670}{100} = 106.7$$

∴ The arithmetic mean of the speeds of cars = 106.7 km/h

Example (2)

The given table shows the number of points scored by one of the teams in the league of basketball games. If the mean of points scored by the team is 90, find the value of k.

Number of points	Frequency
60 –	1
70 –	5
80 –	k
90 –	10
100 –	6

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Intervals	Frequency (f)	Centers of intervals (x_m)	$f.x_m$
60 –	1	65	65
70 –	5	75	375
80 –	k	85	85 k
90 –	10	95	950
100 –	6	105	630
Total	$22 + k$		$2020 + 85\ k$

$$\because \bar{x} = \frac{\Sigma (f.x_m)}{\Sigma f} = \frac{2020 + 85\ k}{22 + k} = 90$$

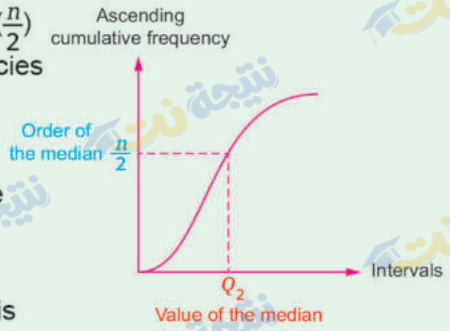
$\therefore 90\ (22 + k) = 2020 + 85\ k$ $\quad \therefore 1980 + 90\ k = 2020 + 85\ k$

$\therefore 90\ k - 85\ k = 2020 - 1980$ $\quad \therefore 5\ k = 40$ $\quad \therefore k = 8$

2 The median of a frequency distribution with intervals

The median can be estimated from the ascending cumulative frequency curve as follows:

- Determine the order of the median $(\frac{n}{2})$ where n is the sum of the frequencies on the vertical axis.
- At the median order, draw a horizontal line that intersects the ascending cumulative curve at a point, so that the value of the median (Q_2) is the projection of this point on the horizontal axis.



Example 3

From the data of the following frequency table:

Intervals	0 –	6 –	12 –	18 –	24 –
Frequency	6	8	12	10	4

Draw the ascending cumulative frequency curve, and from it estimate the median value.

Self-Evaluation 3

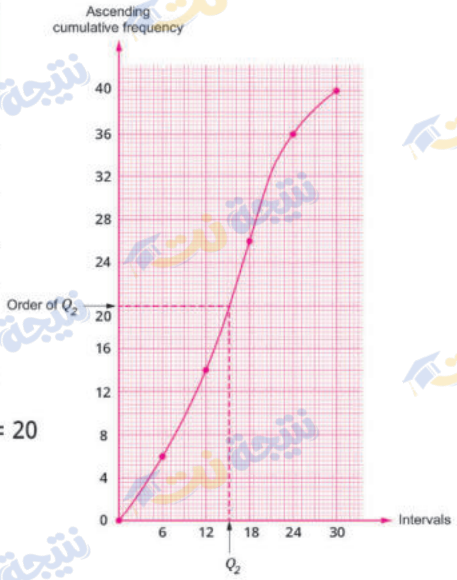
The following table shows the frequency distribution of the marks of 60 students in a mathematics test.

Marks	Frequency
5 –	2
10 –	5
15 –	14
20 –	20
25 –	k
30 –	5
35 –	1
Total	60

Find the value of k, then draw the ascending cumulative frequency curve, and from it estimate the median value.

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Upper limits of intervals	Ascending cumulative frequency
less than 0	0
less than 6	6
less than 12	14
less than 18	26
less than 24	36
less than 30	40



- The order of the median $= \frac{40}{2} = 20$
- The median $(Q_2) = 15$

■ Note that

The value of the median (Q_2) can be found from the ascending cumulative frequency table, as follows:

The order of the median = 20

$$\therefore \frac{Q_2 - 12}{18 - 12} = \frac{20 - 14}{26 - 14}$$

$$\therefore \frac{Q_2 - 12}{6} = \frac{6}{12}$$

$$\therefore Q_2 - 12 = \frac{6 \times 6}{12} = 3$$

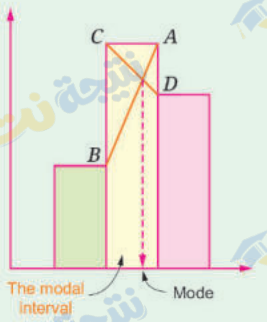
$$\therefore Q_2 = 3 + 12 = 15$$

Upper limits of intervals	Ascending cumulative frequency
less than 0	0
less than 6	6
less than 12	14
less than 18	26
less than 24	36
less than 30	40

Value of the median → Q_2 ; 20 ← Order of the median

3 The mode of a frequency distribution with intervals

You can estimate the mode of a frequency distribution with intervals by drawing the histogram and identifying the interval with the most frequency, it is called the modal interval, and then determine the projection of the intersection point of $\overline{AB}$ and $\overline{CD}$ on the horizontal axis, which is the mode.



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■ Self-Evaluation (4)

Eighty oranges were randomly selected from the quality control department of a juice factory and weighed. The results are shown in the following table:

Weight (grams)	Number of oranges
75 –	4
100 –	8
125 –	16
150 –	32
175 –	14
200 –	6

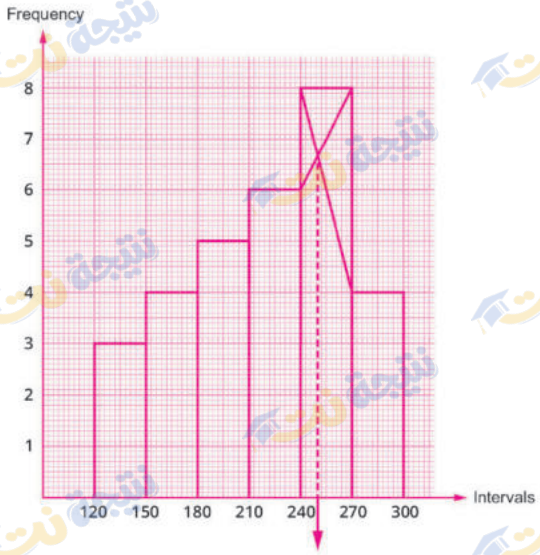
Estimate the mode of the orange weights using the histogram.

Example 4

The following data shows the time spent using the mobile phone (in minutes) for thirty university students on one day.

Time (minutes)	120 –	150 –	180 –	210 –	240 –	270 –
Number of students	3	4	5	6	8	4

Estimate the mode of the time spent using the mobile phone using the histogram.



The mode ≈ 250 minutes

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Lesson Assessment

First Measuring Conceptual Understanding

▸ **Choose the correct answer from the given ones:**

(1) What is the center of the second interval of the intervals:
5 –, 15 –, 25 –, 35 – ?
(a) 5 (b) 10 (c) 15 (d) 20

(2) If $\Sigma f = 50$, $\Sigma (f.x_m) = 800$, what is the arithmetic mean $\overline{x}$?
(a) 16 (b) 160 (c) 750 (d) 850

(3) If the upper limit of an interval is 18 and its center is 15.5, what is the lower limit of this interval?
(a) 12 (b) 12.5 (c) 13 (d) 13.5

(4) The following frequency table shows the marks of 150 students in mathematics:

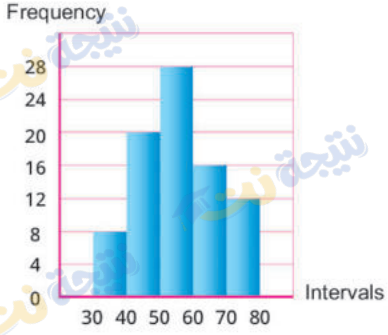
Marks	4 –	12 –	20 –	28 –	36 –	44 –	52 –	Total
Frequency	15	$3k$	$4k$	32	25	22	14	150

(1) What is the value of k?
(a) 4 (b) 5 (c) 6 (d) 7

(2) What is the order of the median?
(a) 37.5 (b) 75 (c) 112.5 (d) 150

(3) What is the value of the median?
(a) 28 (b) 32.5 (c) 34.5 (d) 36

(5) From the given histogram, what is the modal interval?
(a) 30 –
(b) 40 –
(c) 50 –
(d) 70 –



(6) The given ascending cumulative frequency table shows the number of weekly working hours for 40 workers in a factory.
What is the value of the median?
(a) 28.75
(b) 30.25
(c) 24.75
(d) 31.25

Upper limits of intervals	Frequency
less than 20	0
less than 24	2
less than 28	7
less than 32	23
less than 36	35
less than 40	38
less than 44	40

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Second Applying Concepts



(7) The following table shows the mean of daily expenditure of 50 students in the second preparatory grade within a school.

Expenditure (in pounds)	0 –	10 –	20 –	30 –	40 –	50 –
Frequency	7	12	15	9	5	2

Estimate the arithmetic mean of the daily expenditure.

(8) The following table shows the ages of teachers enrolled in a training course on safety and security in educational institutions.

Ages (years)	26 –	30 –	34 –	38 –	42 –	46 –
Frequency	4	10	28	35	15	8

Draw the histogram and estimate the modal age.

(9) The following table shows the masses (in kilograms) of 40 people who joined a weight loss program.

Intervals	70 –	80 –	90 –	100 –	110 –	120 –	130 –
Frequency	2	5	9	12	7	3	2

Estimate the median graphically.

(10) The following table shows the frequency distribution of the marks of 40 students in a test.

Intervals	5 –	10 –	15 –	20 –	25 –	Total
Frequency	k	10	$3k$	8	$k+2$	40

Find the value of k, then estimate the value of the median.

Third Analysis and Subjects Integration



(11) The following table shows the monthly electricity consumption in kilowatt hours for 100 houses.

Intervals (k.w.h)	0 –	100 –	200 –	300 –	400 –	500 –	600 –
Number of houses	4	10	28	35	15	6	2

① Estimate the arithmetic mean of consumption.
② Estimate the median. ③ Estimate the mode.
④ Arrange the values of measurements you obtained in ascending order.

Creative Thinking



(12) The lifespan of two batteries of two different types, A and B, was tested. Then, fifty batteries of each type were randomly selected and tested in the same way. The lifespan of each battery was recorded in hours. The test results are shown in the given table .Estimate the median lifespan of each type and compare them.

Duration (hours)	0 –	5 –	10 –	15 –	20 –	25 –	30 –
Frequency of type A	3	5	8	10	12	7	5
Frequency of type B	1	2	10	20	9	4	4

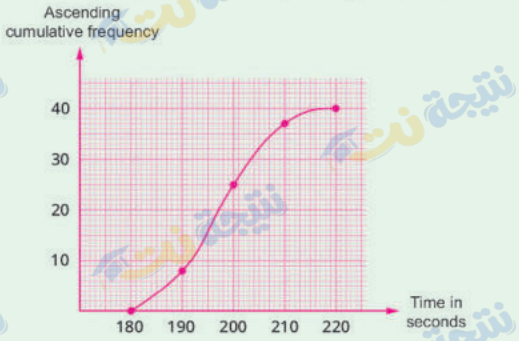
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Unit Four Assessment

Evaluate your understanding!
How well do you understand Unit Four? Tick the right box ☐ ☐ ☐ ☐

▸ Choose the correct answer from the given ones:

(1) 40 students were asked to solve a simple puzzle, and the times they took to solve it were recorded in seconds. The results were presented graphically using the following ascending cumulative frequency curve.



How many students took 3.5 minutes or more?

(a) 30 (b) 35

(c) 37 (d) 3

(2) From the given ascending cumulative frequency table, what is the median value?

Upper limits of intervals	Ascending cumulative frequency
Less than 142	0
Less than 148	4
Less than 154	12
Less than 160	32
Less than 166	44
Less than 172	50

(a) 154.9

(b) 155.9

(c) 157.9

(d) 160.9

(3) If the arithmetic mean $(\bar{X})$ of a frequency table is 24.5 and $\Sigma (f.x_m) = 980$, what is the value of Σf?

(a) 40 (b) 50 (c) 60 (d) 70

(4) What is the center of the third interval of the intervals 6–, 10–, 14–, 18–, ... ?

(a) 12 (b) 14 (c) 16 (d) 18

▸ Complete each of the following with the correct answer using the following frequency table:

Intervals	2–	6–	10–	14–	18–	22–	26–	Total
Frequency	k	6	7	11	8	k^2+1	1	40

(5) The value of k = ...

(6) The arithmetic mean = ...

(7) The order of the median = ...

(8) The median = ...

▸ Answer the following questions:

(9) If the arithmetic mean of the following frequency distribution is 36.6, find the value of each of x and k.

Marks	10–	20–	30–	x–	50–
Frequency	12	20	k	30	15

(10) The following table shows the masses of 50 students in kilograms at a school. Draw the histogram and find the modal mass.

Masses (kg)	30–	35–	40–	45–	50–	55–	60–
Frequency	6	10	12	8	6	5	3

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(11) From the following frequency table:

Intervals	2 –	6 –	10 –	14 –	18 –	22 –	26 –	Total
Frequency	4	6	11	k	14	8	5	60

Form the ascending cumulative frequency table, represent it graphically, and estimate the median value from the graph.

(12) From the given ascending cumulative frequency table, form the frequency table with intervals, then find the modal interval.

Upper limits of intervals	Ascending cumulative frequency
Less than 10	0
Less than 15	3
Less than 20	9
Less than 25	16
Less than 30	24
Less than 35	33
Less than 40	44
Less than 45	50

Unit Four Activity

Measurements of Central Tendency



▸ **Activity Aim:** Form a frequency table with intervals from realistic data and calculate their measurements of central tendency.

▸ **Implementation Steps:**

Collaborate with two of your classmates and try to do the following:

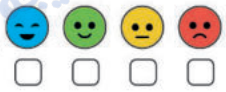
1. Choose one of the life situations you encounter in (the club - a park - the market - the schoolyard), and gather a number of data points about this situation.
2. Organize the data into a frequency table with intervals.
3. Calculate the measurements of central tendency (arithmetic mean - median - mode) from this table, and draw the necessary graphs for each of median and mode.

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Final Assessment

Evaluate your understanding!

How well do you understand the lessons of second preparatory? Tick the right box



First Group

▸ Choose the correct answer from the given ones:

1. What is the multiplicative inverse of the number $-\sqrt{2}$ in its simplest form?

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{2}}{2}$ (d) $-\frac{\sqrt{2}}{2}$

2. What is the result of $\sqrt[3]{2} - 2\sqrt[3]{\frac{1}{4}}$?

(a) 0 (b) 1 (c) $\sqrt[3]{2}$ (d) 2

3. If $x^3 + y^3 = 20$, $x^2 - xy + y^2 = 4$, what is the value of $2x + 2y$?

(a) 5 (b) 10 (c) 15 (d) 20

4. What is the solution set of the equation $x^2 - 4x + 3 = 0$ in R?

(a) {1 , 3} (b) {−1 , −3} (c) {1 , −3} (d) {−1 , 3}

5. If $a + b = 2\sqrt{5}$, $c - d = \sqrt{5}$, what is the value of $ac - ad + bc - bd$?

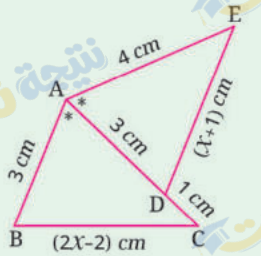
(a) $\sqrt{5}$ (b) $3\sqrt{5}$ (c) 10 (d) 20

6. **In the given figure:**

What is the value of x?

(a) 1 (b) 2

(c) 3 (d) 4



7. What is the number of sides of a polygon whose sum of measures of its interior angles is 1980°?

(a) 10 (b) 11 (c) 12 (d) 13

8. If the lower limit of an interval is 15 and its center is 17.5 , what is the length of the interval?

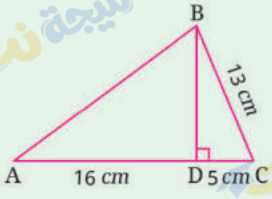
(a) 5 (b) 10 (c) 20 (d) 25

9. **In the given figure:**

What is the length of $\overline{AB}$?

(a) 12 cm (b) 25 cm

(c) 20 cm (d) 16 cm



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Second Group

▸ **Answer the following questions:**

(1) If $X =]-4, 1]$, $Y = [-2, 5[$, find the following in the form of an interval using the number line:

① $X \cap Y$ ② $X \cup Y$ ③ $Y - X$

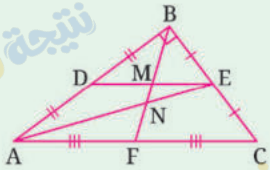
(2) Simplify the expression $\dfrac{(\sqrt{3})^{2n+3}}{(\sqrt{2})^{1-n} \times (\sqrt{6})^{n-1}}$ to its simplest form,

then find the value of the result when $n = 0$

(3) **In the given figure:**

If the length of $\overline{DE} = 6\ cm$,

find with proof the length of $\overline{MN}$.

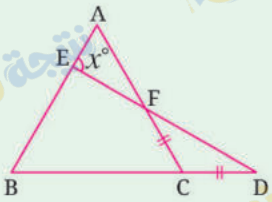


(4) **In the given figure:**

ABC is an equilateral triangle,

$\overline{AC} \cap \overline{ED} = \{F\}$, $CF = CD$

Find with proof the value of X.

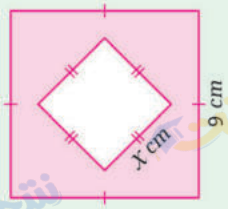


(5) **Factorize each of the following:**

① $4x^2 - 25$ ② $b^3 - 8$ ③ $4x^2 + 14x - 30$

(6) Express the area of the coloured part in the given figure as

a product of two factors.



(7) Find the arithmetic mean of the following frequency distribution:

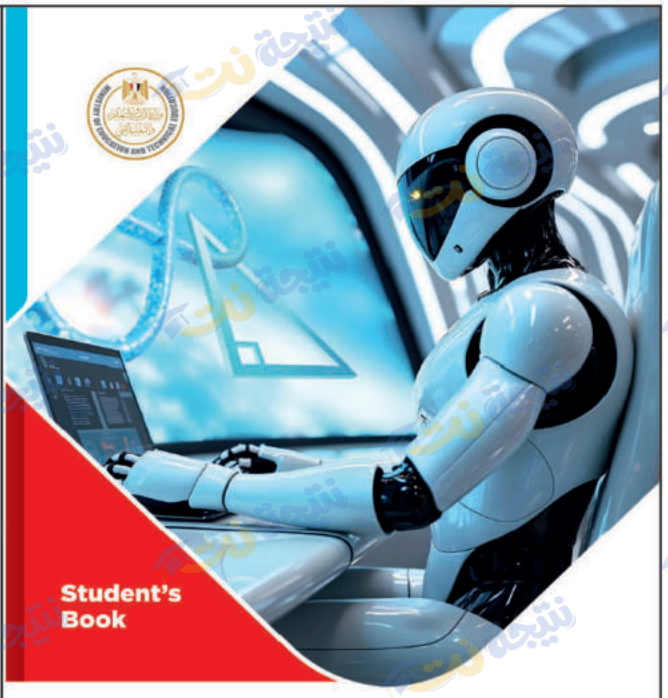
Intervals	5 –	15 –	25 –	35 –	Total
Frequency	6	8	k	2	20

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MATHEMATICS

Second Preparatory - First Term

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